# Communication over X Channel: Signalling and Multiplexing Gain


Mohammad Ali Maddah-Ali, Abolfazl S. Motahari, and Amir K. Khandani

Coding & Signal Transmission Laboratory

Department of Electrical & Computer Engineering

University of Waterloo

Waterloo, Ontario, Canada, N2L 3G1



Technical Report UW-E&CE 2006-12

July. 23, 2006

# Communication over X Channel: Signalling and Multiplexing Gain

Mohammad Ali Maddah-Ali, Abolfazl S. Motahari, and Amir K. Khandani

Coding & Signal Transmission Laboratory(www.cst.uwaterloo.ca)

Dept. of Elec. and Comp. Eng., University of Waterloo

Waterloo, ON, Canada, N2L 3G1

e-mail: {mohammad, abolfazl, khandani}@cst.uwaterloo.ca

**Abstract**

In a multiple antenna system with two transmitters and two receivers, a new scenario of data communication is studied in which each receiver receives data from both transmitters. In this scenario, it is assumed that each transmitter is unaware of the data of the other transmitter (non-cooperative scheme). This system can be considered as a a combination of two broadcast channels (from the transmitters point of view) and two multi-access channels (from receivers point of view). Taking advantage of both perspectives, in [1], we developed a signaling scheme for such a system. In addition, to demonstrate the advantage of such a scenario, we showed that if each transmitter/receiver equipped with three antennas, the multiplexing gain of four is achievable, which outperforms other conventional schemes listed in [2]. In this technical report, we elaborate three signaling schemes for general cases and derive achievable multiplexing gain for each scheme. In addition, we show that for the specific case that both receivers (transmitters) are equipped with $n$ antennas, the total multiplexing gain of $\eta = \lfloor \frac{4n}{3} \rfloor$ is achievable, where the total number of antennas at the transmitters (receivers) side is equal to $\eta$. The extra multiplexing gain is justified as a result of (i) distributing the operation of interference cancelation among receivers/transmitters, and (ii) overlap of the interference terms at the receivers side.

# I. Channel Model

Consider a MIMO system with two transmitters and two receivers. Transmitter $t$, $t = 1, 2$, is equipped with $m_t$ antennas and receiver $r$, $r = 1, 2$, is equipped with $n_r$, $r = 1, 2$ antennas. Assuming flat fading environment, the channel between transmitter $t$ and user $r$ is represented by the channel matrix $\mathbf{H}_{rt}$, where $\mathbf{H}_{rt} \in \mathcal{C}^{n_r \times m_t}$. The received vector $\mathbf{y}_r \in \mathcal{C}^{n_r \times 1}$ by user $r$, $r = 1, 2$, is given by,

$$\mathbf{y}_1 = \mathbf{H}_{11}\mathbf{x}_1 + \mathbf{H}_{12}\mathbf{x}_2 + \mathbf{n}_1, \tag{1}$$

$$\mathbf{y}_2 = \mathbf{H}_{21}\mathbf{x}_1 + \mathbf{H}_{22}\mathbf{x}_2 + \mathbf{n}_2, \tag{2}$$

where $\mathbf{x}_t \in \mathcal{C}^{m_t \times 1}$ represents the transmitted vector by transmitter $t$. The vector $\mathbf{n}_r \in \mathcal{C}^{n_r \times 1}$ is a white Gaussian noise with zero mean and identity covariance matrix.

In the proposed scenario, each transmitter sends two sets of data streams: The transmitter $t$ sends $\mu_{1t}$ data streams $\mathbf{d}_{1t} \in \mathcal{C}^{\mu_{1t} \times 1}$ to user 1 and $\mu_{2t}$ data streams $\mathbf{d}_{2t} \in \mathcal{C}^{\mu_{2t} \times 1}$ to user 2.

# II. Signaling Schemes

In this section, we elaborate three signaling schemes for such channels. As mentioned, these schemes are based on considering the system as a combination of multi-access and broadcast schemes. In the schemes one and two, the zero-forcing (ZF) dirty-paper-coding (DPC) is used at the transmitters side, while in scheme transmitters structure consist of some lineae precoders. For the sake of brevity, the transmitter structure for schemes one and two is explained in one subsection.

## *A. Schemes One and Two*

*1) ZF-Dirty Paper Coding at the Transmitters Side:* In both schemes, the transmitted vectors $\mathbf{x}_t$, $t = 1, 2$, are equal to a linear superposition of some modulation vectors, where the data is embedded in the coefficients. The modulation vectors $\mathbf{v}_{rt}^{(i)}$,

$i = 1, \ldots, \mu_{rt}$, are employed to send $\mu_{rt}$ data streams from transmitter $t$ to receiver $r$. The modulation matrix $\mathbf{V}_{rt} \in \mathcal{C}^{m_t \times \mu_{rt}}$ is defined as

$$\mathbf{V}_{rt} = [\mathbf{v}_{rt}^{(1)}, \mathbf{v}_{rt}^{(2)}, \ldots, \mathbf{v}_{rt}^{(\mu_{rt})}]. \tag{3}$$

The transmitted vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ are equal to

$$\mathbf{x}_1 = \mathbf{V}_{11}\mathbf{d}_{11} + \mathbf{V}_{21}\mathbf{d}_{21}, \tag{4}$$

$$\mathbf{x}_2 = \mathbf{V}_{12}\mathbf{d}_{12} + \mathbf{V}_{22}\mathbf{d}_{22}, \tag{5}$$

where $\mathbf{d}_{rt} \in \mathcal{C}^{\mu_{rt} \times 1}$ contains information of $\mu_{rt}$ streams of independent data. For simplicity, it is assume that the power allocated to each stream is $P$. Therefore, $E[\mathbf{d}_{rt}\mathbf{d}_{rt}^{\dagger}] = \mathbf{P}_{rt} = P\mathbf{I}_{\mu_{rt} \times \mu_{rt}}$, where $\mathbf{I}$ denotes identity matrix.

In both schemes one and two, the interference of $\mathbf{d}_{11}$ over $\mathbf{d}_{21}$, and the interference of $\mathbf{d}_{22}$ over $\mathbf{d}_{12}$ are effectively canceled out based on the DPC theorem. Motivated by the proof of the DPC theorem in [3], we embed data in $\hat{\mathbf{d}}_{21}$ and $\hat{\mathbf{d}}_{12}$, where

$$\mathbf{d}_{21} = \hat{\mathbf{d}}_{21} - \mathbf{\Gamma}_2(\mathbf{V}_{21}^{\dagger}\overline{\mathbf{H}}_{21}^{\dagger}\overline{\mathbf{H}}_{21}\mathbf{V}_{21})^{-1}\mathbf{V}_{21}^{\dagger}\overline{\mathbf{H}}_{21}^{\dagger}\overline{\mathbf{H}}_{21}\mathbf{V}_{11}\mathbf{d}_{11}, \tag{6}$$

$$\mathbf{d}_{12} = \hat{\mathbf{d}}_{12} - \mathbf{\Gamma}_1(\mathbf{V}_{12}^{\dagger}\overline{\mathbf{H}}_{12}^{\dagger}\overline{\mathbf{H}}_{12}\mathbf{V}_{12})^{-1}\mathbf{V}_{12}^{\dagger}\overline{\mathbf{H}}_{12}^{\dagger}\overline{\mathbf{H}}_{12}\mathbf{V}_{22}\mathbf{d}_{22}, \tag{7}$$

$$\mathbf{\Gamma}_1 = \mathbf{P}_{12}\left(\mathbf{P}_{12} + (\mathbf{V}_{12}^{\dagger}\overline{\mathbf{H}}_{12}^{\dagger}\overline{\mathbf{H}}_{12}\mathbf{V}_{12})^{-1}\right)^{-1}, \tag{8}$$

$$\mathbf{\Gamma}_2 = \mathbf{P}_{21}\left(\mathbf{P}_{21} + (\mathbf{V}_{21}^{\dagger}\overline{\mathbf{H}}_{21}^{\dagger}\overline{\mathbf{H}}_{21}\mathbf{V}_{21})^{-1}\right)^{-1}, \tag{9}$$

where $\mathbf{\Gamma}_r$, $r = 1, 2$, are dirty paper matrices ($\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ have the same rule of the coefficient $\alpha$ in scalar case of dirty-paper-coding theorem by Costa [3]). $\overline{\mathbf{H}}_{21}$ and $\overline{\mathbf{H}}_{12}$ are defined later in (21) and (30).

