Iterative Multi-User Turbo-code Receiver for DS-CDMA[1]

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Abstract: A number of different iterative decoding methods are proposed for the multi-user interference cancelation in a Code Division Multiple Access (CDMA) system where Turbo-codes are utilized for Forward Error Correction (FEC). In the proposed methods, the individual users are decoded separately with the operation of iterative interference cancelation being mixed with the iterative decoding of Turbo-code. This results in a modest increase in the overall complexity as compared to a conventional single user receiver utilizing Turbo-code for FEC. Numerical results are presented showing that the proposed iterative decoders show an improvement in the BER performance, and/or a reduction in the computational complexity as compared to similar previously known methods reported in [2], [3].

I. Introduction

The received signal in a DS-CDMA (Direct Sequence Code Division Multiple Access) system is a superposition of the individual transmitted information signals. The conventional CDMA receiver is composed of a bank of single-user detectors, each with a correlator matched to a specific spreading signal. This is the simplest receiver for the CDMA system and is optimum only if the received signals are orthogonal. Such a receiver does not take into account the structure of the Multiple Access Interference (MAI), treating the interference from the other users as additive noise. The performance of the conventional receiver is reasonable when the number of users is small, but deteriorates rapidly as the number of users increases.

The performance of a CDMA system can be greatly enhanced by jointly decoding all the users instead of decoding them separately as done in the conventional receiver. Sergio Verdú first introduced the optimal uncoded multi-user receiver in [4] and discussed it further in [5], [6].

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His derivation of the optimum multi-user receiver is based on a maximum likelihood sequence detection formulation for asynchronous users transmitting over an Additive White Gaussian Noise (AWGN) channel. The complexity of the optimal multi-user receiver increases exponentially with the number of users, making it infeasible for implementation in a practical communication system.

Since the main drawback of the optimal multi-user detector is complexity, most of the recent research works have addressed the problem of simplifying multi-user detection for implementation in a practical communication system with a wide-range of performance versus complexity tradeoffs. Early works on multi-user detection for uncoded synchronous systems are reported in [7], [8], [9]. Some linear complexity sub-optimal receivers that have been proposed include the decorrelating detector [10], [11], the minimum mean-square-error sequence detector [12], and the projection receiver [13]. Several other sub-optimal multi-user detectors that utilize feedback to reduce the effect of the MAI in the received signal include multistage detectors [14], [15], decision-feedback detectors [16], [17], and successive or parallel interference cancellation [18], [19]. Duel-Hallen *et al.* [20] and the references within provide a further description of most of these receivers.

Most of the proposed sub-optimal multi-user detectors in the literature are designed with no consideration to channel coding. Only recently there has been a focus on developing multi-user detectors in combination with various channel coding methods.

Giallorenzi and Wilson [21] extended the optimal uncoded receiver of Verdú [4] to incorporate convolutional coding for FEC. They showed that this receiver could achieve single-user performance for reasonable cross-correlation values typical in CDMA systems. However, the complexity of this receiver is exponentially dependent on the number of users and the number of states of each convolutional code. Due to the high complexity, they proposed a sub-optimal receiver [22] which for a large number of users achieves a performance within a couple of dB of the single-user bound.

The sub-optimal multi-user decoders in the literature that incorporate channel coding can be divided into the categories of non-iterative and iterative receivers. The non-iterative sub-optimal decoders include those proposed in [23] and [24].

Reference [25] describes an iterative detection and decoding scheme in a convolutionally coded DS-CDMA system using MAP decoding and reduces the complexity through an iterative interference cancellation scheme combined with a suboptimal channel decoding algorithm. They also investigate the performance achievable by a low complexity scheme and observe that it is close to optimal.

In [26], Hagenauer proposes an interference cancellation receiver for a synchronous system based on viewing the CDMA channel as a block code. This result led to the proposal of more complex sub-optimal multi-user receivers for asynchronous systems in [27], [28], [29], [30], and [31].

Reed *et al.* [2], [3], [32], [33], [34] also proposed an iterative multi-user decoder for synchronous CDMA systems. These decoders utilize both convolutional code [32], [33], [34], and Turbo-code [2], [3] for FEC. In both cases, near single-user performance was achieved. However, the complexity in both decoders is still exponential with respect to the number of users. Reference [2], [3] also proposes a lower complexity decoder which however results in a noticeable degradation in the performance (in the order of a dB).

Alexander, Grant and Reed [35] further extended [32], [33], [34] to the asynchronous CDMA system using convolutional codes for FEC. Their proposed sub-optimal iterative decoder operates on a sequence of received symbols resulting in a linear per-bit complexity. It views the concatenation of direct-sequence spreading with the asynchronous multiple-access channel as a special form of convolutional code, implementing the interference cancellation operation as part of the decoding. This decoder performed within a fraction of a dB of the single-user bound for a heavily loaded system at high signal to noise ratio values. More recently, Wang *et al.* [36] and El Gamal *et al.* [37] have proposed more advanced iterative sub-optimal receivers using convolutional code for FEC.

Note that most of the iterative interference cancelation schemes based on considering the spreading and the channel coding operations as a concatenated coding scheme require a fairly high signal to noise ratio to perform close to the single user bound. However, this restriction on the admissible range of signal to noise ratio does not apply to the schemes proposed in this article. This feature may be desirable in situations where higher values of error probability are admissible (for example in speech communication).

In 1993, Claude Berrou *et al.* [38] introduced a class of parallel concatenated convolutional codes known as *Turbo-code* that produce results close to the Shannon limit. The Turbo-code technique combines the concepts of iterative decoding, soft-in/soft-out decoding, and pseudo-random interleaving. Independent of [38], channel coding techniques following a similar line of thinking were presented in [39], [40].

This paper proposes a number of different sub-optimal iterative multi-user receivers for CDMA, utilizing Turbo-code for FEC, over an AWGN channel. The iterative structure of these receivers is similar to the iterative structure utilized by Reed *et al.* [2], [3] in the sense that the joint detection is combined with channel decoding. However, unlike [2], [3], the proposed receivers use the individual Turbo decoders to perform the task of interference cancellation, while adding minimal complexity to the Turbo decoding. Throughout the paper, we follow a system model and a set of assumptions similar to [2], [3] to provide a benchmark for comparison.

All the proposed multi-user receivers are based on the same basic iterative decoder construction, incorporating the iterative, soft in/soft out structure of Turbo decoder. The difference between these receivers is a result of varying approximations for the probability distribution of the MAI and the method employed for updating the conditional probability distributions in the system. The first proposed receiver approximates the MAI by a Gaussian random variable while the second proposed receiver represents the MAI as a random variable with a binomial distribution. Another two receiver structures extend the first receiver by also updating the distribution of the noise term. The first of these two receivers uses an estimate of the noise values to provide a better approximation for the conditional noise distribution for each user by exploiting the fact that the noise samples are correlated. In this case, the noise estimates are computed using both a hard and a soft decision on the individual bit values. The last proposed receiver further extends the previous structure by taking the conditional noise distribution over the range of all possible noise estimates.

The paper is organized as follows: Section (II) examines the CDMA system model used in the development of the proposed iterative multi-user receivers. Section (III) presents the derivation of the proposed iterative receivers. Section (IV) describes the simulation setup and presents the BER performance of the proposed schemes, followed by a complexity analysis in Section (V). Finally, Section (VI) concludes with a summary of the results.

