Communication over MIMO X Channel: Signalling and Performance Analysis

Mohammad Ali Maddah-Ali, Abolfazl S. Motahari, and Amir K. Khandani

Coding & Signal Transmission Laboratory

Department of Electrical & Computer Engineering

University of Waterloo

Waterloo, Ontario, Canada, N2L 3G1

Technical Report UW-E&CE 2006-27

December. 20, 2006

Communication over MIMO X Channel: Signalling and Performance Analysis

Mohammad Ali Maddah-Ali, Abolfazl S. Motahari, and Amir K. Khandani

Coding & Signal Transmission Laboratory(www.cst.uwaterloo.ca)

Dept. of Elec. and Comp. Eng., University of Waterloo

Waterloo, ON, Canada, N2L 3G1

e-mail: {mohammad, abolfazl, khandani}@cst.uwaterloo.ca

Abstract

In a multiple antenna system with two transmitters and two receivers, a scenario of data communication, known as the X channel, is studied in which each receiver receives data from both transmitters. In this scenario, it is assumed that each transmitter is unaware of the other transmitter's data (non-cooperative scenario). This system can be considered as a combination of two broadcast channels (from the transmitters point of view) and two multi-access channels (from receivers point of view). Taking advantage of both perspectives, two signaling schemes for such a scenario is developed. In these scheme, some linear filters are employed at the transmitters and the receivers which decompose the system into either two non-interfering multi-antenna broadcast sub-channels or two non-interfering multi-antenna multi-access sub-channels. By using the decomposition schemes,

The material in this paper was presented in part at the IEEE International Symposium on Information Theory (ISIT), Seattle, WA, July 2006, and the technical report number UW-ECE 2006-12, University of Waterloo, Waterloo, July 2006.

This work is financially supported by Communications and Information Technology Ontario (CITO), Nortel Networks, and National Sciences and Engineering Research Council of Canada (NSERC).

the multiplexing gain (MG) of this scenario is derived, which shows improvement as compared with other known non-cooperative schemes. In particular, it is shown that for the specific case that both receivers (transmitters) are equipped with n antennas, the total MG of $\rho = \lfloor \frac{4n}{3} \rfloor$ is achievable, where transmitters (receivers) one and two respectively have $\lceil \frac{\rho}{2} \rceil$ and $\lfloor \frac{\rho}{2} \rfloor$ antennas. The achieved MG is the same as the MG of the system, if the full cooperation is provided either between transmitters or between receivers.

I. Introduction

Wireless technology has been advancing at an exponential rate, due to increasing expectations for multi-media services. This, in turn, necessitates the development of novel techniques of signaling with high spectral efficiency. As a unique solution for such a demand, researchers have proposed employment of multiple antennas at both ends of wireless links [1], [2]. Multiple antenna systems incorporate additional dimension of space to the communication systems, resulting in a multiplicative increase in the overall throughput [2], [3]. The multiplicative increase in the rate is measured by a metric known as *multiplexing gain (MG)*, ρ, defined as the ratio of sum-rate of the system, R, over the the logarithm of the total power P_T in high power regimes, i.e.

$$\rho = \lim_{P_T \to \infty} \frac{R}{\log_2(P_T)}. \tag{1}$$

It is widely known that in a point to point multiple antenna system, with m transmit and n receive antennas, the MG is $\min(m, n)$ [2]. In multi-antenna multi-user systems, when the full cooperation is provided at least at one side of the links (either among transmitters or among receivers), the system still enjoys the multiplicative increase in sum-capacity with the minimum number of the total transmit antennas and the total receive antennas. For example, in a multiple access channel, with two transmitters, equipped with m_1 and m_2 antennas, and one receiver with n antennas, the MG is equal to $\min(m_1 + m_2, n)$ [4]. Similarly, in a multiple-antenna broadcast channel, with one transmitter with m antennas, and two receivers with n_1 and n_2 antennas, the MG is equal to $\min(m, n_1 + n_2)$ [4]. However, for the case that cooperation is not available, the performance of the system can be deteriorated due to the interference of the links over each other. For example in a multiple-antenna interference channel, with two transmitters and two receivers, each equipped with n antennas, the MG of the system is n [4].

Extensive research efforts have been devoted to the multiple antenna interference channels. In [5], the capacity region of the multiple-input single-output (MISO) interference channel with strong interference (see [6]) and the capacity region of the single-input multiple-output (SIMO) interference channel with very strong interference(see [7]) are characterized. In [8], the superposition coding technique is utilized to derive an inner-bound for the capacity of the multiple-input multiple-output (MIMO) interference channels. In [9], several numerical schemes are proposed to compute sub-optimal transmit covariance matrices for the MIMO interference channels. In [4], the MG of the MIMO interference channel with general configuration for the number of transmit and receive antennas is derived. To increase the MG of such systems, the full cooperation among transmitters is proposed in [10], [11], which reduces the system to a single MIMO broadcast channel. To provide such a strong cooperation, an infinite capacity link, connecting the transmitters, is presumed. In [12], the performance of single antenna interference channels is evaluated, where the transmitters or receivers use the same media of transmission for cooperation. It is shown that the resulting MG is still one and this sort of cooperation is not helpful in terms of MG. In [4], a cooperation scheme in the shared communication medium for the MIMO interference channels is proposed and shown that such scheme does not increase the MG.

In this paper, we propose a new scheme of signaling in multiple antenna systems with two transmitters and two receivers. In this scheme, each receiver receives data from both transmitters. It is assumed neither the transmitters nor the receivers cooperate in signaling. In other words, each transmitter is unaware of the data of the other transmitter. Similarly, each receiver is unaware of the signal received by the other receiver. This scenario of signaling has several applications. For example, in a wireless system where two relay nodes are utilized to extend coverage area or in a system where two base stations provide different services to the users. This system can be considered as a a combination of two broadcast channels (from the transmitters point of view) and two multi-access channels (from receivers point of view). By taking advantage of both perspectives, it is shown that by using some linear filters at the transmitters and receivers, the system is decomposed to either two non-interfering multi-antenna broadcast sub-channels or two non-interfering multi-antenna multi-access sub-channels. It is proven that such a scheme outperforms other known non-cooperative schemes in terms of MG. In particular, it is shown that for the specific case that both receivers (transmitters) are equipped with n antennas, the total MG of $\rho = \lfloor \frac{4n}{3} \rfloor$ is achievable, where transmitters (receivers) one and two respectively have $\lceil \frac{\rho}{2} \rceil$ and $\lfloor \frac{\rho}{2} \rfloor$ antennas. Note that even if the full cooperation is provided either between the transmitters or between the receivers the maximum MG is still ρ. In continue, it is argued that such decomposition schemes result in some degradation in the performance of the system. To overcome this problem, a scheme is proposed in which the signaling scheme is jointly designed for both sub-channels (two broadcast or two multi-access sub-channels).

The authors proposed this scenario of signaling and established the possibility of achieving higher MG initially in [13]. Later in [14], we extended the scheme proposed in [13] to more general configurations for the number of transmit and receive antennas, and developed two signaling schemes based on (i) linear operations at the receivers and the dirty paper coding at the transmitters, and (ii) linear operations at the transmitters and the successive decoding at the receivers. In [15], the idea of overlapping the interference terms proposed in [14] was adopted to show that zero-forcing scheme can achieve the multiplexing gain of the X channels for some special configurations for the number of transmit and receive antennas. Furthermore, in [15] an upper-bound on the MG of the X channels, where each transmitter and receiver is equipped with n antennas, is derived.

The rest of the paper is organized as follows. In Section II, the system model is explained. In Section III, the signaling scheme which decomposes the system into two broadcast or two multi-access sub-channels is elaborated. The performance analysis of the scheme, including computing the MG and the power offset(for some special cases) is presented in Section IV. In Section V, the decomposition scheme is modified and joint design for signaling scheme is proposed. Simulation results are represented in Section VI.

Notation: All boldface letters indicate vectors (lower case) or matrices (upper case). $(.)^\dagger$ denotes transpose conjugate operation, and $\mathcal{C}$ represents the set of complex numbers. $\mathcal{OC}^{m\times n}$ represents the set of all $m \times n$ complex matrices with mutually orthogonal and normal columns. $\mathbf{A}\perp\mathbf{B}$ means that every column of the matrix $\mathbf{A}$ is orthogonal to all columns of the matrix $\mathbf{B}$. The sub-space spanned by columns of $\mathbf{A}$ is represented by $\Omega(\mathbf{A})$. The null space of the matrix $\mathbf{A}$ is denoted by $\mathrm{N}(\mathbf{A})$. Identity matrix is represented by $\mathbf{I}$. Adopted from MATLAB notation, $\mathbf{x}(i:j)$ denotes a vector including the entries i to j of the vector $\mathbf{x}$. The i^{th} column of the matrix $\mathbf{A}$ is shown by $\mathbf{a}^{(i)}$.

II. Channel Model

We consider a MIMO system with two transmitters and two receivers. Transmitter t, $t = 1, 2$, is equipped with m_t antennas and receiver r, $r = 1, 2$, is equipped with n_r antennas. This configuration of antennas is shown by (m_1, m_2, n_1, n_2). For simplicity and without loss of generality, it is assumed that $m_1 \geq m_2$ and $n_1 \geq n_2$.

Assuming flat fading environment, the channel between transmitter t and receiver r is represented by the channel matrix $\mathbf{H}_{rt}$, where $\mathbf{H}_{rt} \in \mathcal{C}^{n_r\times m_t}$. The received vector $\mathbf{y}_r \in \mathcal{C}^{n_r\times 1}$ by receiver r, $r = 1, 2$, is given by,

$$\mathbf{y}_1 = \mathbf{H}_{11}\mathbf{s}_1 + \mathbf{H}_{12}\mathbf{s}_2 + \mathbf{w}_1, \tag{2}$$

$$\mathbf{y}_2 = \mathbf{H}_{21}\mathbf{s}_1 + \mathbf{H}_{22}\mathbf{s}_2 + \mathbf{w}_2, \tag{3}$$

where $\mathbf{s}_t \in \mathcal{C}^{m_t\times 1}$ represents the transmitted vector by transmitter t. The vector $\mathbf{w}_r \in \mathcal{C}^{n_r\times 1}$ is a white Gaussian noise with zero mean and identity covariance matrix. The power of $\mathbf{s}_t$ is subject to the constraint $\mathrm{Tr}(E[\mathbf{s}_t\mathbf{s}_t^\dagger]) \leq P_t$, $t = 1, 2$. P_T denotes the total transmit power, i.e. $P_T = P_1 + P_2$.

In the proposed scenario, each transmitter sends two sets of data streams. The transmitter t sends μ_{1t} data streams to receiver 1 and μ_{2t} data streams to receiver 2.

Throughout the paper, we have the following assumptions:

- The perfect information of the entire channel matrices, $\mathbf{H}_{rt}$, $r, t = 1, 2$, is available at both transmitters.
- Each transmitter is unaware of the data sent by the other transmitter, which means that there is no cooperation between transmitters. Similarly, receivers do not cooperate in detection.

III. Decomposition Schemes

In what follows, we propose two signaling schemes. In the first scheme, by using linear transformations at the transmitters and the receivers, the system is decomposed into two non-interfering broadcast sub-channels. Then, any efficient signaling scheme can be employed for each of the resulting broadcast sub-channels.

As a dual of the first scheme, in the second scheme, linear transformations are utilized to break down the system to two non-interfering multi-access sub-channels.

In all of the following schemes, it is assumed that $m_1 < n_1+n_2$ and $n_1 < m_1+m_2$. Otherwise, if $m_1 \geq n_1 + n_2$, the maximum multiplexing gain of $n_1 + n_2$ is achievable by a simple broadcast channel including the first transmitter and the two receivers. Similarly, $n_1 \geq m_1+m_2$, then the maximum multiplexing gain of m_1+m_2 is achievable by a simple multi-access channel including the two transmitters and the first receiver.