The difference between scheme one and two is in the decoding methods applied at the receivers terminals. Unlike the scheme one which is based in linear filters at the receiver, in the scheme two successive decoding (SD) is employed.

*2) Scheme One-Receivers without Successive Decoding (See Fig. 1):* In this scheme, receiver one decodes $\mathbf{d}_{12}$ and $\mathbf{d}_{11}$ in parallel. It decodes $\mathbf{d}_{12}$, while the signals from transmitter one, i.e. $\mathbf{d}_{11}$ and $\mathbf{d}_{21}$, are treated as interference. Due to DPC precoding, the interference of $\mathbf{d}_{22}$ over $\mathbf{d}_{12}$ is effectively canceled out. The filter

$\boldsymbol{\Psi}_{12} = \mathbf{R}_{12}^{-\frac{1}{2}}$ is used to whiten the interference plus noise $\mathbf{H}_{11}(\mathbf{V}_{11}\mathbf{d}_{11} + \mathbf{V}_{21}\mathbf{d}_{21}) + \mathbf{n}_1$ with the variance matrix $\mathbf{R}_{12}$,

$$\mathbf{R}_{12} = \mathbf{H}_{11}[\mathbf{V}_{11}\ \mathbf{V}_{21}] \begin{bmatrix} \mathbf{P}_{11} & 0 \\ 0 & \mathbf{P}_{21} \end{bmatrix} [\mathbf{V}_{11}\ \mathbf{V}_{21}]^{\dagger}\mathbf{H}_{11}^{\dagger} + \mathbf{I}. \tag{10}$$

Equation (10) relies on the fact that $\mathbf{d}_{11}$ and $\mathbf{d}_{21}$ are independent (However, $\mathbf{d}_{11}$ and $\hat{\mathbf{d}}_{21}$ are dependent. See [3]). The output of $\boldsymbol{\Psi}_{12}$ is passed through the filter $\mathbf{U}_{12}$ which maximizes the effective SINR. The design of the precoding and the filter $\mathbf{U}_{12}$ will be explained later.

In parallel, receiver one uses the same method to decode $\mathbf{d}_{11}$, while the signals sent from transmitter two, i.e. $\mathbf{d}_{12}$ and $\mathbf{d}_{22}$, are treated as interference. Due to the applied precoding scheme (explained later in this section), $\mathbf{d}_{21}$ has no interference on $\mathbf{d}_{11}$. The filter $\boldsymbol{\Psi}_{11} = \mathbf{R}_{11}^{-\frac{1}{2}}$ is used to whiten the interference plus noise $\mathbf{H}_{12}(\mathbf{V}_{12}\mathbf{d}_{12} + \mathbf{V}_{22}\mathbf{d}_{22}) + \mathbf{n}_1$ with the variance matrix $\mathbf{R}_{11}$,

$$\mathbf{R}_{11} = \mathbf{H}_{12}[\mathbf{V}_{22}\ \mathbf{V}_{12}] \begin{bmatrix} \mathbf{P}_{22} & 0 \\ 0 & \mathbf{P}_{12} \end{bmatrix} [\mathbf{V}_{22}\ \mathbf{V}_{12}]^{\dagger}\mathbf{H}_{12}^{\dagger} + \mathbf{I}. \tag{11}$$

The output of $\boldsymbol{\Psi}_{11}$ is passed through the filter $\mathbf{U}_{11}$ which maximizes the effective SINR.

At user two terminal, similar scheme is employed to decode $\mathbf{d}_{21}$ and $\mathbf{d}_{22}$, where

$$\mathbf{R}_{21} = \mathbf{H}_{22}[\mathbf{V}_{22}\ \mathbf{V}_{12}] \begin{bmatrix} \mathbf{P}_{22} & 0 \\ 0 & \mathbf{P}_{12} \end{bmatrix} [\mathbf{V}_{22}\ \mathbf{V}_{12}]^{\dagger}\mathbf{H}_{22}^{\dagger} + \mathbf{I}. \tag{12}$$

$$\mathbf{R}_{22} = \mathbf{H}_{21}[\mathbf{V}_{11}\ \mathbf{V}_{21}] \begin{bmatrix} \mathbf{P}_{11} & 0 \\ 0 & \mathbf{P}_{21} \end{bmatrix} [\mathbf{V}_{11}\ \mathbf{V}_{21}]^{\dagger}\mathbf{H}_{21}^{\dagger} + \mathbf{I}. \tag{13}$$

$$\boldsymbol{\Psi}_{21} = \mathbf{R}_{21}^{-\frac{1}{2}}. \tag{14}$$

$$\boldsymbol{\Psi}_{22} = \mathbf{R}_{22}^{-\frac{1}{2}}. \tag{15}$$

Similarly, $\mathbf{U}_{21}^{\dagger}$ and $\mathbf{U}_{22}^{\dagger}$ are used to detect $\mathbf{d}_{21}$ and $\mathbf{d}_{22}$, respectively.

*3) Scheme Two - Receiver with Successive Decoding (See Fig. 2):* In this scheme, at the receivers side, the nonlinear operation of successive decoding is employed. The structure of the receiver is as follows: at user one terminal, first $\hat{\mathbf{d}}_{12}$ is decoded and its effect is subtracted from the received vector $\mathbf{y}_1$. Then, $\mathbf{d}_{11}$ is decoded. Similarly, at user two terminal, first $\hat{\mathbf{d}}_{21}$ is decoded and its effect is subtracted from $\mathbf{y}_2$, then $\mathbf{d}_{22}$ is decoded. To decode $\hat{\mathbf{d}}_{12}$ at user 1 terminal, the signals received from transmitter 1 , i.e. $\mathbf{d}_{11}$ and $\mathbf{d}_{21}$, are treated as interference. The proposed precoding scheme is such that the data stream $\mathbf{d}_{22}$ has no interference on the data stream $\hat{\mathbf{d}}_{12}$. The filter $\mathbf{\Psi}_{12} = \mathbf{R}_{12}^{-\frac{1}{2}}$ is used to whiten the interference plus noise $\mathbf{H}_{11}(\mathbf{V}_{11}\mathbf{d}_{11} + \mathbf{V}_{21}\mathbf{d}_{21}) + \mathbf{n}_1$ with the variance matrix $\mathbf{R}_{12}$,

$$\mathbf{R}_{12} = \mathbf{H}_{11}[\mathbf{V}_{11}\ \mathbf{V}_{21}] \begin{bmatrix} \mathbf{P}_{11} & 0 \\ 0 & \mathbf{P}_{21} \end{bmatrix} [\mathbf{V}_{11}\ \mathbf{V}_{21}]^{\dagger}\mathbf{H}_{11}^{\dagger} + \mathbf{I}. \tag{16}$$