II. Direct-Sequence CDMA System Model

The block diagram of the basic discrete-time CDMA system used in this paper is shown in Figure (1). This model assumes that the channel is both chip and symbol synchronous. Note that a similar model is used in [2], [3] which is the benchmark for the comparison in this paper. Generalization to the asynchronous channel is possible by incorporating an appropriate model to compute the conditional probability distribution for the MAI term.

The K users each transmit M source bits $b_t^{(k)} \in \{0, 1\}$ per each block of data, where $k \in \{1, \ldots, K\}$ specifies the user and $t \in \{1, \ldots, M\}$ is the time index of the uncoded data. The source bits for each user are assumed to be i.i.d. and are independent of the other users. The uncoded bits $b_t^{(k)}$ are coded into $L = M/R$ symbols $d_t^{(k)} \in \{-1, +1\}$ per block, where $k \in \{1, \ldots, K\}$,

$t \in \{1, \ldots, L\}$ is the time index of the coded block, and R is the coding rate. The discrete-time spreading signal utilized by user k to spread the coded symbols at symbol interval t consists of N chips, normalized to unit energy, and is denoted by $\mathbf{s}_t^{(k)} \in \left\{\frac{-1}{\sqrt{N}}, \frac{1}{\sqrt{N}}\right\}^N$. The decoder is assumed to have knowledge of the spreading signals of all the users.

Following the notation used in [2], [3], the matched filter outputs of the K users can be expressed in vector form as,

$$\mathbf{r}_t = \mathbf{H}_t \mathbf{d}_t + \mathbf{n}_t, \tag{1}$$

where $\mathbf{r}_t = \left(r_t^{(1)}, \ldots, r_t^{(K)}\right)^\top$, is the received vector (with $(\cdot)^\top$ denoting the transposition operation), $\mathbf{d}_t = \left(d_t^{(1)}, \ldots, d_t^{(K)}\right)^\top$ is the coded data vector, and $\mathbf{H}_t$ is the $K \times K$ discrete-time cross-correlation matrix of the spreading signals. The noise samples $\mathbf{n}_t = \left(n_t^{(1)}, \ldots, n_t^{(K)}\right)^\top$, as a result of matched filtering, are correlated with autocorrelation matrix $E\left\{\mathbf{n}_t \mathbf{n}_t^\top\right\} = \mathbf{H}_t \sigma^2$, where $E\{\}$ denotes the statistical expectation.

The sampled output of the matched filter for the k^{th} user is expanded as

$$r_t^{(k)} = d_t^{(k)} + \sum_{\substack{i=1 \\ i \neq k}}^{K} d_t^{(i)} \rho^{(ik)} + n_t^{(k)}, \tag{2}$$

where $n_t^{(k)}$ is a Gaussian random process with zero mean and variance $\sigma^2 = N_0/2$ [41]. The cross-correlation value $\rho^{(ik)}$ comprises the ik^{th} element of the cross-correlation matrix $\mathbf{H}_t$.

III. Iterative Multi-User CDMA Receiver

The proposed iterative multi-user CDMA receiver is illustrated in Figure (2). The receiver is composed of a metric generator and K single user Turbo decoders. The metric generator uses the outputs of the matched filters, $\mathbf{r}_t = \left(r_t^{(1)}, \ldots, r_t^{(K)}\right)^\top$, from the front end of the receiver to generate a metric suitable for use in the K single user Turbo decoders. Each Turbo decoder then uses this metric to improve the bit probabilities, which will be subsequently used as the *a-priori* input to the metric generator for use in the next iteration. The iterative process continues in this manner until further iterations yield little or no significant improvement.

The main problem of implementing such an iterative multi-user CDMA receiver is that of generating the correct probability values in the metric generator for use in the Turbo decoders. The key point to the proposed iterative algorithm is to find a proper method to update the marginal conditional probability distributions (metric), $p\left(r_t^{(k)} | d_t^{(k)}\right)$, from iteration to iteration. We explore a variety of methods to extract and update these marginal probabilities.

For the iterative multi-user CDMA receiver, each Turbo decoder is modified to produce both uncoded and coded bit probabilities $Pr\left(b_r^{(k)}=b|\mathbf{r}^{(k)}\right)$, $b\in\{0,1\}$, and $Pr\left(d_t^{(k)}=d|\mathbf{r}^{(k)}\right)$, $d\in\{-1,1\}$, respectively. The metric generator creates the metric $p\left(r_t^{(k)}|d_t^{(k)}\right)$ by assigning the output of the Turbo decoders from iteration i as the *a-priori* input probability for the $i+1$ iteration. For the initial iteration, the metric generator assumes equally likely bit probabilities for all K users. After the required number of iterations have been completed, the systematic bit probabilities $Pr\left(b_r^{(k)}=b|\mathbf{r}^{(k)}\right)$, $b\in\{0,1\}$, generated by the K Turbo decoders are used to make the final hard-decision to calculate

$$\hat{b}_r^{(k)}=sign\left(\frac{Pr\left(b_r^{(k)}=1|\mathbf{r}^{(k)}\right)}{Pr\left(b_r^{(k)}=0|\mathbf{r}^{(k)}\right)}-1\right),\qquad \forall k. \tag{3}$$

To reduce the complexity of the derivations for the probability metric, in most of our discussions, the cross-correlation term $\rho^{(ij)}$ is assumed to be constant and equal to ρ for all $i\neq j, i,j\in\{1,\ldots,K\}$. In such cases, we use $\rho=1/N$ corresponding to maximum length spreading sequences. As a result of this assumption, the discrete cross-correlation matrix $\mathbf{H}_t$ becomes a time invariant matrix, represented as $\mathbf{H}$, which has diagonals equal to one and the non-diagonal elements equal to ρ.

The following proposed receivers explore different methods to update the metric $p\left(r_t^{(k)}|d_t^{(k)}\right)$. The first two receivers only update the probability distribution of the MAI, while the last two receivers also include the updating of the probability distributions of the additive correlated Gaussian noise terms.

A. Continuous Gaussian Approximation of MAI Receiver

In one class of algorithms, the MAI term is approximated by a Gaussian random variable. From (2), the MAI term consists of the sum of $K-1$ discrete binary random variables. From the central limit theorem[2], as the number of users K increases, the distribution of the MAI term approaches that of a Gaussian. Each single user binary probability distribution consists of the coded bit probabilities $Pr\left(d_t^{(k)}=1\right)$ and $Pr\left(d_t^{(k)}=-1\right)$, $k\in\{1,\ldots,K\}$. The mean and variance of these binary distributions for the k^{th} user, $k\in\{1,\ldots,K\}$ are calculated as,

$$m_t^{(k)} = Pr\left(d_t^{(k)}=1\right)-Pr\left(d_t^{(k)}=-1\right) \tag{4}$$

$$v_t^{2\,(k)} = 1-\left[Pr\left(d_t^{(k)}=1\right)-Pr\left(d_t^{(k)}=-1\right)\right]^2. \tag{5}$$

[2]The central limit theorem states that the sum of n statistically independent and identically distributed random variables approach a Gaussian distribution as $n\rightarrow\infty$.