A. Decomposition of the System to Two Broadcast Sub-Channels(See Fig. 1 and Fig. 2)

In this scheme, the precoding matrix $\mathbf{Q}_t \in \mathcal{OC}^{m_t \times (\mu_{1t}+\mu_{2t})}$ is employed at transmitter t, $t = 1, 2$. Therefore, the transmitted vectors $\mathbf{s}_t$, $t = 1, 2$, are equal to

$$\mathbf{s}_t = \mathbf{Q}_t \tilde{\mathbf{s}}_t, \tag{4}$$

where $\tilde{\mathbf{s}}_t \in \mathcal{C}^{(\mu_{1t}+\mu_{2t})\times 1}$ contains μ_{1t} data streams for receiver one and μ_{2t} data streams for receiver two. The matrix $\mathbf{Q}_t$ have two functionalities as (i) confining the transmit signal from transmitter t to a $\mu_{1t} + \mu_{2t}$ dimensional sub-space, which provides the possibility of decomposing the system to two broadcast sub-channels by using zero-forcing filters at the receivers. (ii) exploiting the null space of the channel matrices to achieve the highest multiplexing gain.

At each receiver, two parallel linear filters are employed. The received vector $\mathbf{y}_1$ is passed through the filter $\mathbf{\Psi}_{11}^{\dagger}$, which is used to null out the signal coming from the second transmitter. The μ_{11} data streams, sent by transmitter one intended to receiver one, is available for decoding at the output of $\mathbf{\Psi}_{11}^{\dagger}$. Similarly, to decode μ_{12} data streams, sent by transmitter two to receiver one, the received vector $\mathbf{y}_1$ is passed through the filter $\mathbf{\Psi}_{12}^{\dagger}$, which is used to null out the the signal coming from transmitter one. Receiver two have the similar structure with parallel filters $\mathbf{\Psi}_{21}^{\dagger}$ and $\mathbf{\Psi}_{22}^{\dagger}$. Later, it is shown that if the number of data streams μ_{rt}, $r, t = 1, 2$, satisfy a set of inequalities, then it is possible to deign $\mathbf{Q}_t$ and $\mathbf{\Psi}_{rt}$ to meet the above requirements. In this case, it easy to see that the system is decomposed to two non-interfering MIMO broadcast sub-channels.

In what follows, we step by step explain how to select design parameters including the number of data streams μ_{rt}, $r, t = 1, 2$, the precoding and filter matrices. The primary objective is to prevent saturating the data rate of each data stream in high SNR regimes. In other words, the MG of the system is $\mu_{11} + \mu_{12} + \mu_{21} + \mu_{22}$. To choose μ_{rt}, $r, t = 1, 2$, we first select the integer numbers μ'_{rt}, $r, t = 1, 2$, such that the following set of inequalities holds.

$$\mu'_{11}: \qquad \mu'_{11} + \mu'_{12} + \mu'_{22} \le n'_1 \tag{5}$$

$$\mu'_{12}: \qquad \mu'_{12} + \mu'_{11} + \mu'_{21} \le n'_1 \tag{6}$$

$$\mu'_{22}: \qquad \mu'_{22} + \mu'_{21} + \mu'_{11} \le n'_2 \tag{7}$$

$$\mu'_{21}: \qquad \mu'_{21} + \mu'_{22} + \mu'_{12} \le n'_2 \tag{8}$$

$$\mu'_{11} + \mu'_{21} \le m'_1 \tag{9}$$

$$\mu'_{22} + \mu'_{12} \le m'_2 \tag{10}$$

Later, it is explained how to set the integer parameters n'_r, $r = 1, 2$, and m'_t, $t = 1, 2$, depending on the number of transmit and receive antennas. Each of the first four inequalities corresponds to one of the parameters μ'_{rt}, $r, t = 1, 2$. If any of μ'_{rt}, $r, t = 1, 2$, is zero, the corresponding inequality is removed from the set of constraints. To attain the highest MG, we choose μ'_{rt}, $r, t = 1, 2$, such that $\mu'_{11} + \mu'_{12} + \mu'_{21} + \mu'_{22}$ is maximum, where μ'_{rt}, $r, t = 1, 2$, are subject to constraints (5) to (10).

In what follows, we detail how to select m'_t, $t = 1, 2$, and n'_r, $r = 1, 2$, for different configurations of (m_1, m_2, n_1, n_2), and how μ_{rt}, $r, t = 1, 2$ relate to the selected μ'_{rt}, $r, t = 1, 2$. In addition, we determine how to choose the filters $\mathbf{Q}_1$ and $\mathbf{Q}_2$. We define η_{rt} ,$r, t = 1, 2$, as follows.

- η_{11} denotes the dimension of $\Omega(\mathbf{H}_{12}\mathbf{Q}_2)$.
- η_{21} denotes the dimension of $\Omega(\mathbf{H}_{22}\mathbf{Q}_2)$.
- η_{12} denotes the dimension of $\Omega(\mathbf{H}_{11}\mathbf{Q}_1)$.
- η_{22} denotes the dimension of $\Omega(\mathbf{H}_{21}\mathbf{Q}_1)$.

Case One: $n_1 \geq n_2 \geq m_1 \geq m_2$

In this case, n'_r, $r = 1, 2$, and m'_t, $t = 1, 2$, are given by,

$$n'_1 = n_1, \quad n'_2 = n_2, \quad m'_1 = m_1, \quad m'_2 = m_2. \tag{11}$$

Using the above parameters, we choose μ'_{rt}, $r, t = 1, 2$, subject to (5)-(10) constraints. In this case, μ_{rt}, the number data streams sent from transmitter t to receiver r, is obtained by $\mu_{rt} = \mu'_{rt}$, $r, t = 1, 2$. In addition, $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are randomly chosen from $\mathcal{OC}^{m_1 \times (\mu_{11}+\mu_{21})}$ and $\mathcal{OC}^{m_2 \times (\mu_{12}+\mu_{22})}$, respectively.

Regarding the definition of η_{rt}, $r, t = 1, 2$, it is easy to see that,

$$\eta_{11} = \mu_{12} + \mu_{22}, \quad \eta_{12} = \mu_{11} + \mu_{21}, \quad \eta_{21} = \mu_{12} + \mu_{22}, \quad \eta_{22} = \mu_{11} + \mu_{21}. \tag{12}$$

Case Two: $n_1 \geq m_1 > n_2 \geq m_2$

In this case, we have,

$$\begin{array}{llll} n'_1 = n_1 + n_2 - m_1, & n'_2 = n_2, & m'_1 = n_2, & m'_2 = m_2, \\ \mu_{11} = \mu'_{11} + m_1 - n_2, & \mu_{12} = \mu'_{12}, & \mu_{21} = \mu'_{21}, & \mu_{22} = \mu'_{22}, \\ \eta_{11} = \mu_{12} + \mu_{22}, & \eta_{12} = \mu_{11} + \mu_{21}, & \eta_{21} = \mu_{12} + \mu_{22}, & \eta_{22} = \mu'_{11} + \mu_{21}, \end{array} \quad (13)$$

$\mathbf{Q}_1$ and $\mathbf{Q}_2$ are chosen as,

$$\mathbf{\Sigma}_1 \in \mathcal{OC}^{m_1 \times (n_1 - m_2)}, \quad \mathbf{\Sigma}_1 \in \mathrm{N}(\mathbf{H}_{21}) \quad (14)$$

$$\mathbf{\Sigma}_2 = \mathcal{OC}^{m_1 \times (\mu'_{11} + \mu_{21})}, \quad \mathbf{\Sigma}_2 \perp \mathbf{\Sigma}_1 \quad (15)$$

$$\mathbf{Q}_1 \in \mathcal{OC}^{m_1 \times (\mu_{11} + \mu_{21})}, \quad \mathbf{Q}_1 = [\mathbf{\Sigma}_1, \mathbf{\Sigma}_2] \quad (16)$$

$$\mathbf{Q}_2 \text{ randomly chosen from } \mathcal{OC}^{m_2 \times (\mu_{12} + \mu_{22})}. \quad (17)$$

In fact, in this case, we take advantage of $m_1 - n_2$ dimensions of $\mathrm{N}(\mathbf{H}_{21})$, to exclusively send data from transmitter one to receiver one, without imposing any interference over receiver two. Therefore, the transmitter one and receiver one effectively lose $m_1 - n_2$ of the available space dimensions. Consequently, the resulting system is equivalent to the system with effective number of antennas as $(m'_1, m'_2, n'_1, n'_2) = \Big(m_1 - \{m_1 - n_2\}, m_2, n_1 - \{m_1 - n_2\}, n_2\Big)$. The equivalent system with (m'_1, m'_2, n'_1, n'_2) antennas satisfy the condition of the case one, i.e. $m'_1 \geq m'_2 \geq n'_1 \geq n'_2$. Therefore, this case is categorized in the same category of the case one.

Clearly, in this case, η_{22}, the dimension of $\Omega(\mathbf{H}_{21}\mathbf{Q}_1)$, is equal to $\mu_{11} + \mu_{21} - (m_1 - n_2)$ which is $\mu'_{11} + \mu_{21}$.

Case Three: $n_1 \geq m_1 > m_2 \geq n_2$ and $n_1 + n_2 \geq m_1 + m_2$

In this case, we have,

$$\begin{array}{llll} n'_1 = n_1 + 2n_2 - m_1 - m_2, & n'_2 = n_2, & m'_1 = n_2, & m'_2 = n_2, \\ \mu_{11} = \mu'_{11} + m_1 - n_2, & \mu_{12} = \mu'_{12} + m_2 - n_2, & \mu_{21} = \mu'_{21}, & \mu_{22} = \mu'_{22}, \\ \eta_{11} = \mu_{12} + \mu_{22}, & \eta_{12} = \mu_{11} + \mu_{21}, & \eta_{21} = \mu'_{12} + \mu_{22}, & \eta_{22} = \mu'_{11} + \mu_{21}. \end{array} \quad (18)$$

In addition,

$$\mathbf{\Sigma}_1 \in \mathcal{OC}^{m_1\times(m_1-n_2)}, \quad \mathbf{\Sigma}_1 \in \mathrm{N}(\mathbf{H}_{21}) \tag{19}$$

$$\mathbf{\Sigma}_2 = \mathcal{OC}^{m_1\times(\mu'_{11}+\mu_{21})}, \quad \mathbf{\Sigma}_2 \perp \mathbf{\Sigma}_1 \tag{20}$$

$$\mathbf{Q}_1 \in \mathcal{OC}^{m_1\times(\mu_{11}+\mu_{21})}, \quad \mathbf{Q}_1 = [\mathbf{\Sigma}_1, \mathbf{\Sigma}_2] \tag{21}$$

$$\mathbf{\Sigma}_3 \in \mathcal{OC}^{m_2\times(m_2-n_2)}, \quad \mathbf{\Sigma}_3 \in \mathrm{N}(\mathbf{H}_{22}) \tag{22}$$

$$\mathbf{\Sigma}_4 = \mathcal{OC}^{m_2\times(\mu'_{12}+\mu_{22})}, \quad \mathbf{\Sigma}_4 \perp \mathbf{\Sigma}_3 \tag{23}$$

$$\mathbf{Q}_2 \in \mathcal{OC}^{m_2\times(\mu_{12}+\mu_{22})}, \quad \mathbf{Q}_2 = [\mathbf{\Sigma}_1, \mathbf{\Sigma}_2] \tag{24}$$

In this case, $m_1 - n_2$ and $m_2 - n_2$ dimensions of $\mathrm{N}(\mathbf{H}_{21})$ and $\mathrm{N}(\mathbf{H}_{22})$ are exploited to exclusively transmit $m_1 - n_2$ and $m_2 - n_2$ data streams from transmitter one and to two receiver one, respectively, without imposing any interference on receiver two. Therefore, transmitter one, transmitter two, and receiver one respectively lose $m_1 - n_2$, $m_2 - n_2$, and $m_1 - n_2 + m_2 - n_2$ dimensions. Then, the equivalent system has (m'_1, m'_2, n'_1, n'_2) antennas which satisfies the condition of the case one.