The output of $\mathbf{\Psi}_{12}$ is passed through the filter $\mathbf{U}_{12}$ which maximizes the effective SINR. The design of the precoding and the filter $\mathbf{U}_{12}$ will be explained later. Here, user one decodes $\hat{\mathbf{d}}_{12}$ and then subtracts its effect from the received signal $\mathbf{y}_1$, i.e.

$$\begin{aligned}\widetilde{\mathbf{y}}_1 &= \mathbf{y}_1 - \mathbf{H}_{12}\mathbf{V}_{12}\hat{\mathbf{d}}_{12} \qquad (17)\\ &= \mathbf{Q}_1\mathbf{H}_{12}\mathbf{V}_{22}\mathbf{d}_{22} + \mathbf{H}_{11}\mathbf{V}_{11}\mathbf{d}_{11} + \mathbf{H}_{12}\mathbf{V}_{12}\mathbf{d}_{12} + \mathbf{n}_1.\end{aligned}$$

where regarding (7) we have,

$$\mathbf{Q}_1 = \mathbf{I} - \mathbf{H}_{12}\mathbf{V}_{12}\mathbf{\Gamma}_1(\mathbf{V}_{12}^{\dagger}\overline{\mathbf{H}}_{12}^{\dagger}\overline{\mathbf{H}}_{12}\mathbf{V}_{12})^{-1}\mathbf{V}_{12}^{\dagger}\overline{\mathbf{H}}_{12}^{\dagger}\mathbf{\Psi}_{12}. \tag{18}$$

In the next step, user one decodes $\mathbf{d}_{11}$ from $\widetilde{\mathbf{y}}_1$. First, the filter $\mathbf{\Psi}_{11}$ is used to whiten the interference of $\mathbf{d}_{22}$ over $\mathbf{d}_{11}$. Note that the data stream $\mathbf{d}_{21}$ has no interference over $\mathbf{d}_{11}$ due to the precoding at the transmitter. The interference plus noise is equal to $\mathbf{Q}_1\mathbf{H}_{12}\mathbf{V}_{22}\mathbf{d}_{22}+\mathbf{n}_1$ with the covariance matrix $\mathbf{R}_{11} = \mathbf{Q}_1\mathbf{H}_{12}\mathbf{V}_{22}\mathbf{P}_{22}\mathbf{V}_{22}^{\dagger}\mathbf{H}_{12}^{\dagger}\mathbf{Q}_1^{\dagger}+ \mathbf{I}$. Then, the whitening filter is equal to $\mathbf{\Psi}_{11} = \mathbf{R}_{11}^{-\frac{1}{2}}$. The output of the whitening filter $\mathbf{\Psi}_{11}$ is passed through the filter $\mathbf{U}_{11}^{\dagger}$ which maximizes the SNR of the data stream $\mathbf{d}_{11}$. Similarly, for user 2, there are two whitening filters $\mathbf{\Psi}_{21} = \mathbf{R}_{21}^{-\frac{1}{2}}$ and $\mathbf{\Psi}_{22} = \mathbf{R}_{22}^{-\frac{1}{2}}$

where,

$$\mathbf{R}_{21} = \mathbf{H}_{22}[\mathbf{V}_{12}\ \mathbf{V}_{22}] \begin{bmatrix} \mathbf{P}_{12} & 0 \\ 0 & \mathbf{P}_{22} \end{bmatrix} [\mathbf{V}_{12}\ \mathbf{V}_{22}]^{\dagger}\mathbf{H}_{22}^{\dagger} + \mathbf{I}$$

$$\mathbf{R}_{22} = \mathbf{Q}_2\mathbf{H}_{21}\mathbf{V}_{11}\mathbf{P}_{11}\mathbf{V}_{11}^{\dagger}\mathbf{H}_{21}^{\dagger}\mathbf{Q}_2^{\dagger} + \mathbf{I},$$

$$\mathbf{Q}_2 = \mathbf{I} - \mathbf{H}_{21}\mathbf{V}_{21}\mathbf{\Gamma}_2(\mathbf{V}_{21}^{\dagger}\overline{\mathbf{H}}_{21}^{\dagger}\overline{\mathbf{H}}_{21}\mathbf{V}_{21})^{-1}\mathbf{V}_{21}^{\dagger}\overline{\mathbf{H}}_{21}^{\dagger}\mathbf{\Psi}_{21}.$$

Similarly, at user two terminal, $\mathbf{U}_{21}^{\dagger}$ and $\mathbf{U}_{22}^{\dagger}$ are used to detect $\mathbf{d}_{21}$ and $\mathbf{d}_{22}$, respectively.

In both schemes one and two, we apply the following scheme to design the modulation and demodulation vectors

*4) Designing the Modulation and the Demodulation Vectors in Scheme One and Two:* To derive modulation and demodulation vectors, we consider the second perspective of the system as a set of two broadcast channels. As depicted in Fig. 1 and Fig. 2, the following MIMO broadcast channel is viewed from transmitter one,

$$\widehat{\mathbf{y}}_1 = \overline{\mathbf{H}}_{11}\mathbf{x}_1 + \widehat{\mathbf{n}}_1, \tag{19}$$

$$\check{\mathbf{y}}_2 = \overline{\mathbf{H}}_{21}\mathbf{x}_1 + \check{\mathbf{n}}_2, \tag{20}$$

where $\widehat{\mathbf{n}}_1$ and $\check{\mathbf{n}}_2$ are whitened noise terms and

$$\overline{\mathbf{H}}_{11} = \mathbf{\Psi}_{11}\mathbf{H}_{11}, \tag{21}$$

$$\overline{\mathbf{H}}_{21} = \mathbf{\Psi}_{21}\mathbf{H}_{21}. \tag{22}$$

For signaling, we apply the scheme proposed in [4] for the MIMO broadcast systems with multiple receive antennas. According to [4], the columns of the modulation matrix $\mathbf{V}_{11}$ are equal to the $\mu_{11}$ right singular vectors corresponding to the $\mu_{11}$ largest singular values of the matrix $\overline{\mathbf{H}}_{11}$. Let $\mathbf{v}_{11}^{(i)}$ be the $i^{th}$ column of $\mathbf{V}_{11}$, and define $\sigma_{11}^{(i)} = \| \overline{\mathbf{H}}_{11}\mathbf{v}_{11}^{(i)} \|$, then the $i^{th}$ column of $\mathbf{U}_{11}$ is equal to

$$\mathbf{u}_{11}^{(i)} = \frac{\overline{\mathbf{H}}_{11}\mathbf{v}_{11}^{(i)}}{\sigma_{11}^{(i)}}. \tag{23}$$

In fact, $\sigma_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$, are the $\mu_{11}$ largest singular values of $\overline{\mathbf{H}}_{11}$, and $\mathbf{u}_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$, are the corresponding left singular vectors.