The addition of these binary distributions results in the MAI term $\rho \sum_{\substack{i=1 \\ i \neq k}}^{K} d_t^{(i)}$ for the k^{th} user having a Gaussian distribution with mean and variance:

$$\mu_{t,\,MAI}^{(k)} = \rho \sum_{\substack{i=1 \\ i \neq k}}^{K} \left[Pr\left(d_t^{(i)} = 1\right) - Pr\left(d_t^{(i)} = -1\right) \right] = \rho \sum_{\substack{i=1 \\ i \neq k}}^{K} m_t^{(i)} \tag{6}$$

$$\sigma_{t,\,MAI}^{2\,(k)} = \rho^2 \sum_{\substack{i=1 \\ i \neq k}}^{K} \left(1 - \left[Pr\left(d_t^{(i)} = 1\right) - Pr\left(d_t^{(i)} = -1\right) \right]^2 \right) = \rho^2 \sum_{\substack{i=1 \\ i \neq k}}^{K} v_t^{2\,(i)}. \tag{7}$$

The probability distribution $p\left(r_t^{(k)} | d_t^{(k)} = d\right)$, $d \in \{-1, 1\}$ for the k^{th} user is derived as a Gaussian random variable with mean $\mu_t^{(k)} = d + \mu_{t,\,MAI}^{(k)}$ and variance $\sigma_t^{2\,(k)} = \sigma^2 + \sigma_{t,\,MAI}^{2\,(k)}$.

B. Discrete Analysis of MAI Receiver

The continuous Gaussian approximation of the MAI receiver becomes accurate when the number of users becomes large. Thus, a different approach is needed to determine the probability distribution of the MAI when the number of users is small.

The MAI term $\rho \sum_{\substack{i=1 \\ i \neq k}}^{K} d_t^{(i)}$ for user k is a discrete quantity that can only take on $2K - 1$ different values since each coded bit $d_t^{(i)}$ can only take on values $\{-1, 1\}$. As a result, the MAI term can be represented as a random variable with a binomial distribution. This probability distribution is obtained for user k by convolving the other $K - 1$ sets of bit probabilities to produce,

$$\left\{ P_{t,m}^{(k)} \right\} = \left\{ Pr\left(\sum_{\substack{i=1 \\ i \neq k}}^{K} d_t^{(i)} = m \right) \right\},$$

where $m \in \{-(K-1), -(K-3), \ldots, (K-3), (K-1)\}$. The value of the metric is determined by convolving the distributions of the MAI and the Gaussian noise. The convolution of these two distributions results in,

$$p\left(r_t^{(k)} | d_t^{(k)} = d\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \sum_m \exp\left[-\frac{\left(r_t^{(k)} - d - \rho m\right)^2}{2\sigma^2} \right] \times P_{t,m}^{(k)}, \tag{8}$$

where $m \in \{-(K-1), -(K-3), \ldots, (K-3), (K-1)\}$ and $d \in \{-1, 1\}$.

C. Correlated Gaussian Noise (CGN) Receiver

The correlated Gaussian noise terms $n_t^{(k)}$, $k \in \{1, \ldots, K\}$, in (2) possess information about each other. If these terms were known, the corresponding correlation between the noise terms could be removed. However, the noise terms are not known exactly and only estimates of them can be obtained by the metric generator through the use of the related bit probabilities at the output of the Turbo decoders from the previous iteration. The receiver proposed in this section assumes that the noise estimates are exact while the next proposed receiver uses these estimates as random variables to further extend the calculation of the noise distribution.

The main idea for the CGN receiver is that upon generating an estimate of the noise samples, the metric generator can calculate the conditional distribution $p\left(n_t^{(k)}|n_t^{(1)}, \underset{\neq k}{\dots}, n_t^{(K)}\right)$ to replace the noise distribution $p\left(n_t^{(k)}\right)$ in the calculation of $p\left(r_t^{(k)}|d_t^{(k)}\right)$. The conditional distribution $p\left(n_t^{(k)}|n_t^{(1)}, \underset{\neq k}{\dots}, n_t^{(K)}\right)$ utilizes the $K-1$ other estimated noise values to reduce the effect of the noise for user k. From Bayes' rule, the conditional noise distribution can be calculated from:

$$p\left(n_t^{(k)}|n_t^{(1)}, \underset{\neq k}{\dots}, n_t^{(K)}\right) = \frac{p\left(n_t^{(1)}, \dots, n_t^{(k)}, \dots, n_t^{(K)}\right)}{p\left(n_t^{(1)}, \underset{\neq k}{\dots}, n_t^{(K)}\right)}, \tag{9}$$

where the noise vector $\mathbf{n}_t = \left(n_t^{(1)}, \dots, n_t^{(K)}\right)^\top$ has the multivariate Gaussian distribution

$$p(\mathbf{n}_t) = p\left(n_t^{(1)}, \dots, n_t^{(k)}, \dots, n_t^{(K)}\right) = \frac{1}{(2\pi)^{K/2}\,|\mathbf{H}|^{1/2}} \exp\left[-\frac{1}{2}\mathbf{n}_t^\top \mathbf{H}^{-1}\mathbf{n}_t\right]. \tag{10}$$

The probability distribution of $p\left(n_t^{(1)}, \underset{\neq k}{\dots}, n_t^{(K)}\right)$ will also be a multivariate Gaussian distribution where $\mathbf{H}$ is a $(K-1)\times(K-1)$ matrix. Expanding and simplifying (9), the conditional noise probability distribution takes the form:

$$p\left(n_t^{(k)}|n_t^{(1)}, \underset{\neq k}{\dots}, n_t^{(K)}\right) = \frac{1}{\sqrt{2\pi}\,\tilde{\sigma}_t^{(k)}} \exp\left[-\frac{\left(n_t^{(k)} - \tilde{\mu}_t^{(k)}\right)^2}{2\tilde{\sigma}_t^{2\,(k)}}\right], \tag{11}$$

which is a Gaussian random variable with mean $\tilde{\mu}_t^{(k)} = \frac{\rho}{(K-2)\rho+1} \sum_{\substack{i=1\\ i\neq k}}^{K} n_t^{(i)}$ and variance $\tilde{\sigma}_t^{2\,(k)} = \tilde{\sigma}^2 = \sigma^2\left[\frac{(1-\rho)(1+(K-1)\rho)}{(1+(K-2)\rho)}\right]$, respectively. It is observed that the conditional noise variance $\tilde{\sigma}^2$ is constant while the mean $\tilde{\mu}_t^{(k)}$ depends on the estimates of the noise produced by the metric generator that are updated during the multi-user iterations.

Generating the updated noise probability function is done in combination with the updating of the probability distribution for the MAI term. The continuous Gaussian approximation of the MAI is used in the updating of the input metric for the Turbo decoders in this section. The probability distribution $p\left(r_t^{(k)}|d_t^{(k)} = d\right)$, $d \in \{-1, 1\}$, including the effect of updating the MAI and the noise distribution, is a Gaussian random variable with mean $\mu_t^{(k)} = d + \mu_{t,\,MAI}^{(k)} + \tilde{\mu}_t^{(k)}$ and variance $\sigma_t^{2\,(k)} = \sigma_{t,\,MAI}^{2\,(k)} + \tilde{\sigma}^2$, where $\mu_{t,\,MAI}^{(k)}$ and $\sigma_{t,\,MAI}^{2\,(k)}$ are the MAI mean and variance given in (6) and (7), respectively.