Case Four: $m_1 \geq n_1 > n_2 \geq m_2$ and $n_1 + n_2 \geq m_1 + m_2$

In this case, we have

$$\begin{array}{llll} n'_1 = n_1 + n_2 - m_1, & n'_2 = n_1 + n_2 - m_1, & m'_1 = n_1 + n_2 - m_1, & m'_2 = m_2, \\ \mu_{11} = \mu'_{11} + m_1 - n_2, & \mu_{12} = \mu'_{12}, & \mu_{21} = \mu'_{21} + m_1 - n_1, & \mu_{22} = \mu'_{22}, \\ \eta_{11} = \mu_{12} + \mu_{22}, & \eta_{12} = \mu_{11} + \mu'_{21}, & \eta_{21} = \mu_{12} + \mu_{22}, & \eta_{22} = \mu'_{11} + \mu_{21}, \end{array} \tag{25}$$

In addition,

$$\mathbf{\Sigma}_1 \in \mathcal{OC}^{m_1\times(m_1-n_2+m_1-n_2)}, \quad \mathbf{\Sigma}_1 \in \mathrm{N}(\mathbf{H}_{21}) \cup \mathrm{N}(\mathbf{H}_{11}) \tag{26}$$

$$\mathbf{\Sigma}_2 = \mathcal{OC}^{m_1\times(\mu'_{11}+\mu'_{21})}, \quad \mathbf{\Sigma}_2 \perp \mathbf{\Sigma}_1 \tag{27}$$

$$\mathbf{Q}_1 \in \mathcal{OC}^{m_1\times(\mu_{11}+\mu_{21})}, \quad \mathbf{Q}_1 = [\mathbf{\Sigma}_1, \mathbf{\Sigma}_2] \tag{28}$$

$$\mathbf{Q}_2 \text{ randomly chosen from } \mathcal{OC}^{m_2\times(\mu_{12}+\mu_{22})}. \tag{29}$$

The next steps of the algorithm are the same for all of the aforementioned cases. We define

$$\widetilde{\mathbf{H}}_{rt} = \mathbf{H}_{rt}\mathbf{Q}_t, \;\; r, t = 1, 2. \tag{30}$$

$\mathbf{\Psi}_{rt} \in \mathcal{OC}^{n_t \times (n_t - \eta_{rt})}$, $r, t = 1, 2$, are chosen such that

$$\mathbf{\Psi}_{11} \perp \widetilde{\mathbf{H}}_{12}, \tag{31}$$

$$\mathbf{\Psi}_{12} \perp \widetilde{\mathbf{H}}_{11}, \tag{32}$$

$$\mathbf{\Psi}_{21} \perp \widetilde{\mathbf{H}}_{22}, \tag{33}$$

$$\mathbf{\Psi}_{22} \perp \widetilde{\mathbf{H}}_{21}. \tag{34}$$

According to the definition of η_{rt}, one can always choose such matrices. Clearly, any signal sent by transmitter one does not pass through the filters $\mathbf{\Psi}_{12}^{\dagger}$ and $\mathbf{\Psi}_{22}^{\dagger}$. Similarly, any signal sent by transmitter two does not pass through the filters $\mathbf{\Psi}_{21}^{\dagger}$ and $\mathbf{\Psi}_{11}^{\dagger}$.

We define

$$\overline{\mathbf{H}}_{rt} = \mathbf{\Psi}_{rt}^{\dagger} \widetilde{\mathbf{H}}_{rt}, \quad r, t = 1, 2, \tag{35}$$

$$\mathbf{w}_{rt} = \mathbf{\Psi}_{rt}^{\dagger} \mathbf{w}_{r}, \quad r, t = 1, 2, \tag{36}$$

and

$$\mathbf{y}_{rt} = \mathbf{\Psi}_{rt}^{\dagger} \mathbf{y}_{r}, \quad r, t = 1, 2. \tag{37}$$

Therefore, the system is decomposed to two non-interfering broadcast channels. The MIMO broadcast channel viewed from transmitter 1 is modeled by (see Fig. 2)

$$\begin{cases} \mathbf{y}_{11} = \overline{\mathbf{H}}_{11} \tilde{\mathbf{s}}_1 + \mathbf{w}_{11}, \\ \mathbf{y}_{21} = \overline{\mathbf{H}}_{21} \tilde{\mathbf{s}}_1 + \mathbf{w}_{21}, \end{cases} \tag{38}$$

and the MIMO broadcast channel viewed from transmitter two is modeled by (see Fig. 2)

$$\begin{cases} \mathbf{y}_{12} = \overline{\mathbf{H}}_{12} \tilde{\mathbf{s}}_2 + \mathbf{w}_{12}, \\ \mathbf{y}_{22} = \overline{\mathbf{H}}_{22} \tilde{\mathbf{s}}_2 + \mathbf{w}_{22}. \end{cases} \tag{39}$$

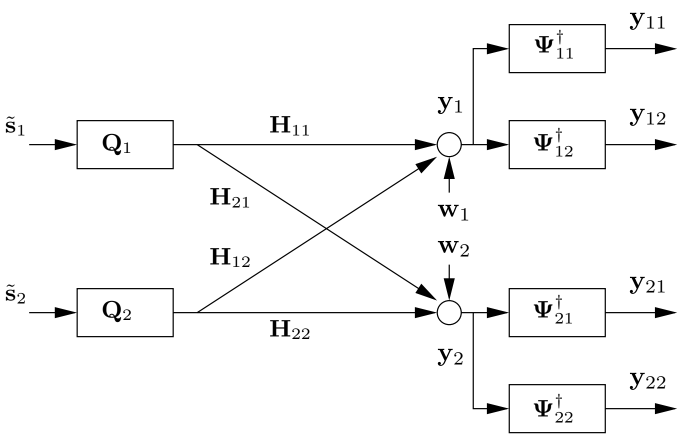

Fig. 1. Decomposition Scheme One

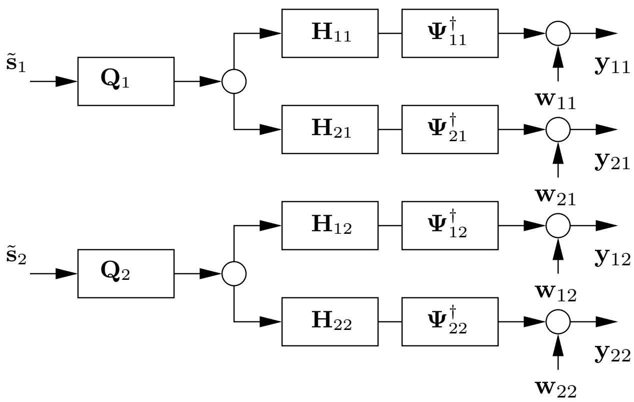

Fig. 2. Scheme One: The Resulting Non-Interfering MIMO Broadcast Sub-Channels

B. Scheme 2 - Decomposition of the System to Two Multi-access Sub-Channels (See Fig. 3 and 4):

This scheme is indeed the dual of the scheme one, detailed in subsection III-A. In this scheme, the parallel precoding matrices $\mathbf{\Psi_{11}}$ and $\mathbf{\Psi_{21}}$ are employed at transmitter one, and the parallel precoding matrices $\mathbf{\Psi_{12}}$ and $\mathbf{\Psi_{22}}$ are employed at transmitter two. Therefore, the transmitted vectors are equal to,

$$\mathbf{s}_1 = \mathbf{\Psi_{11}}\mathbf{s}_{11} + \mathbf{\Psi_{21}}\mathbf{s}_{21}, \tag{40}$$

$$\mathbf{s}_2 = \mathbf{\Psi_{12}}\mathbf{s}_{12} + \mathbf{\Psi_{22}}\mathbf{s}_{22}, \tag{41}$$

where $\mathbf{s}_{rt} \in \mathcal{C}^{\mu_{rt}\times 1}$ contains μ_{rt} data streams from transmitter t intended to receiver r. The precoder matrix $\mathbf{\Psi_{11}}$ nulls out the interference of the μ_{11} data streams, sent from transmitter one to receiver one, at receiver two. Similarly, the precoder matrix $\mathbf{\Psi_{21}}$ nulls out the interference of the μ_{21} data streams sent from transmitter one to receiver two at receiver one. In a similar fashion, at transmitter two, the two parallel precoding matrices $\mathbf{\Psi_{22}}$ and $\mathbf{\Psi_{12}}$ are employed.

At receiver r terminal, the received vector is passed through the filter $\mathbf{Q}_r^\dagger$, where $\mathbf{Q}_r \in \mathcal{OC}^{n_r\times(\mu_{r1}+\mu_{r2})}$,

$$\tilde{\mathbf{y}}_r = \mathbf{Q}_r^\dagger \mathbf{y}_r, \quad r = 1, 2. \tag{42}$$

The functionalities of the matrices $\mathbf{Q}_t$, $t = 1, 2$, include (i) to map the receive signal in a $\mu_{r1} + \mu_{r2}$ sub-space, which allows us to null out the interference terms by using $\mathbf{\Psi_{rt}}$, $r, t = 1, 2$. (ii) to exploit the null space of the channel matrices to provide the highest MG.

By using the aforemention filters at the transmitters and the receivers, the system is decomposed to two non-interfering multi-access sub-channels.

In what follows, we elaborate how to select the number of the data streams μ_{rt}, $r, t = 1, 2$, precoding, and filter matrices.

Similar to the previous sub-section, the objective objective is to prevent saturating the data rate of each data stream in high SNR regimes. In other words, the MG of the system is $\mu_{11} + \mu_{12} + \mu_{21} + \mu_{22}$. To choose μ_{rt}, $r, t = 1, 2$, the integer numbers μ'_{rt}, $r, t = 1, 2$, are chosen subject to the following set of constraints.

$$\mu'_{11} : \qquad \mu'_{11} + \mu'_{21} + \mu'_{22} \leq m'_1 \tag{43}$$

$$\mu'_{21} : \qquad \mu'_{11} + \mu'_{21} + \mu'_{12} \leq m'_1 \tag{44}$$

$$\mu'_{22} : \qquad \mu'_{22} + \mu'_{12} + \mu'_{11} \leq m'_2 \tag{45}$$

$$\mu'_{12} : \qquad \mu'_{22} + \mu'_{12} + \mu'_{21} \leq m'_2 \tag{46}$$

$$\mu'_{11} + \mu'_{12} \leq n'_1 \tag{47}$$

$$\mu'_{22} + \mu'_{21} \leq n'_2 \tag{48}$$

In continue, we explain how to set the integer parameters n'_r, $r = 1,2$, and m'_t, $t = 1,2$, depending on the number of transmit and receive antennas. To attain the highest MG, μ'_{rt}, $r,t = 1,2$ are chosen such that $\mu'_{11} + \mu'_{12} + \mu'_{21} + \mu'_{22}$ is maximum, provided the constraints (43)-(48) are satisfied. Each of the first four inequalities corresponds to one of the parameters μ'_{rt}, $r,t = 1,2$. If any of μ'_{rt}, $r,t = 1,2$, is zero, the corresponding inequality is removed from the set of constraints.

In what follows, we explain how to select m'_t, $t = 1,2$, and n'_r, $r = 1,2$, for each case, and how μ_{rt}, $r,t = 1,2$ relate to the selected μ'_{rt}, $r,t = 1,2$. In addition, we determine how to choose the filters $\mathbf{Q}_1$ and $\mathbf{Q}_2$. Similar to the previous subsection, we define the parameters η_{rt} ,$r,t = 1,2$, as follows.

- η_{11} denotes the dimension of $\Omega(\mathbf{H}_{21}^{\dagger}\mathbf{Q}_2)$.
- η_{21} denotes the dimension of $\Omega(\mathbf{H}_{11}^{\dagger}\mathbf{Q}_1)$.
- η_{12} denotes the dimension of $\Omega(\mathbf{H}_{22}^{\dagger}\mathbf{Q}_2)$.
- η_{22} denotes the dimension of $\Omega(\mathbf{H}_{12}^{\dagger}\mathbf{Q}_1)$.