We define $\boldsymbol{\varphi}_1^{(11)}$, …, $\boldsymbol{\varphi}_{m_1-\mu_{11}}^{(11)}$ such that $[\mathbf{V}_{11}, \boldsymbol{\varphi}_1^{(11)}, \ldots, \boldsymbol{\varphi}_{m_1-\mu_{11}}^{(11)}]$ forms a unitary matrix. Then, we define $\overline{\overline{\mathbf{H}}}_{21}$ as

$$\overline{\overline{\mathbf{H}}}_{21} = \overline{\mathbf{H}}_{21}[\boldsymbol{\varphi}_1^{(11)}, \ldots, \boldsymbol{\varphi}_{m_1-\mu_{11}}^{(11)}]. \tag{24}$$

Then, let the columns of the matrix $\overline{\mathbf{V}}_{21}$ be equal to the $\mu_{21}$ right singular vectors corresponding to the $\mu_{21}$ largest singular values of the matrix $\overline{\overline{\mathbf{H}}}_{21}$. Then, let

$$\mathbf{V}_{21} = [\boldsymbol{\varphi}_1^{(11)}, \ldots, \boldsymbol{\varphi}_{m_1-\mu_{11}}^{(11)}]\overline{\mathbf{V}}_{21}, \tag{25}$$

$$\sigma_{21}^{(i)} = \| \overline{\mathbf{H}}_{21}\mathbf{v}_{21}^{(i)} \|, \ i = 1, \ldots, \mu_{21}, \tag{26}$$

$$\mathbf{u}_{21}^{(i)} = \frac{\overline{\mathbf{H}}_{21}\mathbf{v}_{21}^{(i)}}{\sigma_{21}^{(i)}}, \ i = 1, \ldots, \mu_{21}. \tag{27}$$

It is easy to see that $\sigma_{21}^{(i)}$, $i = 1, \ldots, \mu_{21}$, are the $\mu_{21}$ largest singular values of $\overline{\overline{\mathbf{H}}}_{21}$,

As shown in [4], by using this scheme, the data streams $\mathbf{d}_{21}$ has no interference over the data streams $\mathbf{d}_{11}$. As mentioned, knowing the selected codeword for data streams $\mathbf{d}_{11}$, one can effectively cancel out the interference of the data streams $\mathbf{d}_{11}$ over $\mathbf{d}_{21}$ based on the dirty-paper coding theorem. Consequently, the broadcast channel is reduced to a set of parallel channels with gains $\sigma_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$ and $\sigma_{21}^{(j)}$, $j = 1, \ldots, \mu_{21}$. It is worth mentioning that the modulation vectors used by transmitter one, i.e. columns of $[\mathbf{V}_{11}, \mathbf{V}_{21}]$, are mutually orthogonal.

From transmitter 2, we have a MIMO broadcast channel modeled by

$$\check{\mathbf{y}}_1 = \overline{\mathbf{H}}_{12}\mathbf{x}_2 + \check{\mathbf{n}}_1, \tag{28}$$

$$\widehat{\mathbf{y}}_2 = \overline{\mathbf{H}}_{22}\mathbf{x}_2 + \widehat{\mathbf{n}}_2, \tag{29}$$

where $\check{\mathbf{n}}_1$ and $\widehat{\mathbf{n}}_2$ are whitened noises and

$$\overline{\mathbf{H}}_{12} = \boldsymbol{\Psi}_{12}\mathbf{H}_{12}, \tag{30}$$

$$\overline{\mathbf{H}}_{22} = \boldsymbol{\Psi}_{22}\mathbf{H}_{22}. \tag{31}$$

Similar procedure, employed in transmitted one, is applied to compute the modulation and demodulation vectors for the second transmitter.

*B. Scheme 3 - ZF-Successive Decoding at the Receivers Side, Linear Precoding at the Transmitters Side (See Fig. 3):*

In this part, we introduce another signalling scheme which is dual of the scheme one. In this scheme, linear precoding is employed at the transmitters side, while successive decoding (as the dual of DPC) is used at the receivers side. In this scheme, the columns of the matrix $[\mathbf{U}_{11}, \mathbf{U}_{12}]$ are mutually orthogonal, where $\mathbf{U}_{11}$ and $\mathbf{U}_{12}$ are demodulation matrices for receiver one. Similarly, the columns of the matrix $[\mathbf{U}_{22}, \mathbf{U}_{21}]$ are mutually orthogonal, where $\mathbf{U}_{22}$ and $\mathbf{U}_{21}$ are demodulation matrices for receiver two.

In this scheme, the transmitted vectors are equal to:

$$\mathbf{x}_1 = \mathbf{\Psi_{11}} \mathbf{V}_{11} \mathbf{d}_{11} + \mathbf{\Psi_{21}} \mathbf{V}_{21} \mathbf{d}_{21}, \tag{32}$$

$$\mathbf{x}_2 = \mathbf{\Psi_{12}} \mathbf{V}_{12} \mathbf{d}_{12} + \mathbf{\Psi_{22}} \mathbf{V}_{22} \mathbf{d}_{22}, \tag{33}$$

where $\mathbf{\Psi_{rt}}$, $r = 1, 2$, denote precoding matrices, given by

$$\mathbf{\Psi_{rt}} = \left(\mathbf{R}_{rt}^{-\frac{1}{2}}\right)^{\dagger}, \ r, t = 1, 2. \tag{34}$$

and

$$\mathbf{R}_{21} = P\mathbf{H}_{11}^{\dagger}[\mathbf{U}_{11}, \mathbf{U}_{12}][\mathbf{U}_{11}, \mathbf{U}_{12}]^{\dagger}\mathbf{H}_{11} + \mathbf{I}. \tag{35}$$

$$\mathbf{R}_{22} = P\mathbf{H}_{12}^{\dagger}[\mathbf{U}_{11}, \mathbf{U}_{12}][\mathbf{U}_{11}, \mathbf{U}_{12}]^{\dagger}\mathbf{H}_{12} + \mathbf{I}. \tag{36}$$

$$\mathbf{R}_{12} = P\mathbf{H}_{22}^{\dagger}[\mathbf{U}_{22}, \mathbf{U}_{21}][\mathbf{U}_{22}, \mathbf{U}_{21}]^{\dagger}\mathbf{H}_{22} + \mathbf{I}. \tag{37}$$

$$\mathbf{R}_{11} = P\mathbf{H}_{21}^{\dagger}[\mathbf{U}_{22}, \mathbf{U}_{21}][\mathbf{U}_{22}, \mathbf{U}_{21}]^{\dagger}\mathbf{H}_{21} + \mathbf{I}. \tag{38}$$

Here again, we assume that each data stream has the power of $P$.

It is easy to verify that with these choices of pre-filtering matrices, in high SNR,

- $\mathbf{d}_{11}$ has no interference at the output of the demodulation matrices $\mathbf{U}_{22}$ and $\mathbf{U}_{21}$.
- $\mathbf{d}_{12}$ has no interference at the output of the demodulation matrices $\mathbf{U}_{22}$ and $\mathbf{U}_{21}$.
- $\mathbf{d}_{22}$ has no interference at the output of the demodulation matrices $\mathbf{U}_{11}$ and $\mathbf{U}_{12}$.
- $\mathbf{d}_{21}$ has no interference at the output of the demodulation matrices $\mathbf{U}_{11}$ and $\mathbf{U}_{12}$.

Now, we define

$$\overline{\mathbf{H}}_{rt} = \mathbf{H}_{rt}\boldsymbol{\Psi}_{\mathbf{rt}}, \qquad r,t = 1,2. \tag{39}$$

This system can be considered as a combination of two multiple access channels: (i) the multiple access channel viewed by the receiver one with channels $\overline{\mathbf{H}}_{11}$ and $\overline{\mathbf{H}}_{12}$, (ii) the multiple access channel viewed by receiver two with channels $\overline{\mathbf{H}}_{21}$ and $\overline{\mathbf{H}}_{22}$. At the output terminal of each multiple access channel, we apply the zero-forcing successive-decoding method. In the following, we explain this method for the multiple access channel viewed by receiver one. The columns of $\mathbf{V}_{11}$ are chosen as the $\mu_{11}$ right singular vectors of $\overline{\mathbf{H}}_{11}$, corresponding to the $\mu_{11}$ largest singular values of the matrix. Also,

$$\sigma_{11}^{(i)} = \| \overline{\mathbf{H}}_{11}\mathbf{v}_{11}^{(i)} \|, \tag{40}$$

$$\mathbf{u}_{11}^{(i)} = \frac{\overline{\mathbf{H}}_{11}\mathbf{v}_{11}^{(i)}}{\sigma_{11}^{(i)}}. \tag{41}$$

In fact, $\sigma_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$, are the $\mu_{11}$ largest singular values of $\overline{\mathbf{H}}_{11}$, and $\mathbf{u}_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$, are corresponding left singular vectors. Clearly, $\mathbf{u}_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$, are mutually orthogonal.