Since the coded bit probabilities are used directly in the generation of the noise distributions, reliable bit probabilities are needed before updating the noise distributions. Due to this reason, the distribution of the noise is updated after a few iterations of the multi-user decoder have been completed in which only the MAI distribution is updated. The probability distribution function of the noise is updated with these bit probabilities in two ways: (i) by making a hard decision on the coded bit probabilities, and (ii) by using the soft values of the coded bit probabilities. These two methods will be further examined in the following.

C.1 Hard Decision on Coded Bit Probabilities

For the CGN receiver of this section, the metric generator makes a hard decision on the coded bit probabilities received from the individual Turbo decoders as:

$$d_t^{(k)} = \begin{cases} -1 & : \quad Pr\left(d_t^{(k)} = -1\right) \geq Pr\left(d_t^{(k)} = 1\right) \\ 1 & : \quad Pr\left(d_t^{(k)} = -1\right) < Pr\left(d_t^{(k)} = 1\right). \end{cases}$$

These estimated coded bits are used to produce an estimate of the noise for user k at time t through the equation:

$$n_t^{(k)} = r_t^{(k)} - d_t^{(k)} - \rho \sum_{\substack{i=1 \\ i \neq k}}^{K} d_t^{(i)}, \tag{12}$$

which are used in the calculation of the conditional noise probability distribution in (11).

C.2 Soft Decision on Coded Bit Probabilities

The CGN receiver described in this section uses soft decision information in the updating of the noise probability distribution function. The metric generator, upon receiving the bit probabilities from the previous iteration, calculates the average value of the bits from (4) as $m_t^{(k)} = E\left\{d_t^{(k)}\right\} = Pr\left(d_t^{(k)} = 1\right) - Pr\left(d_t^{(k)} = -1\right), \quad k \in \{1, \ldots, K\}$. These average values are used to generate a soft estimate of the noise through an equation similar to (12) as:

$$\hat{n}_t^{(k)} = r_t^{(k)} - m_t^{(k)} - \rho \sum_{\substack{i=1 \\ i \neq k}}^{K} m_t^{(i)}. \tag{13}$$

These soft noise estimates are then used in the calculation of (11).

D. Extending The CGN Receiver

The CGN receiver of the previous receiver updates the correlated noise distribution assuming the noise values of the other $K-1$ users are known exactly. However, the estimates of the noise computed in the previous section are not exact but are random variables since they are computed from bit probabilities of the previous iteration. As such, it can be shown [1] that the noise estimate $\bar{n}_t^{(k)}$ can be modeled by a Gaussian distribution:

$$\bar{n}_t^{(k)} \sim N\left(\hat{n}_t^{(k)}, v_t^{2\,(k)} + \rho^2 \sum_{\substack{i=1 \\ i\neq k}}^{K} v_t^{2\,(i)}\right),$$

where $v_t^{2\,(k)}$ is the variance of the information bits given in (5) and $\hat{n}_t^{(k)}$ is given by (13), independent of whether hard or soft estimates of the coded bits are utilized to create the noise estimates.

The joint probability distribution $p\left(\bar{n}_t^{(1)}, \underset{\neq k}{\cdots}, \bar{n}_t^{(K)}\right)$ of these noise random variables is used to extend the conditional probability distribution (11) of the CGN receiver. The updated noise probability distribution $p\left(n_t^{(k)}\right)$ for user k is obtained as:

$$\begin{aligned} p\left(n_t^{(k)}\right) &= \int_{\bar{n}_t^{(1)}} \underset{\neq k}{\cdots} \int_{\bar{n}_t^{(K)}} p\left(\bar{n}_t^{(1)}, \underset{\neq k}{\cdots}, \bar{n}_t^{(K)}\right) \times \\ & \quad p\left(n_t^{(k)} | n_t^{(1)} = \bar{n}_t^{(1)}, \underset{\neq k}{\cdots}, n_t^{(K)} = \bar{n}_t^{(K)}\right) d\bar{n}_t^{(1)} \underset{\neq k}{\cdots} d\bar{n}_t^{(K)}. \end{aligned} \tag{14}$$

Since both the conditional and joint probabilities distributions in (14) are Gaussian, the probability distribution $p\left(n_t^{(k)}\right)$ will also be Gaussian. Thus, only the mean and variance of the distribution need to be computed. The mean and variance of (14) are found to be [1],

$$\bar{\mu}_t^{(k)} = \frac{\rho}{(K-2)\rho+1} \sum_{\substack{i=1 \\ i\neq k}}^{K} \hat{n}_t^{(i)} \tag{15}$$

$$\begin{aligned} \bar{\sigma}_t^{2\,(k)} &= \sigma^2 \left[\frac{(1-\rho)(1+(K-1)\rho)}{(1+(K-2)\rho)}\right] + \left[\frac{\rho}{(K-2)\rho+1}\right]^2 \times \\ & \left[\sum_{\substack{i=1 \\ i\neq k}}^{K} \left(v_t^{2\,(i)} + \rho^2 \sum_{\substack{j=1 \\ j\neq i}}^{K} v_t^{2\,(j)}\right) + 2 \sum_{\substack{i=1 \\ i\neq k}}^{K-1} \sum_{\substack{j=2 \\ j\neq k \\ j>i}}^{K} E\left\{\bar{n}_t^{(i)} \bar{n}_t^{(j)}\right\} \right. \\ & \left. -2 \sum_{\substack{i=1 \\ i\neq k}}^{K-1} \sum_{\substack{j=2 \\ j\neq k \\ j>i}}^{K} \hat{n}_t^{(i)} \hat{n}_t^{(j)} \right]. \end{aligned} \tag{16}$$

The probability distribution of the metric $p\left(r_t^{(k)}|d_t^{(k)}=d\right)$, $d \in \{-1,1\}$, for the k^{th} user, with the generation of the updated noise probability distribution and the continued updating of the MAI, is a Gaussian random variable with the mean $\mu_t^{(k)} = d + \mu_{t,\,MAI}^{(k)} + \bar{\mu}_t^{(k)}$ and variance $\sigma_t^{2\,(k)} = \sigma_{t,\,MAI}^{2\,(k)} + \bar{\sigma}_t^{2\,(k)}$, where the continuous Gaussian approximation receiver has been utilized to update probability distribution of the MAI.

IV. Numerical Results

The Bit Error Rate (BER) performance of the various iterative multi-user CDMA receivers derived in this paper is presented in this section. The figures shown in this section are the average BER performance taken over all the users present in the system.

The results presented are obtained by simulating the communication system over an AWGN channel. The simulations are performed using a block size of $M = 192$ bits[3] and $K = 5$ users. The Turbo encoder utilized for simulation consists of two parallel recursive systematic encoders of 4 memory elements with generator polynomials of 37_8 and 21_8, respectively. The two encoder outputs are properly punctured to produce the desired effective bit rate for each case. The interleaver is S-random interleaver [42] with $S = 9$.

Figure (3) shows the BER performance of the continuous Gaussian approximation of the MAI receiver versus E_b/N_0 for various values of ρ after the 5^{th} iteration of the multi-user receiver, each with only 1 iteration of the Turbo decoder. The multi-user receiver consists of two different types of iterations, the Turbo decoder iterations and the overall receiver iterations. The iterations of the overall receiver are referred to as "MUit" in the figures while the iterations for the individual Turbo decoders are referred to as "TDit". Figure (3) shows that as the amount of correlation increases, the performance of the continuous Gaussian approximation receiver becomes worse. This is expected since there is more interference as ρ increases.