Case Five: $m_1 \geq m_2 \geq n_1 \geq n_2$

In this case, n'_r, $r = 1,2$, and m'_t, $t = 1,2$, are given by,

$$n'_1 = n_1, \quad n'_2 = n_2, \quad m'_1 = m_1, \quad m'_2 = m_2, \tag{49}$$

Using the above parameters, we choose μ'_{rt}, $r,t = 1,2$, subject to (43)-(48). In this case, μ_{rt}, the number of data streams sent from transmitter t to receiver r, is obtained by $\mu_{rt} = \mu'_{rt}$, $r,t = 1,2$. $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are randomly chosen from $\mathcal{OC}^{n_1\times(\mu_{11}+\mu_{12})}$ and $\mathbf{Q}_2 \in \mathcal{OC}^{n_2\times(\mu_{21}+\mu_{22})}$, respectively.

According to the definition of η_{rt}, $r,t = 1,2$, it is easy to see that,

$$\eta_{11} = \mu_{21} + \mu_{22}, \quad \eta_{12} = \mu_{21} + \mu_{22}, \quad \eta_{21} = \mu_{12} + \mu_{11}, \quad \eta_{22} = \mu_{11} + \mu_{12}. \tag{50}$$

Case Six: $m_1 \geq n_1 > m_2 \geq n_2$

In this case, we have,

$$\begin{aligned} &m_1' = m_1 + m_2 - n_1, && m_2' = m_2, && n_1' = m_2, && n_2' = n_2, \\ &\mu_{11} = \mu_{11}' + n_1 - m_2, && \mu_{12} = \mu_{12}', && \mu_{21} = \mu_{21}', && \mu_{22} = \mu_{22}', \\ &\eta_{11} = \mu_{21} + \mu_{22}, && \eta_{12} = \mu_{21} + \mu_{22}, && \eta_{21} = \mu_{12} + \mu_{11}, && \eta_{22} = \mu_{11}' + \mu_{12} \end{aligned} \tag{51}$$

$\mathbf{Q}_1$ and $\mathbf{Q}_2$ are chosen as,

$$\mathbf{\Sigma}_1 \in \mathcal{OC}^{n_1 \times (n_1 - m_2)}, \quad \mathbf{\Sigma}_1 \in \mathrm{N}(\mathbf{H}_{12}^{\dagger}) \tag{52}$$

$$\mathbf{\Sigma}_2 = \mathcal{OC}^{n_1 \times (\mu_{11}' + \mu_{12})}, \quad \mathbf{\Sigma}_2 \perp \mathbf{\Sigma}_1 \tag{53}$$

$$\mathbf{Q}_1 \in \mathcal{OC}^{n_1 \times (\mu_{11} + \mu_{21})}, \quad \mathbf{Q}_1 = [\mathbf{\Sigma}_1, \mathbf{\Sigma}_2] \tag{54}$$

$$\mathbf{Q}_2 \text{ randomly selected from } \mathcal{OC}^{n_2 \times (\mu_{21} + \mu_{22})} \tag{55}$$

At the receiver one, the signal coming from transmitter two does not have any component in the $n_1 - m_2$ subspace $\mathrm{N}(\mathbf{H}_{12}^{\dagger})$. With the above choice of $\mathbf{Q}_1$, this sub-space is exploited to exclusively send data streams from transmitter one to receiver one. Therefore, each of transmitter one and receiver one lose $n_1 - m_2$ spacial dimensions. Consequently, the system is reduced to a system with $(m_1', m_2', n_1', n_2') = \big(m_1 - \{n_1 - m_2\}, m_2, n_1 - \{n_1 - m_2\}, n_2\big)$, which satisfy the condition of the case five.

Case Seven: $m_1 \geq n_1 > n_2 \geq m_2$ and $m_1 + m_2 \geq n_1 + n_2$

In this case, we have,

$$\begin{aligned} &m_1' = m_1 + 2m_2 - n_1 - n_2 && m_2' = m_2 && n_1' = m_2 && n_2' = m_2 \\ &\mu_{11} = \mu_{11}' + n_1 - m_2 && \mu_{12} = \mu_{12}' && \mu_{21} = \mu_{21}' + n_2 - m_2 && \mu_{22} = \mu_{22}' \\ &\eta_{11} = \mu_{21} + \mu_{22} && \eta_{12} = \mu_{21}' + \mu_{22} && \eta_{21} = \mu_{12} + \mu_{11} && \eta_{22} = \mu_{11}' + \mu_{12} \end{aligned} \tag{56}$$

where,

$$\mathbf{\Sigma}_1 \in \mathcal{OC}^{n_1 \times (n_1 - m_2)} \quad \mathbf{\Sigma}_1 \in \mathrm{N}(\mathbf{H}_{21}^{\dagger}) \tag{57}$$

$$\mathbf{\Sigma}_2 = \mathcal{OC}^{n_1 \times (\mu_{11}' + \mu_{12})} \quad \mathbf{\Sigma}_2 \perp \mathbf{\Sigma}_1 \tag{58}$$

$$\mathbf{Q}_1 \in \mathcal{OC}^{n_1 \times (\mu_{11} + \mu_{12})}, \quad \mathbf{Q}_1 = [\mathbf{\Sigma}_1, \mathbf{\Sigma}_2] \tag{59}$$

$$\mathbf{\Sigma}_3 \in \mathcal{OC}^{n_2 \times (n_2 - m_2)} \quad \mathbf{\Sigma}_3 \in \mathrm{N}(\mathbf{H}_{22}^{\dagger}) \tag{60}$$

$$\mathbf{\Sigma}_4 = \mathcal{OC}^{n_2 \times (\mu_{21}' + \mu_{22})} \quad \mathbf{\Sigma}_4 \perp \mathbf{\Sigma}_3 \tag{61}$$

$$\mathbf{Q}_2 \in \mathcal{OC}^{n_2 \times (\mu_{21} + \mu_{22})}, \quad \mathbf{Q}_2 = [\mathbf{\Sigma}_3, \mathbf{\Sigma}_4] \tag{62}$$

Case Eight: $n_1 \geq m_1 > m_2 \geq n_2$ and $m_1 + m_2 \geq n_1 + n_2$

In this case, we have

$$\begin{array}{llll} m'_1 = m_1 + m_2 - n_1 & m'_2 = m_1 + m_2 - n_1 & n'_1 = m_1 + m_2 - n_1 & n'_2 = n_2 \\ \mu_{11} = \mu'_{11} + n_1 - m_2 & \mu_{12} = \mu'_{12} + n_1 - m_1 & \mu_{21} = \mu'_{21} & \mu_{22} = \mu'_{22} \\ \eta_{11} = \mu_{21} + \mu_{22} & \eta_{12} = \mu_{21} + \mu_{22} & \eta_{21} = \mu'_{12} + \mu_{11} & \eta_{22} = \mu'_{11} + \mu_{12} \end{array} \quad (63)$$

$$\mathbf{\Sigma}_1 \in \mathcal{OC}^{n_1 \times (n_1 - m_2 + n_1 - m_2)}, \quad \mathbf{\Sigma}_1 \in \mathrm{N}(\mathbf{H}_{12}^\dagger) \cup \mathrm{N}(\mathbf{H}_{11}^\dagger) \quad (64)$$

$$\mathbf{\Sigma}_2 = \mathcal{OC}^{n_1 \times (\mu'_{11} + \mu'_{12})}, \quad \mathbf{\Sigma}_2 \perp \mathbf{\Sigma}_1 \quad (65)$$

$$\mathbf{Q}_1 \in \mathcal{OC}^{n_1 \times (\mu_{11} + \mu_{12})}, \quad \mathbf{Q}_1 = [\mathbf{\Sigma}_1, \mathbf{\Sigma}_2] \quad (66)$$

$$\mathbf{Q}_2 \text{ randomly chosen from } \mathcal{OC}^{n_2 \times (\mu_{21} + \mu_{22})}. \quad (67)$$

The next steps of the algorithm are the same for the cases five to eight. We define

$$\widetilde{\mathbf{H}}_{rt} = \mathbf{Q}_r^\dagger \mathbf{H}_{rt}, \ \ r, t = 1, 2. \quad (68)$$

$\mathbf{\Psi}_{rt} \in \mathcal{OC}^{m_r \times (m_r - \eta_{rt})}$, $r, t = 1, 2$, are chosen such that,

$$\mathbf{\Psi}_{11} \perp \widetilde{\mathbf{H}}_{21}^\dagger, \quad (69)$$

$$\mathbf{\Psi}_{21} \perp \widetilde{\mathbf{H}}_{11}^\dagger, \quad (70)$$

$$\mathbf{\Psi}_{12} \perp \widetilde{\mathbf{H}}_{22}^\dagger, \quad (71)$$

$$\mathbf{\Psi}_{22} \perp \widetilde{\mathbf{H}}_{12}^\dagger. \quad (72)$$

According to the definition of η_{rt}, we can always choose such matrices. Clearly, any signal passed through the filters $\mathbf{\Psi}_{11}^\dagger$ and $\mathbf{\Psi}_{12}^\dagger$ has no interference at the output of the filter $\mathbf{Q}_2$. Similarly, any signal passed through the filters $\mathbf{\Psi}_{21}^\dagger$ and $\mathbf{\Psi}_{22}^\dagger$ has no interference at the output of the filter $\mathbf{Q}_2$. We define

$$\overline{\mathbf{H}}_{rt} = \widetilde{\mathbf{H}}_{rt} \mathbf{\Psi}_{rt}, \quad r, t = 1, 2, \quad (73)$$

and

$$\tilde{\mathbf{w}}_r = \mathbf{Q}_r^\dagger \mathbf{w}_r, \quad r, t = 1, 2. \quad (74)$$

This system is decomposed into two non-interfering multiple-access channels: (i) the multi-access channel viewed by receiver one with channels $\overline{\mathbf{H}}_{11}$ and $\overline{\mathbf{H}}_{12}$, modeled by (see Fig. 4),

$$\tilde{\mathbf{y}}_1 = \overline{\mathbf{H}}_{11}\mathbf{s}_{11} + \overline{\mathbf{H}}_{12}\mathbf{s}_{12} + \tilde{\mathbf{w}}_1, \tag{75}$$

and, (ii) the multi-access channel viewed by receiver two with channels $\overline{\mathbf{H}}_{21}$ and $\overline{\mathbf{H}}_{22}$, modeled by (see Fig. 4),

$$\tilde{\mathbf{y}}_2 = \overline{\mathbf{H}}_{21}\mathbf{s}_{21} + \overline{\mathbf{H}}_{22}\mathbf{s}_{12} + \tilde{\mathbf{w}}_2. \tag{76}$$

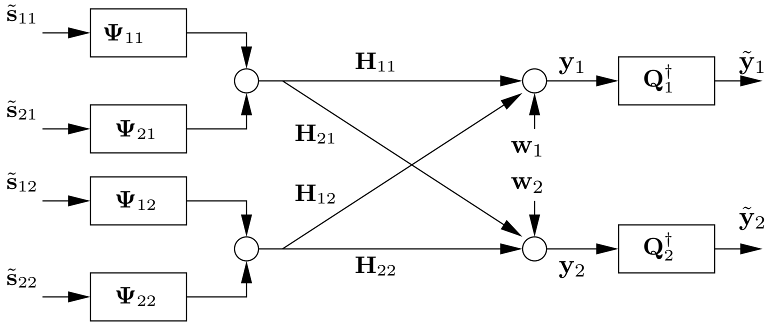

Fig. 3. Decomposition Scheme Two

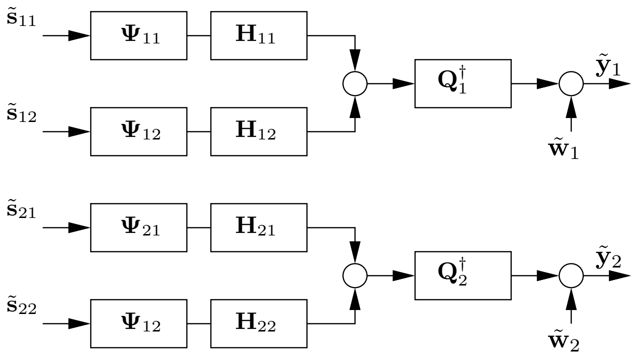

Fig. 4. Scheme Two: The Resulting Non-Interfering MIMO Multi-Access Sub-Channels

IV. Performance Evaluation

The decomposition schemes, presented in Section III, simplify the performance evaluation of the X channels, specially for high SNR regime. In what follows, the MG of the X channel is studied. In addition, for some special cases, a metric known as *power offset*, is evaluated.