We define $\boldsymbol{\omega}_1^{(11)}$, …, $\boldsymbol{\omega}_{m_1-\mu_{11}}^{(11)}$ such that $[\mathbf{U}_{11}, \boldsymbol{\omega}_1^{(11)}, \ldots, \boldsymbol{\omega}_{m_1-\mu_{11}}^{(11)}]$ forms a unitary matrix. Then, define $\overline{\overline{\mathbf{H}}}_{21}$ as,

$$\overline{\overline{\mathbf{H}}}_{21} = [\boldsymbol{\omega}_1^{(11)}, \ldots, \boldsymbol{\omega}_{m_1-\mu_{11}}^{(11)}]\overline{\mathbf{H}}_{21}. \tag{42}$$

The columns of $\mathbf{V}_{21}$ are equal to the $\mu_{21}$ right singular vectors of $\overline{\overline{\mathbf{H}}}_{21}$, corresponding to the $\mu_{21}$ largest singular values of the matrix. In addition,

$$\sigma_{21}^{(i)} = \| \overline{\overline{\mathbf{H}}}_{21}\mathbf{v}_{21}^{(i)} \|, \tag{43}$$

$$\overline{\mathbf{u}}_{21}^{(i)} = \frac{\overline{\overline{\mathbf{H}}}_{21}\mathbf{v}_{21}^{(i)}}{\sigma_{21}^{(i)}}. \tag{44}$$

and finally,

$$\mathbf{U}_{21} = [\boldsymbol{\omega}_1^{(11)}, \ldots, \boldsymbol{\omega}_{m_1-\mu_{11}}^{(11)}]^\dagger \overline{\mathbf{U}}_{21}. \tag{45}$$

It is easy to see that with these choices of the modulation and demodulation vectors, $\mathbf{d}_{11}$ has no interference over $\mathbf{d}_{12}$, while the interference of $\mathbf{d}_{12}$ over $\mathbf{d}_{11}$ is canceled out by using successive decoding. As a result, the multiple access channel is reduced to a set of parallel channels with gains $\sigma_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$, and $\sigma_{12}^{(j)}$, $j = 1, \ldots, \mu_{12}$.

Similar technique is applied to determine the modulation and demodulation vectors for the multiple access channel viewed by the second receiver.

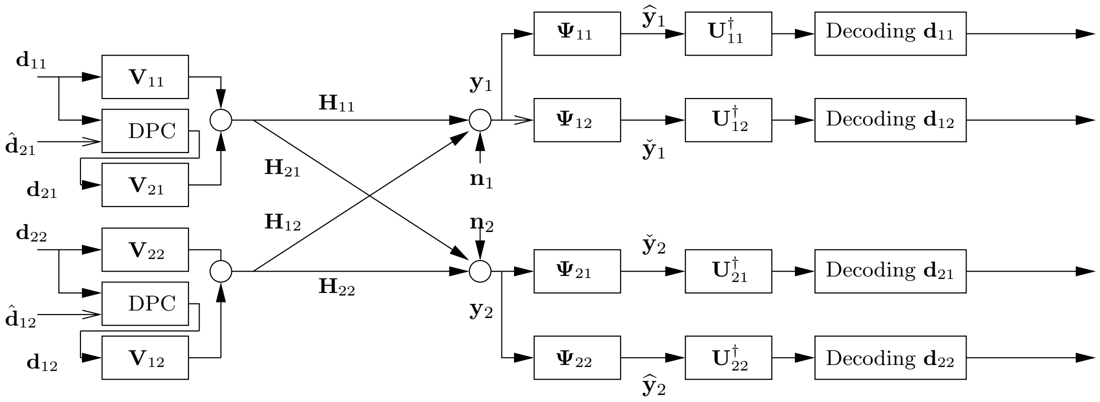


Fig. 1. Scheme One: ZF-DPC at the Transmitters Side and ZF at the Receivers Side

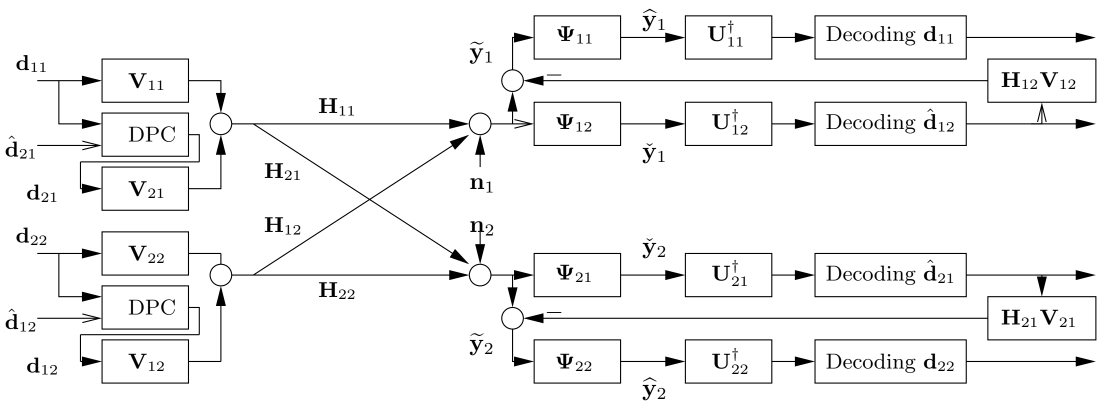


Fig. 2. Scheme Two ZF-DPC at the Transmitters Side and Successive Decoding at the Receivers Side

## III. Achievable Multiplexing Gain

In this section, we investigate the achievable multiplexing gain of the proposed schemes.

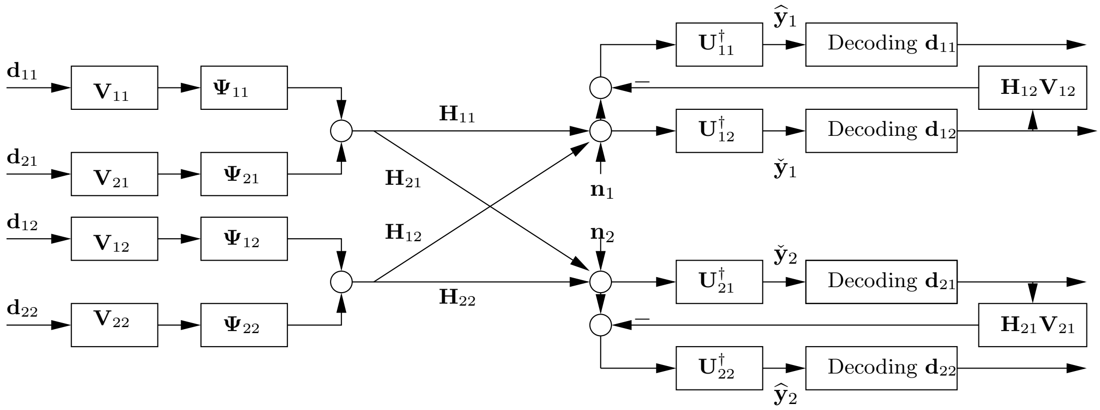


Fig. 3. Scheme Three: ZF at the Transmitters Side and Successive Decoding at the Receivers Side