Figures (4) and (5) show the average BER performance of the continuous Gaussian approximation and discrete analysis of the MAI receivers after the 5^{th} and 10^{th} system iterations, each with only 1 Turbo decoding iteration. The continuous Gaussian approximation and discrete analysis of the MAI receivers give almost similar results with the continuous Gaussian approximation receiver being much less complex (refer to [1] for a detailed complexity analysis) and only slightly inferior.

Figures (6) present the average BER performance of the CGN receiver and the extended CGN receiver. The results presented show the performance of these receivers after 5 iterations of the

[3]The block size of $M = 192$ bits corresponds to a delay specification of 20 ms, assuming a data rate of 9.6 Kbits/sec.

continuous Gaussian approximation receiver with the 6^{th} iteration including the updating of the noise distributions according to the respective model. In these figures, the iterations that only produce an update of the MAI probability distribution are referred to as "MAIit" while the iterations that update both the MAI and noise distributions are referred to as "MUit". The results show that the CGN receiver and the extended CGN receiver perform slightly better than the continuous Gaussian approximation receiver after the same number of iterations at a cost of slightly higher complexity (refer to [1] for a detailed complexity analysis.). As expected, the difference in the results between these receivers is small since the results are already very close to the single user BER performance bound.

Reed *et al.* [2], [3] derived an iterative multi-user decoder using Turbo-code for a CDMA system with the same assumptions on the system synchronization resulting in a formulation similar to the one presented in Section (II). In the following, we compare the proposed continuous Gaussian approximation MAI receiver to the receiver introduced in [2], [3]. Reference [2], [3] presents results assuming random spreading for $K = 5$ users in $N = 7$ chips using a Turbo-code of rate 1/2 (with the same generator as used in our work, namely 37_8 and 21_8) with a block length of 200 information bits. To provide an appropriate comparison with reference [2], [3], we have extended and applied our method based on a continuous Gaussian approximation to the case of random spreading sequences. In this case, equations (6) and (7) should be modified as follows,

$$\mu_{t,\,MAI}^{(k)} = \sum_{\substack{i=1\\ i\neq k}}^{K} \rho^{(ik)} \left[Pr\left(d_t^{(i)}=1\right) - Pr\left(d_t^{(i)}=-1\right)\right] = \sum_{\substack{i=1\\ i\neq k}}^{K} \rho^{(ik)} m_t^{(i)} \tag{17}$$

$$\sigma_{t,\,MAI}^{2\,(k)} = \sum_{\substack{i=1\\ i\neq k}}^{K} [\rho^{(ik)}]^2 \left(1-\left[Pr\left(d_t^{(i)}=1\right) - Pr\left(d_t^{(i)}=-1\right)\right]^2\right) = \sum_{\substack{i=1\\ i\neq k}}^{K} [\rho^{(ik)}]^2 v_t^{2\,(i)}, \tag{18}$$

where $\rho^{(ik)}$, $i, k = 1, \ldots, K$ are the elements of the cross correlation matrix $\mathbf{H}_t$ of the spreading signals.

In [2], [3], simulation results are presented for three multi-user system iterations, each with four Turbo-code iterations and the corresponding average BER performance results are compared with a single-user system using Turbo-code for FEC, with four iterations. At an average BER of 10^{-3}, the Reed *et al.* iterative multi-user decoder is about 0.25dB worse than the single-user performance. The receiver in [2], [3] performs the updating operation of the iterative decoder over a set of received bits for different users resulting in an exponential complexity. However, the proposed receivers in this paper perform the updating operation on a per bit basis, resulting in a much lower complexity. Reference [2], [3] also presents a reduced complexity method which has a polynomial complexity. In the following, we provide a more detailed comparison between our proposed method based on a continuous Gaussian approximation and the reduced complexity method of [2], [3].

Figure (7) provides a comparison between different methods using random spreading. In this figure, cases (1) and (3) correspond to single user Turbo-code of rates 1/3 and 1/2 (four iterations decoding) with a block length of 192 bits, respectively. Case (2) corresponds to the proposed method based on a continuous Gaussian approximation with $K = 5$ users in $N = 7$ chips with rate 1/3 Turbo-code (effective rate of 5/21 bits/chip), and with a block length of 192 bits. Case (4) corresponds to the proposed method based on a continuous Gaussian approximation with $K = 5$ users in $N = 6$ chips with rate 3/7 Turbo-code (effective rate of 5/14 bits/chip), and with a block length of 192 bits. Case (5) corresponds to the proposed method based on a continuous Gaussian approximation with $K = 5$ users in $N = 7$ chips with rate 1/2 Turbo-code (effective rate of 5/14 bits/chip), and with a block length of 192 bits. Case (6) corresponds to the reduced complexity method of [2], [3] with $K = 5$ users in $N = 7$ chips with rate 1/2 Turbo-code (effective rate of 5/14 bits/chip), and with a block length of 200 bits (curve extracted from [2], [3]). The total number of Turbo decoding iterations for the Cases (2), (4), (5) and (6) is equal to 12. It is observed that the method proposed in this work results in about 0.7 dB improvement with respect to the reduced complexity method of [2], [3] with a similar number of users and similar band-width efficiency. We will later present a comparison between the complexity of these methods showing that our method has a smaller complexity as compared to the reduced complexity method of [2], [3].

Reference [2], [3] also presents a method with an exponential complexity which is about 0.2 to 0.5 dB away from the single user bound. Referring to Figure (7), the example of our method specified as Case (4) (which is about 0.5 dB away from the single user bound) offers a performance very close the full complexity method of [2], [3], however, with a much lower complexity.

Performance comparisons with a decorrelator receiver [11] concatenated with a Turbo-decoder is available from [3] which shows a substantial performance improvement for the iterative interference cancellation methods proposed in the current article, or in [2], [3].

V. Complexity Analysis

In the following, we compare the complexity of the proposed method based on a continuous Gaussian approximation (assuming random spreading) with [2], [3]. As already mentioned, reference [2], [3] presents two decoding methods: one method with an exponential complexity in terms of the number of users, and a second method with a complexity which is only polynomial in the number of users. Reference [2], [3] expresses the complexity in terms of the number of computations required to compute the likelihood values and to generate the metrics required in each iteration of a Turbo decoder. This complexity per Turbo-code iteration and per information bit is reported in [2], [3] to be equal to:

$$\frac{2K^2 + 4K + 4}{R} \tag{19}$$

where K is the number of users and R is the code rate.

In our case, using the method based on a continuous Gaussian approximation, the complexity required to compute the likelihood values and to generate the metrics required in each iteration of a Turbo decoder is easily shown to be equal to (per Turbo-code iteration and per information bit):

$$\frac{6K}{R} \tag{20}$$

This is mainly the complexity of computing the mean and the variance of the MAI term using equations (17) and (18), respectively. Note that we have not included the complexity of computing the elements $\rho^{(ik)}$, namely the entries of the cross matrix $\mathbf{H}_t$ and this is consistent with the complexity analysis used in reference [2], [3] (as quoted from [2], [3] in equation (19)). Table (I) presents numerical values for equations (19) and (20) for curves corresponding to the Cases (2), (4), (5) and (6) given in Figure (7).