A. Multiplexing Gain

Theorem 1 *The MIMO X channel with* (m_1, m_2, n_1, n_2) *antennas achieves the multiplexing gain of* $\mu_{11} + \mu_{21} + \mu_{12} + \mu_{22}$*, if* μ_{rt}*,* $r, t = 1, 2$*, are selected according to the schemes presented in Section III.*

Proof: As explained in Sub-Section III-A, the X channel is decomposed into two non-interfering broadcast sub-channels (38) and (39). The first broadcast sub-channel forms with the channel matrices $\overline{\mathbf{H}}_{11} \in \mathcal{C}^{(\mu_{11}+\mu_{21})\times(n_1-\eta_{11})}$, and $\overline{\mathbf{H}}_{21} \in \mathcal{C}^{(\mu_{11}+\mu_{21})\times(n_2-\eta_{21})}$. The inequalities (5) and (8) guarantee that $n_1 - \eta_{11} \geq \mu_{11}$ and $n_2 - \eta_{21} \geq \mu_{21}$. Therefore, as long as the matrix $[\overline{\mathbf{H}}_{11}^{\dagger}, \overline{\mathbf{H}}_{21}^{\dagger}]^{\dagger}$ is full rank, the broadcast sub-channel achieves the MG of $\mu_{11} + \mu_{21}$ by sending μ_{11} data streams to receiver one and μ_{21} data streams to receiver two. It is easy to see that the $[\overline{\mathbf{H}}_{11}^{\dagger}, \overline{\mathbf{H}}_{21}^{\dagger}]^{\dagger}$ is full rank almost everywhere. Similarly, the second broadcast sub-channel forms with the channel matrices $\overline{\mathbf{H}}_{12} \in \mathcal{C}^{(\mu_{12}+\mu_{22})\times(n_1-\eta_{12})}$, and $\overline{\mathbf{H}}_{22} \in \mathcal{C}^{(\mu_{12}+\mu_{22})\times(n_2-\eta_{22})}$. Constraints 8 and 7 respectively guarantee that $n_1 - \eta_{21} \geq \mu_{21}$ and $n_2 - \eta_{22} \geq \mu_{22}$. Therefore, as long as the matrix $[\overline{\mathbf{H}}_{12}^{\dagger}, \overline{\mathbf{H}}_{22}^{\dagger}]^{\dagger}$ is full rank, the second broadcast sub-channel achieves the MG of $\mu_{12} + \mu_{22}$, by sending μ_{12} data streams to receiver one and μ_{22} data streams to receiver two.

Similar argument for the scheme presented in Sub-Section III-B is valid. ■

To have a better insight about the MG of the X channels, in the following corollaries, the MG of some special cases is computed in a closed-form.

Corollary 1 *For the special case of* $n_1 = n_2 = n$*, in the scheme of Sub-Section III-A, the MG of* $\rho = \lfloor \frac{4n}{3} \rfloor$ *is achievable, where the total number of transmit antennas is* *equal to ρ, which are almost equally divided between transmitters, i.e. $m_1 = \lceil \frac{\rho}{2} \rceil$ and $m_2 = \lfloor \frac{\rho}{2} \rfloor$.*

Proof: This configuration of antennas falls in the case one. By adding the four inequalities (5), (6), (7), and (8), and dividing both sides of the resulting inequality to four, and considering the fact that μ_{rt}, $r, t = 1, 2$, are integer values, we have,

$$\mu_{11} + \mu_{21} + \mu_{12} + \mu_{22} \leq \lfloor \frac{4n}{3} \rfloor, \tag{77}$$

which provides us with an upper bound on the total multiplexing gain. Let $n = 3k + l$, where $0 \leq l \leq 2$. It is easy to prove that by choosing m_t, and μ_{rt}, $r, t = 1, 2$, as listed in Table I, all the constraints (5) to (10) are satisfied and the upper bound is attained. ■

TABLE I

TABLE OF CHOICES FOR COROLLARY 1 ($n = 3k + l,\ 0 \leq l \leq 2\ k \geq 0$)

l	m_1	m_2	μ_{11}	μ_{12}	μ_{21}	μ_{22}	Multiplexing Gain
0	$2k$	$2k$	k	k	k	k	$4k$
1	$2k+1$	$2k$	$k+1$	k	k	k	$4k+1$
2	$2k+1$	$2k+1$	$k+1$	k	k	$k+1$	$4k+2$

Corollary 2 *In the special case of $m_1 = m_2 = m$ in the scheme presented in Sub-Section III-B, the MG of $\rho = \lfloor \frac{4m}{3} \rfloor$ is achievable, where the total number of receive antennas is equal to ρ, which are almost equally divided between transmitters, i.e. $n_1 = \lceil \frac{\rho}{2} \rceil$ and $n_2 = \lfloor \frac{\rho}{2} \rfloor$.*

Proof: The proof is similar to that of Corollary 1 with the choices listed in Table II. ■

Regarding Theorem 1, the MG of X channels outperforms the MG of the interference channel with the same number of antennas. For example, the multiplexing gains of a X channels with $(3, 3, 3, 3)$, $(4, 3, 4, 3)$, $(9, 5, 8, 7)$ antennas are 4, 5, and 11 respectively, while the MG of the interference channels with the same number of antennas are

TABLE II

Table of Choices for Corollary 2 ($m = 3k + l,\ 0 \leq l \leq 2\ k \geq 0$)

l	n_1	n_2	μ_{11}	μ_{12}	μ_{21}	μ_{22}	Multiplexing Gain
0	$2k$	$2k$	k	k	k	k	$4k$
1	$2k+1$	$2k$	$k+1$	k	k	k	$4k+1$
2	$2k+1$	$2k+1$	$k+1$	k	k	$k+1$	$4k+2$

respectively 3, 4, and 9. For all the cases listed in Colorations 1 and 2, the MG of the X channel is even the same as the MG of the system where there is full cooperation between transmitters or between receivers. For example, the multiplexing gains of X channels with $(2, 2, 3, 3)$, $(3, 3, 2, 2)$, $(3, 2, 4, 4)$, and $(3, 3, 5, 5)$ antennas are respectively 4, 4, 5, and 6.

The improvement in MG of X channels as compared with that of interference channels can be explained from two perspectives.

1) Managing the Interference Terms to Occupy an Overlapped Space: For simplicity, we consider an X channel with $(2, 2, 3, 3)$ antennas, and assume that the transmitter t sends one data stream d_{rt} to transmitter r, $r = 1, 2$. Therefore, there are four data streams in a shared wireless medium. At receiver one, we are interested to decode d_{11} and d_{12}, while d_{22} and d_{21} are treated as interference. The signaling scheme is designed such that at the receiver one terminal, d_{21} and d_{22} are received in directions for which the distractive components be along each other. Therefore, at receiver one with three antennas, one direction is occupied with interference, while we have two interference free dimensions to receive d_{11} and d_{12}. The design scheme provides similar condition at the receiver two terminal, while d_{22} and d_{21} are desired data streams and d_{22} and d_{21} are interference terms. Such an overlap of interference terms saves the available spacial dimensions to exploit higher MG.

2) Exploiting the Possibility of Cooperation: It is well-known that the MG for a point-to-pint MIMO channel, a MIMO broadcast channel, and a MIMO multi-access channel is the same, as long as the total number of transmit antennas and the total number of receive antennas are the same in all three systems. The immediate conclusion is that to attain the maximum MG, cooperation in one side of communication link is enough.

Now, consider an interference channel with $m_1 = m_2 = 2$ and $n_1 = n_2 = 3$, and assume that two data streams d_1 and d_2 sent from transmitter one to receiver one and two data streams d_3 and d_4 are sent from transmitter two to receiver two. In this scenario, the cooperation between d_1 and d_2 is provided at *both* transmitter one and receiver one. Similarly, the cooperation between d_3 and d_4 is provided at *both* transmitter two and receiver two. While the system do not gain MG because of cooperation at both sides, it loses MG since there is no possibility of cooperation between (d_1, d_2) and (d_3, d_4). On the the other hand, in X channel, the cooperation between d_{11} and d_{21} is provided at transmitter one, and the cooperation between d_{12} and d_{22} is provided at transmitter two. Similarly, the cooperation between d_{11} and d_{12} is provided at receiver one, and the cooperation between d_{21} and d_{22} is provided at receiver two. In fact, the maximum possibility of cooperation among the data streams are exploited, which results in more MG, if the space dimension allows.

B. Power Offset

In Corollaries 1 and 2, some spacial cases are listed for which the MG of the X channel is the same as the MG of a point-to-point MIMO system resulting from full cooperation between transmitters and between receivers. However, it does not mean that the system does not gain any improvement by cooperation. The gain of the cooperation is reflected in a metric known as the *power offset*. The power offset is the negative of the the zero-order term in the expansion of the sum-rate, normalized with multiplexing gain, with respect to the total power, i.e.

$$R = \rho(\log_2(P_T) - L_\infty) + o(1), \tag{78}$$

where P_T denotes the total power, and L_∞ denotes the power offset in 3dB unit. In this definition, it is assumed that the noise is normalized as in system model 2. The power offset was first introduced in [16] to evaluate the performance of the different CDMA schemes. Later, the power offset for MIMO channels in [17] and some special cases of MIMO broadcast channels in [18] were computed. In what follows, the result of [18] is adopted to compute the power offset of some special cases of MIMO X channels.

Theorem 2 *In an X channel with* $(m_1, m_2, n_1, n_2) = (2k, 2k, 3k, 3k)$ *antennas, where the entries of channel matrices have Rayleigh distribution, if the decomposition scheme is employed, the power offset is equal to,*

$$L_\infty(m_1, m_2, n_1, n_2) = L_\infty(2k, 2k) - \frac{1}{3}\Big(\log_2(\alpha) + \log_2(1-\alpha)\Big), \tag{79}$$

in $3dB$ *units, where* $P_1 = \alpha P_T$, $P_2 = (1-\alpha)P_T$, $0 \leq \alpha \leq 1$,

$$L_\infty(m, n) = \log_2 m + \frac{1}{\ln(2)}\left(\gamma + 1 - \sum_{i=1}^{\tilde{m}-\tilde{n}} \frac{1}{i} - \frac{\tilde{m}}{\tilde{n}} \sum_{i=\tilde{m}-\tilde{n}+1}^{\tilde{m}} \frac{1}{i}\right), \tag{80}$$

$\gamma = 0.5772$, $\tilde{m} = \max\{m, n\}$, *and* $\tilde{n} = \min\{m, n\}$. *Furthermore, the power offset of the X channels with* $(2k, 2k, 3k, 3k)$ *antennas with respect to a MIMO Rayleigh Channel with* $4k$ *transmit antennas and* $6k$ *receive antennas is equal to,*

$$\frac{3}{2\ln(2)} \sum_{i=2k+1}^{6k} \frac{1}{i} - 1 - \frac{1}{2}\Big(\log_2(\alpha) + \log_2(1-\alpha)\Big) \tag{81}$$

in 3dB unit.