**Theorem 1** *The scheme (without successive decoding), $\mathbf{d}_{rt}$, for all $r, t = 1, 2$, achieves multiplexing gain of $\mu_{rt}$ if,*

$$\mu_{11}: \qquad \mu_{11} + \mu_{12} + \mu_{22} \leq n_1 \tag{46}$$

$$\mu_{12}: \qquad \mu_{11} + \mu_{12} + \mu_{21} \leq n_1 \tag{47}$$

$$\mu_{22}: \qquad \mu_{22} + \mu_{21} + \mu_{11} \leq n_2 \tag{48}$$

$$\mu_{21}: \qquad \mu_{22} + \mu_{21} + \mu_{12} \leq n_2 \tag{49}$$

$$\mu_{11} + \mu_{21} \leq m_1 \tag{50}$$

$$\mu_{22} + \mu_{12} \leq m_2 \tag{51}$$

*If $\mu_{rt} = 0$ for a pair $r$ and $t$, then the corresponding inequality is eliminated from the set of constraints.*

*Proof:* From (10), we have

$$\mathbf{R}_{12} = P\mathbf{H}_{11}[\mathbf{V}_{11}\ \mathbf{V}_{21}][\mathbf{V}_{11}\ \mathbf{V}_{21}]^\dagger \mathbf{H}_{11}^\dagger + \mathbf{I}.$$

Applying the SVD decomposition, we have

$$\mathbf{H}_{11}[\mathbf{V}_{11}\ \mathbf{V}_{21}][\mathbf{V}_{11}\ \mathbf{V}_{21}]^\dagger \mathbf{H}_{11}^\dagger = \sum_{i=1}^{\mu_{11}+\mu_{21}} \lambda_i^2 \boldsymbol{\varpi}_i \boldsymbol{\varpi}_i^\dagger \tag{52}$$

where $\lambda_i, \geq 0$, $i = 1, \ldots, \mu_{11} + \mu_{21}$, and $\boldsymbol{\varpi}_i$, $i = 1, \ldots, \mu_{11} + \mu_{21}$, are unit orthogonal vectors. Consider $\boldsymbol{\varpi}_j$, $j = \mu_{11}+\mu_{21}+1, \ldots, n_1$ such that the matrix $[\boldsymbol{\varpi}_1, \boldsymbol{\varpi}_2, \ldots, \boldsymbol{\varpi}_{n_1}]$ forms a unitary matrix, then we can show that

$$\boldsymbol{\Psi}_{12} = \begin{bmatrix} \mathbf{A}_{12} & 0 \\ 0 & \mathbf{B}_{12} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varpi}_1^\dagger \\ \\ \cdots \\ \boldsymbol{\varpi}_{\mu_{11}+\mu_{21}}^\dagger \\ \boldsymbol{\varpi}_{\mu_{11}+\mu_{21}+1}^\dagger \\ \\ \cdots \\ \boldsymbol{\varpi}_{n_1}^\dagger \end{bmatrix}, \tag{53}$$

where,

$$\mathbf{A}_{12} = \text{diag}\left\{ \left[ \frac{1}{\sqrt{P\lambda_1^2 + 1}}, \ldots, \frac{1}{\sqrt{P\lambda_{\mu_{11}+\mu_{21}}^2 + 1}} \right] \right\}. \tag{54}$$

$$\mathbf{B}_{12} = \text{diag}\{[1, \ldots, 1]\}. \tag{55}$$

According to (30), $\overline{\mathbf{H}}_{12} = \boldsymbol{\Psi}_{12}\mathbf{H}_{12}$. As mentioned, $\sigma_{12}^{(i)}$, $i = 1, \ldots, \mu_{12}$, are equal to the $\mu_{12}$ largest singular values of $\overline{\overline{\mathbf{H}}}_{12}$, where $\overline{\overline{\mathbf{H}}}_{12} = \overline{\mathbf{H}}_{12}[\boldsymbol{\varphi}_1^{(22)}, \ldots, \boldsymbol{\varphi}_{m_2-\mu_{22}}^{(22)}]$, and $\boldsymbol{\varphi}_1^{(22)}, \ldots, \boldsymbol{\varphi}_{m_2-\mu_{22}}^{(22)}$ are a set of unit vectors such that $[\mathbf{V}_{22}, \boldsymbol{\varphi}_1^{(22)}, \ldots, \boldsymbol{\varphi}_{m_2-\mu_{22}}^{(22)}]$ forms a unitary matrix. In high SNR, $\mathbf{A} \longrightarrow \mathbf{0}$. Therefore, the first $\mu_{11}+\mu_{21}$ rows of $\overline{\overline{\mathbf{H}}}_{12}$ of the matrix converges zero. The remaining rows of the matrix form a $(n_1-\mu_{11}-\mu_{21})\times(m_2-\mu_{22})$ full rank matrix. Since $\sigma_{12}^{(i)}$, $i = 1, \ldots, \mu_{12}$, are equal to the $\mu_{12}$ largest singular values of $\overline{\overline{\mathbf{H}}}_{12}$, if $\mu_{12} \leq \min\{(m_2 - \mu_{22}), (n_1 - \mu_{11} - \mu_{21})\}$, then for all $i$, $i = 1, \ldots, \mu_{12}$, $\sigma_{12}^{(i)}$ is a positive number which is non-vanishing with increasing power. Note that $\sigma_{12}^{(i)}$, $i = 1, \ldots, \mu_{12}$, are the gain of set of equivalent parallel channels from transmitter 2 to receiver 1. This means that if $\mu_{22} + \mu_{12} \leq m_2$ and $\mu_{12} \leq n_1 - \mu_{11} - \mu_{21}$, $\mathbf{d}_{12}$ achieves the multiplexing gain of $\mu_{12}$. Similarly, we can show that if $\mu_{21} \leq n_2 - \mu_{22} - \mu_{12}$ and $\mu_{11} + \mu_{21} \leq m_1$, $\mathbf{d}_{21}$ achieves the multiplexing gain of $\mu_{21}$.

From (11), we have

$$\mathbf{R}_{11} = P\mathbf{H}_{12}[\mathbf{V}_{22}\ \mathbf{V}_{12}][\mathbf{V}_{22}\ \mathbf{V}_{12}]^{\dagger}\mathbf{H}_{12}^{\dagger} + \mathbf{I}.$$