Referring to Figure (7) and Table (I), we observe that the method proposed in the current article results in a reduction in the complexity, as well as an improvement in the performance as compared to the reduced complexity method of [2], [3]. As already mentioned, reference [2], [3] also presents a method with an exponential complexity, $O(2^K)$, which is about 0.2 to 0.5 dB away from the single user bound. Referring to Figure (7), the example of our method specified as case (4) is about 0.5 dB away from the single user bound which is very close the full complexity method of [2], [3], however, with a much lower complexity, i.e., $O(K)$.

Method	Current article			References [2],[3]
Case	(2)	(4)	(5)	(6)
Code rate	1/3	3/7	1/2	1/2
Bits/chip	5/21	5/14	5/14	5/14
Complexity	90	70	60	148

TABLE I

COMPLEXITY COMPARISONS FOR DIFFERENT CURVES IN FIGURE 9.

VI. Conclusions

This paper has studied a number of different sub-optimal iterative multi-user receivers for CDMA, utilizing Turbo-code for FEC purposes. These iterative decoders show an improvement in the BER performance, and/or a reduction in the computational complexity as compared to similar previously known research works [2], [3].

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References

[1] D. Stienstra, "Iterative Multi-User Turbo-Code Receiver for DS-CDMA", M.A.Sc. Thesis, E&CE Dept., Univ. of Waterloo, Jan. 2000.

[2] M. C. Reed, P. D. Alexander, J. A. Asenstorfer and C. B. Schlegel, "Near single user performance using iterative multi-user detection for CDMA with Turbo-code decoders," *The 8th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, 1997 (PIMRC'97)*, pp. 740 -744 vol.2

[3] M. C. Reed, C. B. Schlegel, P. D. Alexander and J. A. Asenstorfer, "Iterative Multiuser Detection for CDMA with FEC: Near-Single-User Performance," *IEEE Transactions on Communications*, vol. 46, pp. 1693-1699, December 1998.

[4] S. Verdú, "Optimal Multi-User Signal Detection," Ph.D Thesis, University of Illinois at Urbana-Champaign, Coordinated Science Laboratory, Urbana, U.S.A., September 1984.

[5] S. Verdú, "Minimum Probability of Error for Asynchronous Gaussian Multiple-Access Channels," *IEEE Transactions on Information Theory*, pp. 85-96, January 1986.

[6] S. Verdú, "Computational Complexity of Optimum Multiuser Detection," *Algorithmica*, vol. 4, pp. 303-312, 1989.

[7] D. Horwood and R. Gagliardi, "Signal Design For Digital Multiple Access Communications," *IEEE Transactions on Communications*, vol. 23, pp. 378-385, March 1975.

[8] K. Schneider, "Optimum Detection of Code Division Multiplexed Signals," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-15, pp. 181-185, January 1979.

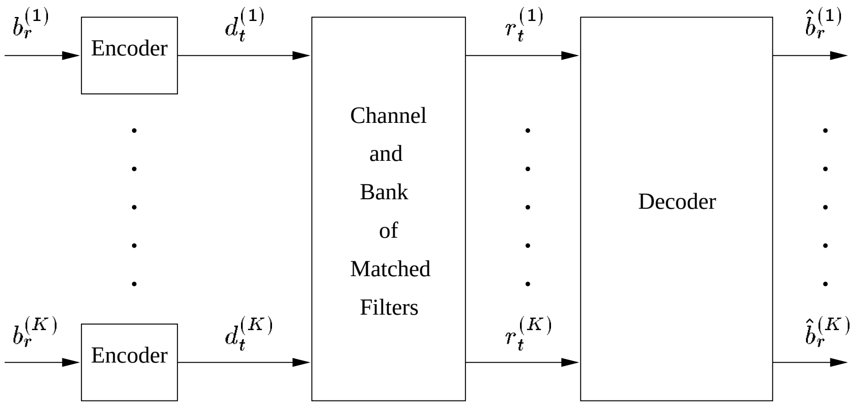

Fig. 1. Discrete-Time CDMA System Model

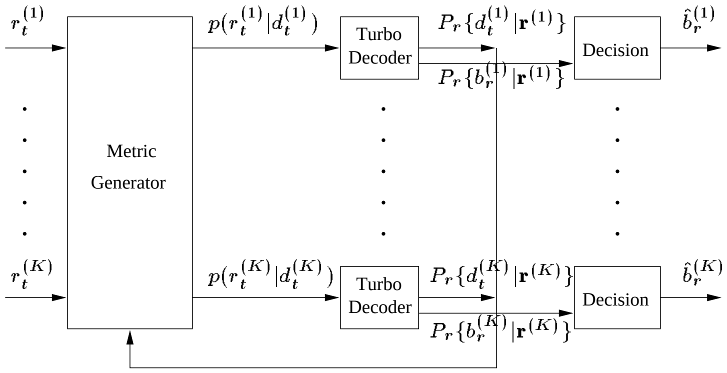

Fig. 2. Iterative Multi-User Decoder

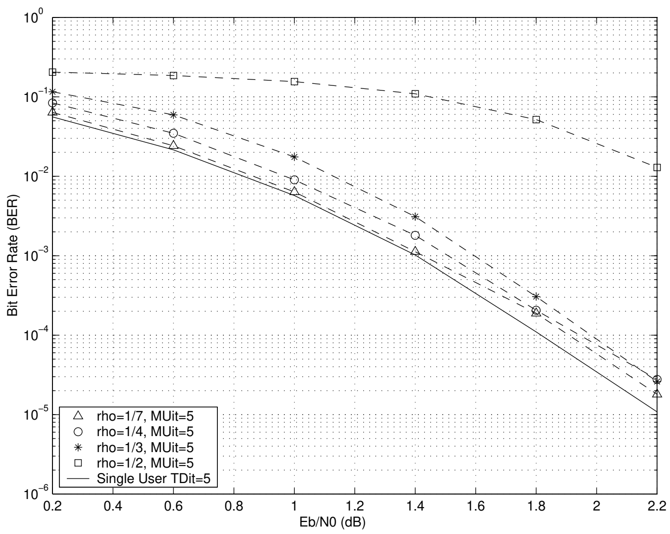

Fig. 3. Continuous Approximation of MAI with $K = 5$ Users and Block Size $M = 192$ for Various Values of $\rho = 1/N$, Code Rate $R = 1/3$ (corresponding to an effective rate of $5\rho/3$ bits/chip).

[9] K. Schneider, "Crosstalk Resistant Receiver for M-ary Multiplexed Communications," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-16, pp. 426-433, July 1980.

[10] P. Jung and J. Blanz, "Joint Detection With Coherent Receiver Antenna Diversity in CDMA Mobile Radio Systems," *IEEE Transactions on Vehicular Technology*, vol. 44, pp. 76-88, February 1995.

[11] R. Lupas and S. Verdú, "Linear Multiuser Detectors for Synchronous Code-Division Multiple-Access Channels," *IEEE Transactions on Information Theory*, vol. 35, pp. 123-136, January 1989.

[12] Z. Xie, R. T. Short, and C. K. Rushforth, "A Family of Suboptimum Detectors for Coherent Multi-User Communications," *IEEE Journal on Selected Areas in Communications*, vol. 12, pp. 683-690, May 1990.

[13] P. D. Alexander, L. K. Rasmussen, and C. B. Schlegel, "A Linear Receiver for Coded Multiuser CDMA," *IEEE Transactions on Communications*, vol. 45, pp. 605-610, May 1997.