Proof: In this case, the matrix $\mathbf{Q}_1$, is randomly chosen from $\mathcal{OC}^{2k\times 2k}$, independent of $\mathbf{H}_{11}$, and $\mathbf{H}_{21}$. In addition, the filters $\mathbf{\Psi}_{11} \in \mathcal{OC}^{2k\times 2k}$ and $\mathbf{\Psi}_{21} \in \mathcal{OC}^{2k\times 2k}$ are independent of $\mathbf{H}_{11}$, and $\mathbf{H}_{21}$, respectively. Therefore, the matrices $\overline{\overline{\mathbf{H}}}_{11}$, and $\overline{\overline{\mathbf{H}}}_{21}$, defined in (35), have Rayleigh distribution. Similar argument is valid for $\overline{\overline{\mathbf{H}}}_{12}$, and $\overline{\overline{\mathbf{H}}}_{22}$. Therefore, the system is decomposed to two broadcast sub-channels, each with the Rayleigh distribution. Therefore, the sum-rate of the MIMO broadcast sub-channel, viewed from transmitter t, is approximated by [18]

$$2k[\log_2(P_t) - L_\infty(2k, 2k)] + o(1). \tag{82}$$

By summation of the approximated formulas for the two MIMO broadcast sub-channels, (79) is obtained.

In [17], it is proven that power offset for a MIMO Rayleigh channel with m transmit and n receive antennas is obtain by (80). By substituting $m = 4k$ and $n = 6k$ (80), and subtracting (80) from (79), (81) is derived. ■

V. Joint Design

The decomposition schemes, proposed in Section III, simplify the signaling and performance evaluation for X channels. However, such schemes deteriorate the performance of the system because (i) $\mathbf{\Psi}_{rt}$, $r, t = 1, 2$ are chosen such that the interference terms are forced to be zero, rather than exploiting the statistical properties of the interference, (ii) the matrices $\mathbf{Q}_t$, $t = 1, 2$ are randomly chosen, rather than choosing them according to the channel matrices. For example, consider an X channel with $(2, 2, 3, 3)$ antennas. In Section III, the filters $\mathbf{\Psi}_{rt}$, $r, t = 1, 2$, are chosen such that the interference of each broadcast sub-channels over the other one is forced to be zero. In low SNR regimes, the performance of the system is improved by choosing whitening filter for $\mathbf{\Psi}_{rt}$, $r, t = 1, 2$, instead of zero-forcing filters. In high SNR, the whitening filters converge to zero-forcing filters, and the resulting improvement diminishes. Note that in the X channel with $(2, 2, 3, 3)$, the matrices $\mathbf{Q}_t$, $t = 1, 2$, are such that the entire two dimensional spaces available at transmitter one and two are used for signaling. Therefore, there is no improvement in modifying $\mathbf{Q}_t$, $t = 1, 2$.

In a system with $(3, 3, 3, 3)$ antennas, the same argument for $\mathbf{\Psi}_{rt}$, $r, t = 1, 2$ is valid. In this case, the matrices $\mathbf{Q}_t$, $t = 1, 2$, are chosen randomly, therefore the signaling space is confined in a randomly-selected two dimensional sub-space of a three dimensional space. One can take advantage of the degrees of freedom available for choosing $\mathbf{Q}_t$ to find the signaling sub-spaces at transmitter one and two for which the channels offer higher gains.

Optimizing the filters $\mathbf{Q}_t$ and $\mathbf{\Psi}_{rt}$, $r, t = 1, 2$, depends on the signaling scheme used for the MIMO broadcast or multi-access sub-channels. On the other hand, it is not possible to design a signaling scheme for each sub-channel separately, but we have to jointly develop the design parameters. In what follows, we elaborate a joint design scheme based on a generalized version of Zero-Forcing Dirty Paper Coding (ZF-DPC) scheme, presented in [19] , for broadcast sub-channels for the cases one to four. In this scheme, the number of data streams μ_{rt}, $r, t = 1, 2$, and also integer parameters μ'_{rt}, $r, t = 1, 2$ are selected as explained in Sub-section III-A. In addition, we use filters $\mathbf{Q}_t$ and $\mathbf{\Psi}^{\dagger}_{rt}$, r, t, in a similar fashion as shown in Fig. 1 , but with a new scheme of design.

According to the generalized ZF-DPC, explained in [19] for MIMO broadcast channels, the vector $\tilde{\mathbf{s}}_t$, $t = 1, 2$, are equal to linear superpositions of some modulation vectors, where the data is embedded in the coefficients. The modulation matrix $\mathbf{V}_t \in \mathcal{OC}^{(\mu_{1t}+\mu_{2t})\times(\mu_{1t}+\mu_{2t})}$ is defined as

$$\mathbf{V}_t = [\mathbf{v}_t^{(1)}, \mathbf{v}_t^{(2)}, \ldots, \mathbf{v}_t^{(\mu_{1t}+\mu_{2t})}], \tag{83}$$

where $\mathbf{v}_t^{(i)}$, $i = 1, \ldots, \mu_{1t} + \mu_{2t}$, denote the normal modulation vectors, employed by transmitter t, to send μ_{1t} data streams to receiver one and μ_{2t} data streams to receiver two. The vectors $\tilde{\mathbf{s}}_1$ and $\tilde{\mathbf{s}}_2$ are equal to

$$\tilde{\mathbf{s}}_1 = \mathbf{V}_1\mathbf{d}_1, \tag{84}$$

$$\tilde{\mathbf{s}}_2 = \mathbf{V}_2\mathbf{d}_2, \tag{85}$$

where the vector $\mathbf{d}_t \in \mathcal{C}^{(\mu_{1t}+\mu_{2t})\times 1}$ represents the $\mu_{1t} + \mu_{2t}$ streams of independent data. The covariance of the vector $\mathbf{d}_t$ is denoted by the diagonal matrix $\mathbf{P}_t$, i.e. $E[\mathbf{d}_t\mathbf{d}_t^{\dagger}] = \mathbf{P}_t$, $t = 1, 2$. At transmitter t, the data streams which modulated over the vectors $\mathbf{v}_t^{(i)}$, $i = 1, \ldots, \mu'_{1t}$ and $i = \mu'_{1t} + \mu'_{21} + 1, \ldots, \mu_{1t} + \mu'_{2t}$, are intended for the receiver one, and the data streams which modulated over the vectors $\mathbf{v}_t^{(i)}$, $i = \mu'_{1t} + 1, \ldots, \mu'_{1t} + \mu'_{2t}$ and $i = \mu_{1t} + \mu'_{2t} + 1, \ldots, \mu_{1t} + \mu_{2t}$, are intended for receiver two. We define $\mathbf{d}_{1t}$ and $\mathbf{d}_{2t}$ as

$$\mathbf{d}_{1t} = \begin{bmatrix} \mathbf{d}_1(1 : \mu'_{11}) \\ \mathbf{d}_1(\mu'_{11} + \mu'_{21} + 1 : \mu_{11} + \mu'_{21}) \end{bmatrix}, \tag{86}$$

and

$$\mathbf{d}_{2t} = \begin{bmatrix} \mathbf{d}_1(\mu'_{11} + 1 : \mu'_{11} + \mu'_{21}) \\ \mathbf{d}_1(\mu_{11} + \mu'_{21} + 1 : \mu_{11} + \mu_{21}) \end{bmatrix}, \tag{87}$$

which represent the data streams, send by transmitter t to receiver one and two, respectively. The modulation and demodulation vectors are designed such that the data stream i has no interference over the data stream j, where $j < i$. Choosing the codeword for the data stream j, the interference of the data stream j over data stream i is non-causally known, and therefore can be effectively canceled out based on the dirty paper coding (DPC) theorem [20]. However, if the data streams i and j are sent to the same receiver, none of them has interference over the other, and DPC is not needed in this case.

At receiver one, to decode $\mathbf{d}_{11}$, the signal coming from transmitter two, i.e. $\widetilde{\mathbf{H}}_{12}\mathbf{V}_{12}\mathbf{d}_2$ is treated as interference, therefore the covariance of the interference plus noise, $\mathbf{R}_{11}$, is equal to,

$$\mathbf{R}_{11} = \widetilde{\mathbf{H}}_{12}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^{\dagger}\widetilde{\mathbf{H}}_{12} + \mathbf{I}, \tag{88}$$

where $\widetilde{\mathbf{H}}_{12}$ is defined in (68). The received vector $\mathbf{y}_1$ is passed through the whitening filter $\mathbf{\Psi}_{11}^{\dagger} = \mathbf{R}_{11}^{-\frac{1}{2}}$. The output of $\mathbf{\Psi}_{11}^{\dagger}$ is passed through the filter $\mathbf{U}_{11}^{\dagger}$ which maximizes the effective SNR. The design of $\mathbf{U}_{rt}^{\dagger}$, $r, t = 1, 2$, is explained later. Similarly, to decode $\mathbf{d}_{21}$ at receiver two terminal, the signal from transmitter two, i.e. $\widetilde{\mathbf{H}}_{22}\mathbf{V}_2\mathbf{d}_2$ is treated as interference. The received vector $\mathbf{y}_1$ is passed through the whitening filter $\mathbf{\Psi}_{21}^{\dagger} = \mathbf{R}_{21}^{-\frac{1}{2}}$, where

$$\mathbf{R}_{21} = \widetilde{\mathbf{H}}_{22}\mathbf{V}_2\mathbf{P}_2\mathbf{V}_2^{\dagger}\widetilde{\mathbf{H}}_{22} + \mathbf{I}. \tag{89}$$

The output of $\mathbf{\Psi}_{12}^{\dagger}$ is passed through the filter $\mathbf{U}_{12}^{\dagger}$ which maximizes the effective SNR.

Let us assume that the modulation matrix $\mathbf{V}_2$, the covariance matrix $\mathbf{P}_2$, and the precoding matrix $\mathbf{Q}_2$ are known, therefore one can compute $\mathbf{\Psi}_{11}^{\dagger}$ and $\mathbf{\Psi}_{12}^{\dagger}$. In the sequel, we explain how to choose $\mathbf{Q}_1$, $\mathbf{V}_1$, $\mathbf{P}_1$, $\mathbf{U}_{11}$, and $\mathbf{U}_{21}$.

The following algorithm is proposed to compute the columns of the matrix $\mathbf{Q}_1 \in \mathcal{OC}^{m_1\times(\mu_{11}+\mu_{21})}$.

1) Choose $\mathbf{q}_1^{(i)}$, $i = 1, \ldots, \mu'_{11}$, as μ'_{11} right singular vectors (RSV) corresponding to the μ'_{11} largest singular values of the matrix $\mathbf{\Psi}_{11}^{\dagger}\mathbf{H}_{11}$.
2) Choose $\mathbf{\Phi}_1 = [\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_{\mu_{11}+\mu_{21}-\mu'_{11}}]$ such that $[\mathbf{\Phi}_1, \mathbf{q}_1^{(1)}, \ldots, \mathbf{q}_1^{(\mu'_{11})}]$ forms a unitary matrix.
3) Choose $\mathbf{q}'^{(i)}_1$, $i = 1, \ldots, \mu'_{21}$, as the μ'_{21} RSVs corresponding to the μ'_{21} largest singular values of the matrix $\mathbf{\Psi}_{21}^{\dagger}\mathbf{H}_{21}\mathbf{\Phi}_1$.
4) Let $\mathbf{q}_1^{(\mu'_{11}+i)} = \mathbf{\Phi}_1\mathbf{q}'^{(i)}_1$, $i = 1, \ldots, \mu'_{21}$.
5) If $\mu_{11} - \mu'_{11} \neq 0$, then choose $\mathbf{q}_1^{(i)}$, $i = \mu'_{11} + \mu'_{21} + 1, \ldots, \mu_{11} + \mu'_{21}$, such that $\Omega([\mathbf{q}_1^{(1)}, \ldots, \mathbf{q}_1^{(\mu_{11}+\mu'_{21})}]) = \Omega([\mathbf{q}_1^{(1)}, \ldots, \mathbf{q}_1^{(\mu'_{11}+\mu'_{21})}, \mathrm{N}(\mathbf{H}_{21})])$.
6) If $\mu_{21} - \mu'_{21} \neq 0$, then choose $\mathbf{q}_1^{(i)}$, $i = \mu_{11} + \mu'_{21} + 1, \ldots, \mu_{11} + \mu_{21}$, such that $\Omega([\mathbf{q}_1^{(1)}, \ldots, \mathbf{q}_1^{(\mu_{11}+\mu_{21})}]) = \Omega([\mathbf{q}_1^{(1)}, \ldots, \mathbf{q}_1^{(\mu_{11}+\mu'_{21})}, \mathrm{N}(\mathbf{H}_{11})])$.