Applying SVD decomposition, we have

$$\mathbf{H}_{12}[\mathbf{V}_{22}\ \mathbf{V}_{12}][\mathbf{V}_{22}\ \mathbf{V}_{12}]^{\dagger}\mathbf{H}_{12}^{\dagger} = \sum_{i=1}^{\mu_{22}+\mu_{12}} \kappa_i^2 \boldsymbol{\xi}_i \boldsymbol{\xi}_i^{\dagger}, \tag{56}$$

where $\kappa_i, \geq 0$, $i = 1, \ldots, \mu_{22} + \mu_{12}$, and $\boldsymbol{\xi}_i$, $i = 1, \ldots, \mu_{22} + \mu_{12}$ are unit orthogonal vectors. Consider $\boldsymbol{\xi}_j$, $j = \mu_{22} + \mu_{12} + 1, \ldots, n_1$ such that $[\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \ldots, \boldsymbol{\xi}_{n_1}]$ forms a unitary matrix, then we can show that

$$\boldsymbol{\Psi}_{11} = \begin{bmatrix} \mathbf{A}_{11} & 0 \\ 0 & \mathbf{B}_{11} \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}_1^{\dagger} \\ \\ \cdots \\ \boldsymbol{\xi}_{\mu_{22}+\mu_{12}}^{\dagger} \\ \boldsymbol{\xi}_{\mu_{22}+\mu_{12}+1}^{\dagger} \\ \\ \cdots \\ \boldsymbol{\xi}_{n_1}^{\dagger} \end{bmatrix}, \tag{57}$$

where,

$$\mathbf{A}_{11} = \mathrm{diag}\left\{\left[\frac{1}{\sqrt{P\kappa_1^2 + 1}}, \ldots, \frac{1}{\sqrt{P\kappa_{\mu_{22}+\mu_{12}}^2 + 1}}\right]\right\}, \tag{58}$$

$$\mathbf{B}_{11} = \mathrm{diag}\{[1, \ldots, 1]\}. \tag{59}$$

According to (21), $\overline{\mathbf{H}}_{11} = \boldsymbol{\Psi}_{11}\mathbf{H}_{11}$. In high SNR, $\mathbf{A}_{11} \longrightarrow \mathbf{0}$. Consequently, the first $\mu_{22} - \mu_{12}$ rows of $\overline{\mathbf{H}}_{11}$ converges to zeros, while the remaining rows of $\overline{\mathbf{H}}_{11}$ forms a $(n_1 - \mu_{22} - \mu_{12}) \times m_1$ full rank matrix. If $\mu_{11} \leq \min\{m_1, (n_1 - \mu_{22} - \mu_{12})\}$, $\sigma_{11}^{(i)}$, $i = 1, \ldots, \mu_{11}$, the $\mu_{11}$ largest singular values of $\overline{\mathbf{H}}_{11}$, are some positive numbers which are non-vanishing with power increasing. Therefore, if $\mu_{11} \leq n_1 - \mu_{22} - \mu_{12}$, then $\mathbf{d}_{11}$ achieves multiplexing gain of $\mu_{11}$. Similarly, if $\mu_{22} \leq n_2 - \mu_{11} - \mu_{21}$, $\mathbf{d}_{22}$ achieves multiplexing gain of $\mu_{22}$. ■

**Theorem 2** *The scheme (without successive decoding), $\mathbf{d}_{rt}$ for all $r, t = 1, 2$, achieves multiplexing gain of $\mu_{rt}$ if*

$$\begin{aligned}
\mu_{11} : &\qquad \mu_{11} + \mu_{12} + \mu_{22} \leq n_1 && (60)\\
\mu_{12} : &\qquad \mu_{11} + \mu_{12} + \mu_{21} \leq n_1 && (61)\\
\mu_{22} : &\qquad \mu_{22} + \mu_{21} + \mu_{11} \leq n_2 && (62)\\
\mu_{21} : &\qquad \mu_{22} + \mu_{21} + \mu_{12} \leq n_2 && (63)\\
&\qquad \mu_{11} + \mu_{21} \leq m_1 && (64)\\
&\qquad \mu_{22} + \mu_{12} \leq m_2 && (65)
\end{aligned}$$

*If $\mu_{rt} = 0$ for a pair $r$ and $t$, then the corresponding inequality is eliminated from the set of constraints.*

*Proof:* The discussion for $\mathbf{d}_{12}$ and $\mathbf{d}_{21}$ is the same as theorem 1. The only difference is for $\mathbf{d}_{11}$ and $\mathbf{d}_{22}$. As mentioned, the covariance of the interference plus noise for the data streams $\mathbf{d}_{11}$ is equal to

$$\mathbf{R}_{11} = \mathbf{Q}_1\mathbf{H}_{12}\mathbf{V}_{22}\mathbf{P}_{22}\mathbf{V}_{22}^{\dagger}\mathbf{H}_{12}^{\dagger}\mathbf{Q}_1^{\dagger} + \mathbf{I}. \tag{66}$$

where $\mathbf{Q}_1$ is obtained by,

$$\mathbf{Q}_1 = \mathbf{I} - \mathbf{H}_{12}\mathbf{V}_{12}\mathbf{\Gamma}_1(\mathbf{V}_{12}^{\dagger}\overline{\mathbf{H}}_{12}^{\dagger}\overline{\mathbf{H}}_{12}\mathbf{V}_{12})^{-1}\mathbf{V}_{12}^{\dagger}\overline{\mathbf{H}}_{12}^{\dagger}\mathbf{\Psi}_{12}.$$

The second term of the matrix $\mathbf{Q}_1$ has the degree of at most $\min\{\mu_{12}, n_1 - \mu_{11} - \mu_{21}\}$. Therefore, the output space of the matrix $\mathbf{Q}_1$ has degree of at least $\max\{n_1 - \mu_{12}, \mu_{11} + \mu_{21}\}$, and therefore greater than $n_1 - \mu_{12}$. On the other hand, it is easy to see that for all $i$, $i = 1, \ldots, \mu_{11} + \mu_{21}$, $\mathbf{Q}_1\boldsymbol{\varpi}_i = \boldsymbol{\varpi}_i$, where $\boldsymbol{\varpi}_i$ is defined in (52). This implies that the columns of $\mathbf{H}_{11}\mathbf{V}_{11}$ are in the output space of the matrix $\mathbf{Q}_1$. On the other hand, the interference $\mathbf{Q}_1\mathbf{H}_{12}\mathbf{V}_{22}$ occupy $\mu_{22}$ of the output space of $\mathbf{Q}_1$. Therefore, if $\mu_{11} + \mu_{22} \leq n_1 - \mu_{12}$, then we can choose columns of $\mathbf{V}_{11}$ such that the columns of $\mathbf{H}_{11}\mathbf{V}_{11}$ have term on the orthogonal space of $\mathbf{Q}_1\mathbf{H}_{12}\mathbf{V}_{22}$ and therefore achieve full multiplexing gain. Similar statements are valid for the data streams $\mathbf{d}_{22}$. ■

Comparing theorem 1 and 2, we conclude that successive decoding at the receivers side does not increase the multiplexing gain.

**Theorem 3** *In the scheme three,* $\mathbf{d}_{rt}$ *for all* $r, t = 1, 2$*, achieves multiplexing gain of* $\mu_{rt}$*, if*

$$\mu_{11}: \qquad \mu_{11} + \mu_{21} + \mu_{22} \leq m_1 \tag{67}$$

$$\mu_{21}: \qquad \mu_{11} + \mu_{21} + \mu_{12} \leq m_1 \tag{68}$$

$$\mu_{22}: \qquad \mu_{22} + \mu_{12} + \mu_{11} \leq m_2 \tag{69}$$

$$\mu_{12}: \qquad \mu_{22} + \mu_{12} + \mu_{21} \leq m_2 \tag{70}$$

$$\mu_{11} + \mu_{12} \leq n_1 \tag{71}$$

$$\mu_{22} + \mu_{21} \leq n_2 \tag{72}$$

*If* $\mu_{rt} = 0$ *for a pair* $r$ *and* $t$*, then the corresponding inequality is eliminated from the set of constraints.*

*Proof:* Proof is similar to the proof of the Theorem 1. ■

**Theorem 4** *In the special case of* $n_1 = n_2 = n$ *in the schemes one and two, the multiplexing gain of* $\eta = \lfloor \frac{4n}{3} \rfloor$ *is achievable, where the total number of transmit antennas is equal* $\eta$*, which are almost equally divided between transmitters, i.e. if* $\eta$ *is even* $m_1 = m_2 = \frac{\eta}{2}$*, otherwise* $m_1 = \lfloor \frac{\eta}{2} \rfloor$ *and* $m_2 = \lfloor \frac{\eta}{2} \rfloor + 1$ *or vise versa.*