[14] M. K. Varanasi and B. Aazhang, "Near-Optimum Detection in Synchronous Code-Division Multiple-Access Systems," *IEEE Transactions on Communications*, vol. 39, pp. 725-736, May 1991.

[15] M. K. Varanasi and B. Aazhang, "Multistage Detection in Asynchronous Code Division Multiple-Access Communications," *IEEE Transactions on Communications*, vol. 38, pp. 509-519, April 1990.

[16] A. Duel-Hallen, "Decorrelating Decision-Feedback Multiuser Detector for Synchronous Code-Division Multiple Access Channel," *IEEE Transactions on Communications*, vol. 41, pp. 285-290, February 1993.

[17] R. T. Short, "Multiple User Receiver Structures," Ph.D. Dissertation, University of Utah, Department of Electrical Engineering, Utah, USA, December 1988.

[18] P. Patel and J. Holtzman, "Analysis of a Simple Successive Interference Cancellation Scheme in a DS-CDMA System," *IEEE Journal on Selected Areas in Communications*, vol. 12, pp. 796-807, June 1994.

[19] A. Johansson and A. Svensson, "Multistage Interference Cancellation in Multi-rate DS/CDMA Systems," *IEEE* 5^{th} *International Symposium on Personal, Indoor and Mobile Radio Communications*, Toronto, Canada, pp. 965-969, September 1995.

[20] A. Duel-Hallen, J. Holtzman, and Z. Zvonar, "Multiuser Detection for CDMA Systems," *IEEE Personal Communications*, vol.2, pp. 46-58, April 1995.

[21] T. R. Giallorenzi and S. G. Wilson, "Multiuser ML Sequence Estimator for Convolutionally Coded Asynchronous DS-CDMA Systems," *IEEE Transactions on Communications*, vol. 44, pp. 997-1008, August 1996.

[22] T. R. Giallorenzi and S. G. Wilson, "Suboptimum Multiuser Receivers for Convolutionally Coded Asynchronous DS-CDMA Systems," *IEEE Transactions on Communications*, vol. 44, pp. 1183-1196, September 1996.

[23] B. R. Vojcic, Y. Shama and R. L. Pickholtz, "Optimum Soft Output MAP Detector for Coded Multiuser Communications," *Proceedings of the IEEE International Symposium on Information Theory*, Ulm, Germany, pp. 229, June 1997.

[24] P. Hoeher, "On Channel Coding and Multiuser Detection for DS-CDMA," *Proceedings of the IEEE International Conference on Universal Personal Communications*, Ottawa, Canada, pp. 641-646, October 1993.

[25] S. Das, E. Erkip and B. Aazhang, "Computationally efficient iterative multiuser detection and decoding," *Thirty-Second Asilomar Conference on Signals, Systems & Computers,* 1998, vol 1, pp. 631 -634

[26] J. Hagenauer, "Forward Error Correcting for CDMA Systems," *IEEE International Symposium on Spread Spectrum Techniques and Applications*, Mainz, Germany, pp. 566-569, September 1996.

[27] M. L. Moher, "Turbo-Based Multiuser Detection," *Proceedings of the IEEE International Symposium on Information Theory*, Ulm, Germany, pp. 195, June 1997.

[28] M. L. Moher, "An Iterative Multiuser Decoder for Near-Capacity Communications," *IEEE Transactions on Communications*, vol. 46, pp. 870-880, July 1998.

[29] F. Tarkoy, "Iterative Multi-User Decoding for Asynchronous Users," *Proceedings of the IEEE International Symposium on Information Theory*, Ulm, Germany, pp. 30, June 1997.

[30] M. C. Reed and P. D. Alexander, "Iterative Multiuser Detection Using Antenna Arrays and FEC on Multi-path Channels," *IEEE Journal on Selected Areas in Communications*, vol. 17, pp. 2082-2089, December 1999.

[31] M. C. Vanlenti and B. D. Woerner, " Iterative Multiuser Detection for Convolutionally Coded Asynchronous DS-CDMA," *Proceedings of the IEEE International Symposium on Personal, Indoor, and Mobile Radio Communications*, Boston, MA, pp. 213-217, 1998.

[32] M. C. Reed, P. D. Alexander, J. A. Asenstorfer, and C. B. Schlegel, "Iterative Multi-User Detection for DS-CDMA with FEC," *The International Symposium on Turbo Codes & Related Topics*, Brest, France, pp. 162-165, September 1997.

[33] M. C. Reed, P. D. Alexander, J. A. Asenstorfer, and C. B. Schlegel, "Reduced Complexity Iterative Multi-User Detection for DS/CDMA with FEC," *International Conference on Universal Personal Communications*, San Diego, U.S.A., pp. 10-14, October 1997.

[34] P. D. Alexander, M. C. Reed, J. A. Asenstorfer, and C. B. Schlegel, "Iterative Multi-User Interference Reduction: Turbo CDMA," *IEEE Transactions on Communications*, vol. 47, pp. 1008-1014, July 1999.

[35] P. D. Alexander, A. J. Grant, and M. C. Reed, "Iterative Detection In Code-Division Multiple-Access With Error Control Coding," *European Transactions on Telecommunications, Special Issue on CDMA Techniques for Wireless Communications Systems*, vol.9, no.5, September-October 1998.

[36] X. Wang and H. V. Poor, "Iterative (Turbo) Soft Interference Cancellation and Decoding for Coded CDMA," *IEEE Transactions on Communications*, vol. 47, pp. 1046-1061, July 1999.

[37] H. El Gamal and E. Gerantiotis, "Iterative Multiuser Detection for Coded CDMA Signals in AWGN and Fading Channels," *IEEE Journal on Selected Areas in Communications*, vol. 18, pp. 30-41, January 2000.

[38] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo-Codes," *Proceedings IEEE International Conference on Communications*, Geneva, Switzerland, pp. 1064-1070, May 1993.

[39] J. Lodge, P. Hoeher and J. Hagenauer, "The decoding of multi-dimensional codes using separable MAP filters," *Proc. of the 16th Biennial Symposium on Communications,* Kingston, Canada, pp. 343-346, May 1992.

[40] J. Lodge, R. Young, P. Hoeher and J. Hagenauer, "Separable MAP "filters" for the decoding of product and concatenated codes", *Proc. of IEEE International Conference on Communications, 1993 (ICC'93)*, Geneva, pp. 1740 -1745, vol.3

[41] S. Verdú, "Multiuser Detection," 1st Ed., Cambridge University Press, 1998.

[42] S. Dolinar and D. Divsalar, "Weight Distributions for Turbo Codes Using Random and Nonrandom Permutations", *The Telecommunications and Data Acquisition Progress Report 42-122, April-June 1995*, Jet Propulsion Laboratory, Pasadena, California, pp. 56-65, August 15 1995.