After computing $\mathbf{\Psi}_{11}^{\dagger}$, $\mathbf{\Psi}_{12}^{\dagger}$, and $\mathbf{Q}_1$, the broadcast sub-channel with $\overline{\mathbf{H}}_{11}$ and $\overline{\mathbf{H}}_{21}$, defined in Sub-section III-A as $\overline{\mathbf{H}}_{r1} = \mathbf{\Psi}_{r1}^{\dagger}\mathbf{H}_{r1}\mathbf{Q}_1$, $r = 1, 2$, is developed. Here, we explain how to choose the modulation and demodulation vectors for this broadcast sub-channel.

1) Respectively choose $\mathbf{v}_1^{(i)}$ and $\mathbf{u}_{11}^{(i)}$, $i = 1, \ldots, \mu'_{11}$, as RSV and left singular vector (LSV), corresponding to the i^{th} largest singular value, $\sigma_{11}^{(i)}$, of the matrix $\overline{\mathbf{H}}_{11}$. Therefore, we have [21]

$$\sigma_{11}^{(i)} = \| \overline{\mathbf{H}}_{11}\mathbf{v}_1^{(i)} \|, \ i = 1, \ldots, \mu'_{11}, \tag{90}$$

$$\mathbf{u}_{11}^{(i)} = \frac{\overline{\mathbf{H}}_{11}\mathbf{v}_{11}^{(i)}}{\sigma_{11}^{(i)}}, \ i = 1, \ldots, \mu'_{11}. \tag{91}$$

With the above choice of the matrix $\mathbf{Q}_1$, it is easy to see that $\mathbf{v}_1^{(i)}$ is equal to the column i of the identity matrix $\mathbf{I}_{(\mu_{11}+\mu_{21})\times(\mu_{11}+\mu_{21})}$, for $i = 1, \ldots, \mu'_{11}$.

2) Define $\boldsymbol{\varphi}_1^{(1)}, \ldots, \boldsymbol{\varphi}_1^{(\mu_{11}+\mu_{21}-\mu'_{11})}$ such that $[\mathbf{v}_1^{(1)}, \ldots, \mathbf{v}_1^{(\mu'_{11})}, \boldsymbol{\varphi}_1^{(1)}, \ldots, \boldsymbol{\varphi}_1^{(\mu_{11}+\mu_{21}-\mu'_{11})}]$ forms a unitary matrix. Then, define $\widehat{\overline{\mathbf{H}}}_{21}$ as

$$\widehat{\overline{\mathbf{H}}}_{21} = \overline{\mathbf{H}}_{21}[\boldsymbol{\varphi}_1^{(1)}, \ldots, \boldsymbol{\varphi}_1^{(\mu_{11}+\mu_{21}-\mu'_{11})}]. \tag{92}$$

3) Respectively choose $\overline{\mathbf{v}}_{21}^{(i)}$ and $\mathbf{u}_{21}^{(i)}$ as the RSV and LSV, corresponding to the i^{th} largest singular value $\sigma_{21}^{(i)}$ of the matrix $\widehat{\widehat{\mathbf{H}}}_{21}$. Therefore, we have,

$$\sigma_{21}^{(i)} = \| \widehat{\widehat{\mathbf{H}}}_{21}\overline{\mathbf{v}}_{21}^{(i)} \|, \; i = 1, \ldots, \mu'_{21}, \tag{93}$$

$$\mathbf{u}_{21}^{(i)} = \frac{\widehat{\widehat{\mathbf{H}}}_{21}\overline{\mathbf{v}}_{21}^{(i)}}{\sigma_{21}^{(i)}}, \; i = 1, \ldots, \mu'_{21}. \tag{94}$$

Then, let

$$\mathbf{v}_1^{(\mu'_{11}+i)} = [\boldsymbol{\varphi}_1^{(1)}, \ldots, \boldsymbol{\varphi}_1^{(\mu_{11}+\mu_{21}-\mu'_{11})}]\overline{\mathbf{v}}_{21}^{(i)}, \; i = 1, \ldots, \mu'_{21} \tag{95}$$

It is easy to see that with the aforementioned choice of $\mathbf{Q}_1$, $\mathbf{v}_1^{(\mu'_{11}+i)}$ is equal to the column $\mu'_{11} + i$ of the matrix $\mathbf{I}_{(\mu_{11}+\mu_{21})\times(\mu_{11}+\mu_{21})}$, for $i = 1, \ldots, \mu'_{21}$.

4) Define $\boldsymbol{\varphi}_2^{(1)}, \ldots, \boldsymbol{\varphi}_2^{(\mu_{11}+\mu_{21}-\mu'_{11}-\mu'_{21})}$ such that $[\mathbf{v}_1^{(1)}, \ldots, \mathbf{v}_1^{(\mu'_{11}+\mu'_{21})}, \boldsymbol{\varphi}_2^{(1)}, \ldots, \boldsymbol{\varphi}_2^{(\mu_{11}+\mu_{21}-\mu'_{11}-\mu'_{21})}]$ forms a unitary matrix. Then, define $\widehat{\widehat{\mathbf{H}}}_{11}$ as

$$\widehat{\widehat{\mathbf{H}}}_{11} = \overline{\mathbf{H}}_{11}[\boldsymbol{\varphi}_2^{(1)}, \ldots, \boldsymbol{\varphi}_2^{(\mu_{11}+\mu_{21}-\mu'_{11}-\mu'_{21})}]. \tag{96}$$

5) Respectively choose $\overline{\mathbf{v}}_{11}^{(i)}$ and $\mathbf{u}_{11}^{(i+\mu'_{11})}$ as the RSV and LSV, corresponding to the i^{th} largest singular value of the matrix $\widehat{\widehat{\mathbf{H}}}_{11}$, denoted by $\sigma_{11}^{(i+\mu'_{11})}$, for $i = 1, \ldots, \mu_{11} - \mu'_{11}$. Therefore, we have,

$$\sigma_{11}^{(i+\mu'_{11})} = \| \widehat{\widehat{\mathbf{H}}}_{11}\overline{\mathbf{v}}_{11}^{(i)} \|, \; i = 1, \ldots, \mu_{11} - \mu'_{11}, \tag{97}$$

$$\mathbf{u}_{11}^{(i+\mu'_{11})} = \frac{\widehat{\widehat{\mathbf{H}}}_{11}\overline{\mathbf{v}}_{11}^{(i)}}{\sigma_{11}^{(i+\mu'_{11})}}, \; i = 1, \ldots, \mu_{11} - \mu'_{11}. \tag{98}$$

Then,

$$\mathbf{v}_1^{(\mu'_{11}+\mu'_{21}+i)} = [\boldsymbol{\varphi}_2^{(1)}, \ldots, \boldsymbol{\varphi}_2^{(\mu_{11}+\mu_{21}-\mu'_{11}-\mu'_{21})}]\overline{\mathbf{v}}_{11}^{(i)}, \; i = 1, \ldots, \mu_{11} - \mu'_{11}. \tag{99}$$

6) Define $\boldsymbol{\varphi}_3^{(1)}, \ldots, \boldsymbol{\varphi}_3^{(\mu_{21}-\mu'_{21})}$ such that $[\mathbf{v}_1^{(1)}, \ldots, \mathbf{v}_1^{(\mu_{11}+\mu'_{21})}, \boldsymbol{\varphi}_3^{(1)}, \ldots, \boldsymbol{\varphi}_3^{(\mu_{21}-\mu'_{21})}]$ forms a unitary matrix. Then, define $\widehat{\widehat{\widehat{\mathbf{H}}}}_{21}$ as

$$\widehat{\widehat{\widehat{\mathbf{H}}}}_{21} = \overline{\mathbf{H}}_{21}[\boldsymbol{\varphi}_3^{(1)}, \ldots, \boldsymbol{\varphi}_3^{(\mu_{21}-\mu'_{21})}]. \tag{100}$$

7) Respectively choose $\overline{\overline{\mathbf{v}}}_{21}^{(i)}$ and $\mathbf{u}_{21}^{(i+\mu'_{21})}$ as RSV and LSV, corresponding to the i^{th} largest singular value of the matrix $\widehat{\widehat{\mathbf{H}}}_{11}$, denoted by $\sigma_{21}^{(i+\mu'_{21})}$, for $i = 1, \ldots, \mu_{21} - \mu'_{21}$. Therefore, we have,

$$\sigma_{21}^{(i+\mu'_{21})} = \| \widehat{\widehat{\mathbf{H}}}_{21} \overline{\overline{\mathbf{v}}}_{21}^{(i)} \|, \; i = 1, \ldots, \mu_{21} - \mu'_{21}, \tag{101}$$

$$\mathbf{u}_{21}^{(i+\mu'_{21})} = \frac{\widehat{\widehat{\mathbf{H}}}_{21} \overline{\overline{\mathbf{v}}}_{21}^{(i)}}{\sigma_{21}^{(i+\mu'_{21})}}, \; i = 1, \ldots, \mu_{21} - \mu'_{21}. \tag{102}$$

Then, let

$$\mathbf{v}_1^{(\mu_{11}+\mu'_{21}+i)} = [\boldsymbol{\varphi}_3^{(1)}, \ldots, \boldsymbol{\varphi}_3^{(\mu_{21}-\mu'_{21})}] \overline{\overline{\mathbf{v}}}_{21}^{(i)}, \; i = 1, \ldots, \mu_{11} - \mu'_{11}, \tag{103}$$

As shown in [19], by using this scheme, the broadcast channel, viewed from transmitter one is reduced to a set of parallel channels with gains $\sigma_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$ and $\sigma_{21}^{(j)}$, $j = 1, \ldots, \mu_{21}$. For power allocation, the power P_1 can equally be divided among the data streams or the water-filling algorithm can be used for optimal power allocation [22].

Similar procedure is applied for transmitter two to compute $\mathbf{Q}_2$, $\mathbf{V}_2$, $\mathbf{U}_{12}$, $\mathbf{U}_{22}$, $\mathbf{P}_2$, where

$$\mathbf{R}_{22} = \mathbf{H}_{21} \mathbf{V}_1 \mathbf{P}_1 \mathbf{V}_1^\dagger \mathbf{H}_{21}^\dagger + \mathbf{I}, \tag{104}$$

$$\boldsymbol{\Psi}_{22}^\dagger = \mathbf{R}_{22}^{-\frac{1}{2}}, \tag{105}$$

$$\mathbf{R}_{21} = \mathbf{H}_{11} \mathbf{V}_1 \mathbf{P}_1 \mathbf{V}_1^\dagger \mathbf{H}_{11}^\dagger + \mathbf{I}, \tag{106}$$

$$\boldsymbol{\Psi}_{12}^\dagger = \mathbf{R}_{12}^{-\frac{1}{2}}. \tag{107}$$

Note that to compute $\mathbf{Q}_1$, $\mathbf{V}_1$, and $\mathbf{P}_1$, we need to know $\mathbf{Q}_2$, $\mathbf{V}_2$, and $\mathbf{P}_2$ ($\boldsymbol{\Psi}_{11}$, and $\boldsymbol{\Psi}_{21}$ are functions of $\mathbf{Q}_2$, $\mathbf{V}_2$, and $\mathbf{P}_2$), and vice versa. To derive the modulation vectors, we can randomly initialize the matrices, and iteratively follow the scheme, until the resulting matrices converge. Simulation results show that the algorithm converges very fast.

The dual of the the proposed scheme here can be employed for the cases five to eight.

VI. Simulation Results

In the simulation part, we assume that the entries of the channel matrices have complex normal distribution with zero mean and unit variance.