*Proof:* By adding the four inequalities (46), (47), (48), (49), and dividing both sides of the resulting inequality to four, we have,

$$\mu_{11} + \mu_{21} + \mu_{12} + \mu_{22} \leq \lfloor \frac{4n}{3} \rfloor, \tag{73}$$

which provides us with an upper bound on the total multiplexing gain. By assume that $n = 3k + l$, where $0 \leq l \leq 2$, it is easy to prove that with choices of $m_t$, and $\mu_{rt}$, $r, t = 1, 2$, listed in Table I, all the constraints in theorems 1 and 2 are satisfied. ■

TABLE I

TABLE OF CHOICES FOR THEOREM 4 ($n = 3k + l,\ 0 \le l \le 2$)

| $l$ | $m_1$ | $m_2$ | $\mu_{11}$ | $\mu_{12}$ | $\mu_{21}$ | $\mu_{22}$ | Multiplexing Gain |
|---|---|---|---|---|---|---|---|
| 0 | $2k$ | $2k$ | $k$ | $k$ | $k$ | $k$ | $4k$ |
| 1 | $2k+1$ | $2k$ | $k+1$ | $k$ | $k$ | $k$ | $4k+1$ |
| 2 | $2k+1$ | $2k+1$ | $k+1$ | $k$ | $k$ | $k+1$ | $4k+2$ |

**Theorem 5** *In the special case of $m_1 = m_2 = m$ in the scheme three, the multiplexing gain of $\eta = \lfloor \frac{4m}{3} \rfloor$ is achievable, where the total number of receive antennas is equal $\eta$, which are almost equally divided between transmitters, i.e. if $\eta$ is even $n_1 = n_2 = \frac{\eta}{2}$, otherwise $n_1 = \lfloor \frac{\eta}{2} \rfloor$ and $n_2 = \lfloor \frac{\eta}{2} \rfloor + 1$ or vise versa.*

*Proof:* The proof is similar to that of theorem 4 with the choices listed in Table II. ■

TABLE II

TABLE OF CHOICES FOR THEOREM 5 ($m = 3k + l,\ 0 \le l \le 2$)

| $l$ | $n_1$ | $n_2$ | $\mu_{11}$ | $\mu_{12}$ | $\mu_{21}$ | $\mu_{22}$ | Multiplexing Gain |
|---|---|---|---|---|---|---|---|
| 0 | $2k$ | $2k$ | $k$ | $k$ | $k$ | $k$ | $4k$ |
| 1 | $2k+1$ | $2k$ | $k+1$ | $k$ | $k$ | $k$ | $4k+1$ |
| 2 | $2k+1$ | $2k+1$ | $k+1$ | $k$ | $k$ | $k+1$ | $4k+2$ |

**Remark 1:** Theorems 1 to 3 characterize the achievable multiplexing gain of the some channels as special cases:

- *Multiple Access Channels:* By choosing $m_1$ or $m_2$ equal to zero.
- *Broadcast Channels:* By choosing $n_1$ or $n_2$ equal zero.
- *Interference Channels:* By choosing $\mu_{12}$ and $\mu_{12}$ equal to zero.

The multiplexing gain of the above channels have been reported in in [2].

## IV. EXPLANATION

Theorems 1 to 5 state that the proposed scenario outperforms other conventional scenarios in terms of achievable multiplexing gain. For example, in the special case of $m_1 = m_2 = n_1 = n_2 = n$, the proposed scheme achieves multiplexing gain of $\lfloor \frac{4n}{3} \rfloor$, while for interference channel it is proven in [2] that the achievable multiplexing gain is $n$. The extra multiplexing gain can be justified from two perspectives:

### A. Distributed Interference Cancelation

In multiple antenna systems, cooperation is one side of communication link is enough to achieve the maximum multiplexing gain. For example in all the following channels, the multiplexing gain of $n$ is achievable: (i) a MIMO point to point system with $n$ antennas at each side, (ii) a MIMO broadcast system with $n$ transmit antennas and $n$ single-antenna users, (iii) a MIMO multiple access system with $n$ receive antennas and $n$ single-antenna users.

In MIMO system with two transmitters and two receivers, if we treat the system as an interference channel, then each receiver is designated to one transmitter. In this case, for the data streams sent from transmitter one to receiver one, there are full cooperation in both sides, which is waste of recourses (cooperation in one side is enough). Similar statement is valid for data streams, sent from transmitter two to receiver two.

In the proposed scheme, each two sets of data streams have cooperation in only transmitter side or receiver side. For example, data streams $\mathbf{d}_{11}$ and $\mathbf{d}_{21}$ have cooperation only at the transmitter side, and not at the receiver side. As another example, data streams $\mathbf{d}_{11}$ and $\mathbf{d}_{12}$ have cooperation only at the receiver side, and not at the transmitter side. Using this scheme, recourses in the system are efficiently exploited to increase multiplexing gain.

### B. Overlap of the interference terms at the receivers side

In this part, reviewing the spacial dimensions at the receivers side provides us a better understanding of the idea behind the proposed schemes. Here, we specially focus on the schemes one and two. The third scheme is the dual of the scheme one.

At receiver one, it is needed to decode $\mathbf{d}_{11}$ and $\mathbf{d}_{12}$, without any destructive effects from interference. Therefore, $\mathbf{H}_{11}\mathbf{V}_{11}$, $\mathbf{H}_{12}\mathbf{V}_{12}$, and $\mathbf{H}_{11}\mathbf{V}_{21}$ respectively occupy $\mu_{11}$, $\mu_{12}$, and $\mu_{21}$ orthogonal dimensions at the terminal of receiver one. Otherwise, it is not possible for $\mathbf{d}_{11}$ and $\mathbf{d}_{12}$ to achieve full multiplexing gain.

On the other hand, the space occupied by $\mathbf{H}_{12}\mathbf{V}_{22}$ has mainly overlap with the space of $\mathbf{H}_{11}\mathbf{V}_{21}$, which is not important, because both of them are interference and are not supposed to be decoded at receiver one terminal. In addition, $\mathbf{H}_{12}\mathbf{V}_{22}$ may have – although it is very unlikely – overlap with $\mathbf{H}_{12}\mathbf{V}_{12}$. This event is not important as well, because the dirty-paper-coding theorem guarantees that it does not reduce the capacity. Therefore, if $\mu_{12} + \mu_{11} \leq n_1 - \max\{\mu_{21}, \mu_{22}\}$, we can satisfy all the conditions required to decode $\mathbf{d}_{11}$ and $\mathbf{d}_{12}$ with full multiplexing gain. Similar argument can be applied for the second receiver.

## References


[1] M. A. Maddah-Ali, S. A. Motahari, and Amir K. Khandani, “Signaling over mimo multi-base systems: Combination of multi-access and broadcast schemes,” in *IEEE International Symposium on Information Theory*, Seattle, WA, USA, 2006.

[2] S. A. Jafar, “Degrees of freedom in distributed mimo communications,” in *IEEE Communication Theory Workshop*, 2005.

[3] M. Costa, “Writing on dirty paper,” *IEEE Trans. Inform. Theory*, vol. 29, pp. 439–441, May 1983.

[4] M. A. Maddah-Ali, M. Ansari, and Amir K. Khandani, “An efficient signaling scheme for MIMO broadcast systems: Design and performance evaluation,” *IEEE Transactions on Information Theory*, July 2005, Submitted for Publication.