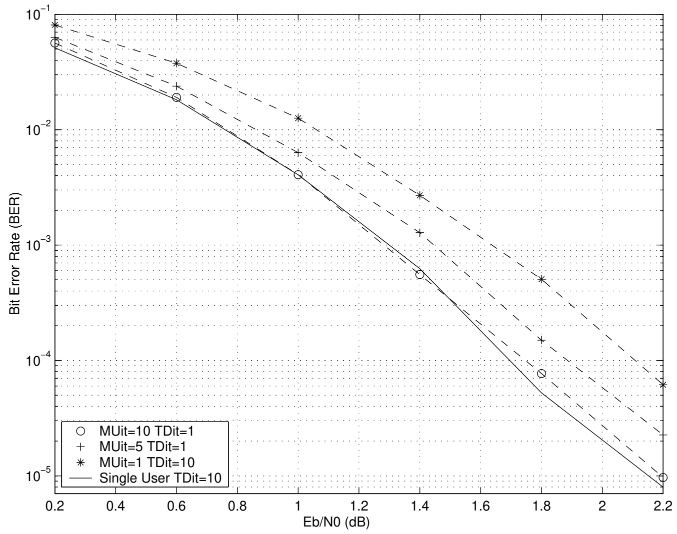

Fig. 4. Continuous Approximation of MAI with $K = 5$ Users, Block Size $M = 192$, Chip Length $N = 7$, Code Rate $R = 1/3$ (corresponding to an effective rate of 5/21 bits/chip).

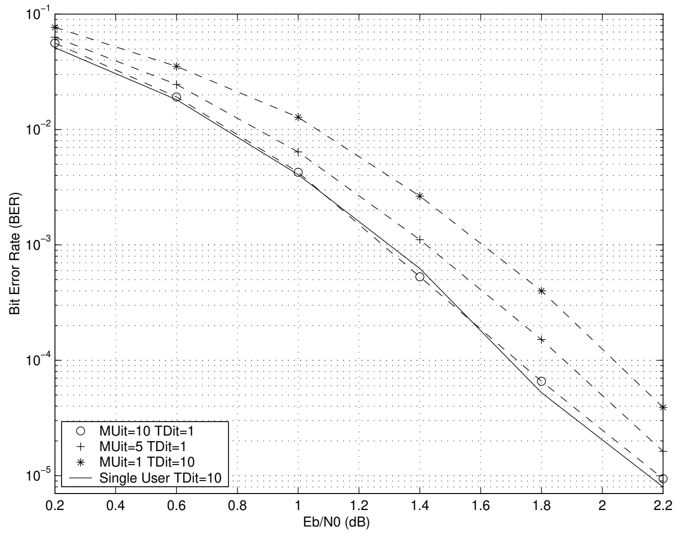

Fig. 5. Discrete Analysis of MAI with $K = 5$ Users, Block Size $M = 192$, Chip Length $N = 7$, Code Rate $R = 1/3$ (corresponding to an effective rate of 5/21 bits/chip).

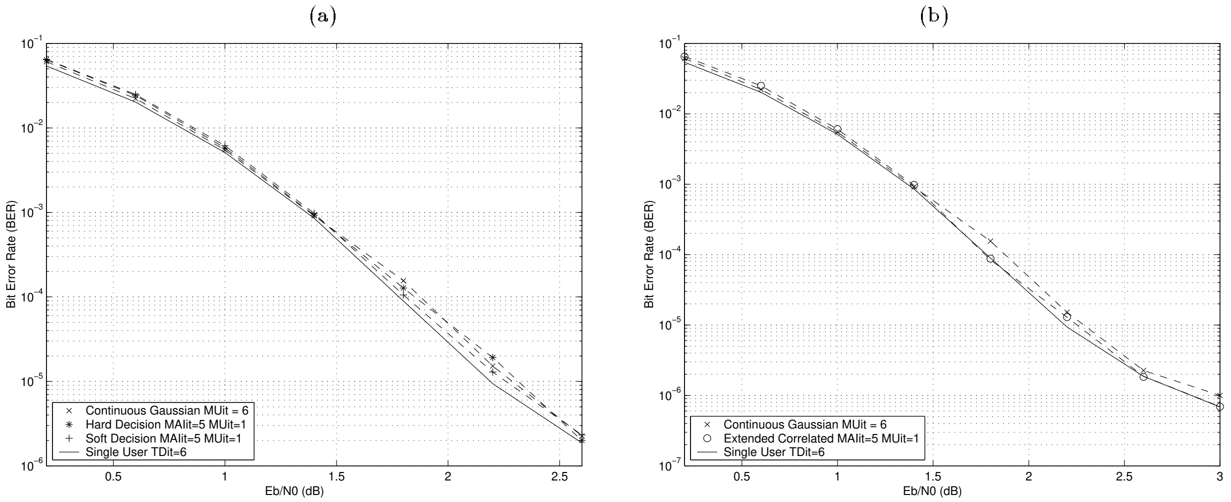

Fig. 6. (a) Hard Versus Soft Decision of Noise, and (b)Extended Correlated Gaussian Noise Model, both with $K = 5$ Users, Block Size $M = 192$, Chip Length $N = 7$, Code Rate $R = 1/3$ (corresponding to an effective rate of 5/21 bits/chip).

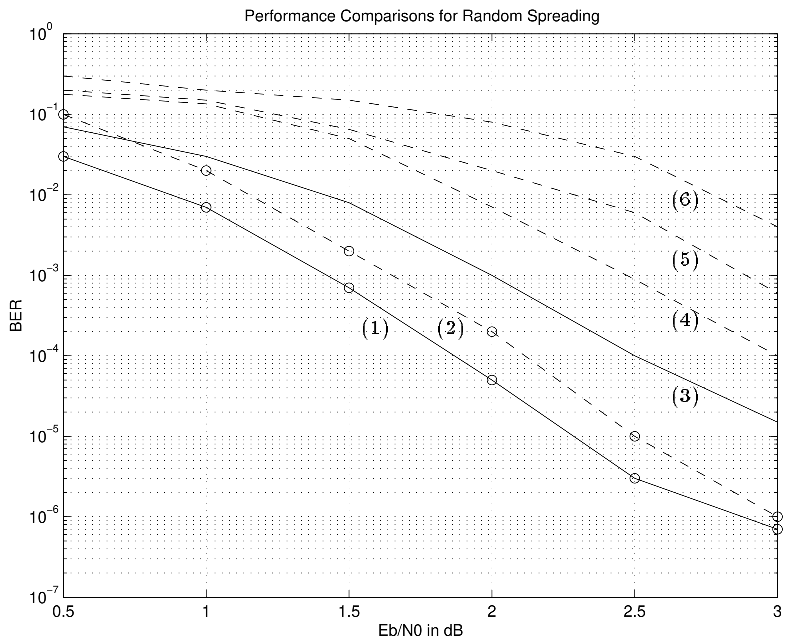

Fig. 7. Numerical comparisons for $K = 5$ users. Case (1): Single user Turbo-code with rate 1/3. Case (2): Proposed method based on a continuous Gaussian approximation with $K = 5$ users in $N = 7$ chips with rate 1/3 Turbo-code (effective rate of 5/21 bits/chip). Case (3): Single user Turbo-code with rate 1/2. Case (4): Proposed method based on a continuous Gaussian approximation with $K = 5$ users in $N = 6$ chips with rate 3/7 Turbo-code (effective rate of 5/14 bits/chip). Case (5): Proposed method based on a continuous Gaussian approximation with $K = 5$ users in $N = 7$ chips with rate 1/2 Turbo-code (effective rate of 5/14 bits/chip). Case (6): Reduced complexity method of [2],[3] for $K = 5$ users in $N = 7$ chips with rate 1/2 Turbo-code (effective rate of 5/14 bits/chip).