Fig. 5 shows the sum-rate versus power for a X channel with $(2, 2, 3, 3)$ antennas, where the decomposition scheme presented in Section III is employed. Therefore, the achievable sum-rate is indeed equal the twice of the sum-capacity of a MIMO broadcast channel with 2 transmit antennas, and two user each with one antennas. The sum-capacity of the MIMO broadcast channel is fully characterized in [23]–[25]. To compute the sum-capacity, the effective algorithm presented in [26] is utilized. As a comparison, the capacity of a point-to-point MIMO channel with 4 transmit and 6 receive antennas is depicted. It is easy to see that both curves have the same slope (multiplexing gain). In addition, as expected by (81), the sum-rate of the X channel has 6.2 dB power lost in comparison with that of the MIMO channel.

Figure 6 shows the sum-rate versus power for a X channel with $(2, 2, 3, 3)$ and $(3, 3, 3, 3)$ antennas, where ZD-DPC scheme is used. As it is shown in Fig. 6, for the case of $(2, 2, 3, 3)$ antennas, the jointed design scheme has better performance than the decomposition scheme in low SNR regimes. The improvement is mainly due to utilizing whitening filters instead of zero-forcing filters. It is easy to see that in high SNR, the whitening filters converges to zero-forcing filters. Note that in this case, optimizing the $\mathbf{Q}_t$, $t = 1, 2$, has no improving effect. The reason is that the entire two-dimensional space available at each transmitter is utilized and there is no room for improvement. As depicted in Fig. 6, for the case of $(3, 3, 3, 3)$-antenna X channel, the jointed design scheme has better performance as compared with the decomposition scheme in both high and low SNR regimes. The improvement relies on the fact that in this case at each transmitter, a two-dimensional sub-space of three dimensional space is needed for signaling. By using the scheme presented in Section V, a sub-space for which the channel gains are optimal is chosen.

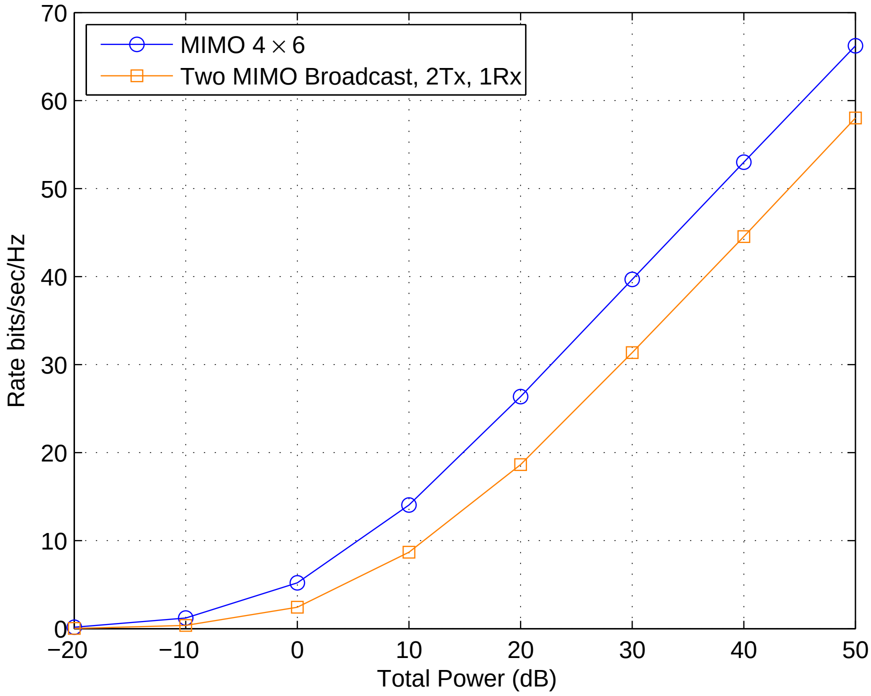

Fig. 5. The Sum capacity of Point-to-Point MIMO Channel with 4 Transmit and 6 Receive Antennas, and the Sum-Rate of the X Channel with (2,2,3,3) Antennas Achieved based on Decommission Scheme

VII. Conclusion

In a multiple antenna system with two transmitters and two receivers, a new non-cooperative scenario of data communication is studied in which each receiver receives data from both transmitters. For such a system a scheme of preceding and filtering is proposed which decomposes the system to two broadcast or two multi-access sub-channels. Using the decomposition scheme, it is shown that this method of signaling outperforms other known schemes of non-cooperative schemes in terms of the multiplexing gain. In particular, it is shown that for a system with $(\lceil \frac{1}{2} \lfloor \frac{4n}{3} \rfloor \rceil, \lfloor \frac{1}{2} \lfloor \frac{4n}{3} \rfloor \rfloor, n, n)$ and $(n, n, \lceil \frac{1}{2} \lfloor \frac{4n}{3} \rfloor \rceil, \lfloor \frac{1}{2} \lfloor \frac{4n}{3} \rfloor \rfloor)$ antennas, the multiplexing gain of $\lfloor \frac{4n}{3} \rfloor$ is achievable, which is the MG of the system where full-cooperation between transmitters or between receivers is provided.

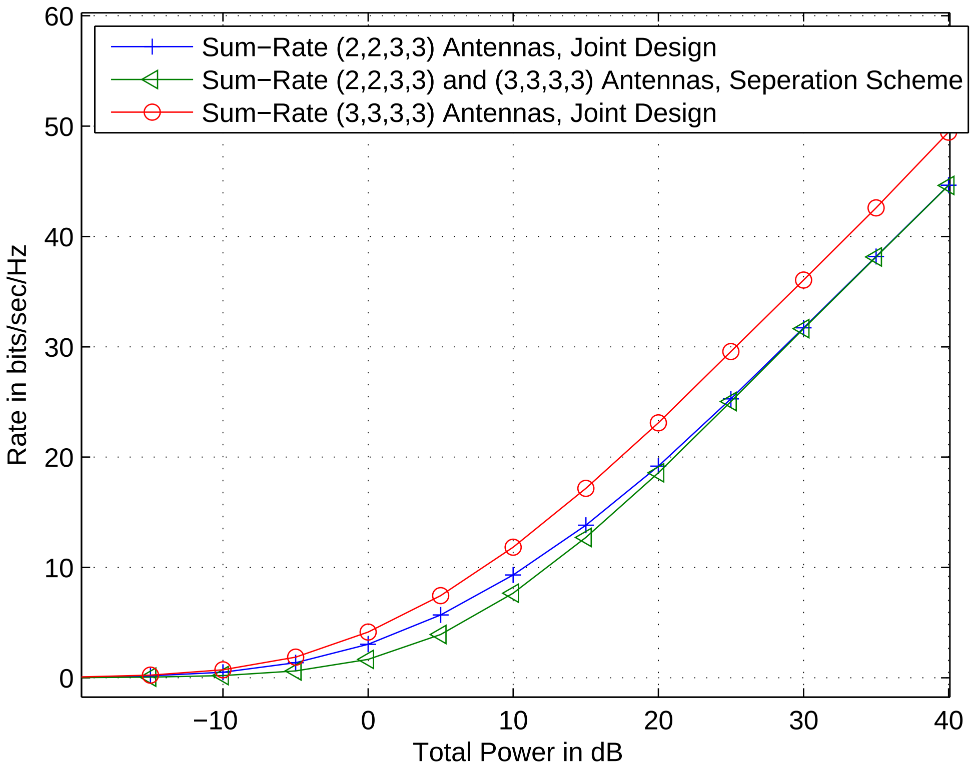

Fig. 6. The Sum-Rate of the X Channels using ZF-DPC Scheme over the Decomposed Channels and the Sum-Rate of the X Channels achieved by Jointly Designed ZF-DPC Scheme

References

[1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311–335, 1998.

[2] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Europ. Trans. Telecommun.*, pp. 585–595, Nov. 1999.

[3] W. Yu and W. Rhee, "Degrees of freedom in multi-user spatial multiplex systems with multiple antennas," *IEEE Transactions on Communications*, 2004, submitted for Publication.

[4] S. A. Jafar, "Degrees of freedom in distributed mimo communications," in *IEEE Communication Theory Workshop*, 2005.

[5] S. Vishwanath and S.A. Jafar, "On the capacity of vector Gaussian interference channels," in *IEEE Information Theory Workshop*, Austin, TX, USA, 2004, pp. 365– 369.

[6] M. Costa and A.E. Gamal, "The capacity region of the discrete memoryless interference channel with strong interference (corresp.)," *IEEE Transactions on Information Theory*, vol. 33, pp. 710– 711, 1987.

[7] A. Carleial, "A case where interference does not reduce capacity (corresp.)," *IEEE Transactions on Information Theory*, vol. 21, pp. 569– 570, 1975.

[8] X. Shang, B. Chen, and M.J. Gans, "On the achievable sum rate for MIMO interference channels," *IEEE Transactions on Information Theory*, vol. 52, pp. 4313 – 4320, 2006.

[9] S. Ye and R.S. Blum, "Optimized signaling for MIMO interference systems with feedback," *IEEE Transactions on Signal Processing*, vol. 51, pp. 2839– 2848, 2003.

[10] S. Shamai (Shitz) and B.M. Zaidel, "Enhancing the cellular downlink capacity via co-processing at the transmitting end," in *IEEE Vehicular Technology Conference*, May 2001, vol. 3, pp. 1745 – 1749.

[11] G. J. Foschini, H. Huang, K. Karakayali, R. A. Valenzuela, and S. Venkatesan, "The value of coherent base station coordination," in *Conference on Information Sciences and Systems (CISS)*, March 2005.

[12] A. Host-Madsen, "Capacity bounds for cooperative diversity," *IEEE Transactions on Information Theory*, vol. 52, pp. 1522– 1544, 2006.

[13] M. A. Maddah-Ali, S. A. Motahari, and Amir K. Khandani, "Signaling over mimo multi-base systems: Combination of multi-access and broadcast schemes," in *IEEE International Symposium on Information Theory*, Seattle, WA, USA, 2006.

[14] M. A. Maddah-Ali, S. A. Motahari, and Amir K. Khandani, "Communication over x channel: Signalling and multiplexing gain," Tech. Rep. UW-ECE-2006-12, University of Waterloo, July 2006.

[15] S.A. Jafar, "Degrees of freedom on the MIMO X channel - the optimality of the MMK scheme," Tech. Rep., Sept. 2006, available at http://arxiv.org/abs/cs.IT/0607099.

[16] S. Shamai and S. Verdu, "The impact of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Transactions on Information Theory*, vol. 47, pp. 1302–1327, 2001.

[17] A. Lozano, A.M. Tulino, and S Verdu, "High-SNR power offset in multiantenna communication," *IEEE Transactions on Information Theory*, vol. 51, pp. 4134– 4151, 2005.

[18] N. Jindal, "High SNR analysis of MIMO broadcast channels," in *International Symposium on Information Theory (ISIT)*, Adelaide, Australia, Sept. 2005, pp. 2310– 2314.

[19] M. A. Maddah-Ali, M. Ansari, and Amir K. Khandani, "An efficient signaling scheme for MIMO broadcast systems: Design and performance evaluation," *IEEE Transactions on Information Theory*, July 2005, Submitted for Publication.

[20] M. Costa, "Writing on dirty paper," *IEEE Trans. Inform. Theory*, vol. 29, pp. 439–441, May 1983.

[21] R.G. Horn and C.A. Johnson, *Matrix Analysis*, Cambridge University Press, 1985.

[22] R. G. Gallager, *Information Theory and Reliable Communication*, John Wiley & Sons, New York, 1968.

[23] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2658–2668, Oct. 2003.

[24] P. Viswanath and D.N.C. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1912 – 1921, Aug. 2003.

[25] W. Yu and J. Cioffi, “Sum capacity of vector Gaussian broadcast channels,” *IEEE Trans. Inform. Theory*, submitted for Publication.

[26] W. Yu, “A dual decomposition approach to the sum power Gaussian vector multiple access channel sum capacity problem,” in *37th Annual Conference on Information Sciences and Systems (CISS)*, March 2003.