Soft Decision Decoding of Fixed Rate Entropy Constrained Quantizer over a Noisy Channel

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Abstract: Variable-length codes (e.g. Huffman codes) are commonly employed to minimize the average binary codeword length in the quantization of discrete sources. However, the effect of error propagation limits the usefulness of variable length codes for the transmission over noisy channels. To avoid this shortcoming, one can use a fixed-rate entropy constrained quantizer, limiting the propagation of error within a single block. This paper presents the optimal decoder for a fixed-rate entropy constrained scalar quantizer. We model all the possible combinations of codewords in a given block by a trellis structure where the Viterbi algorithm is used to choose the most likely path through this trellis. Numerical results are presented indicating a substantial reduction in the end-to-end source distortion with respect to the conventional decoding methods.

I. Introduction

Optimum fixed-rate scalar quantizers, introduced by Max [1] and Lloyd [2] (LMQ), minimize the average distortion for a given number of reconstruction levels. LMQ performs worse than the optimal Entropy Constrained Scalar Quantizer (ECSQ) in the absence of channel noise. ECSQ is known to perform close to the rate-distortion bound for a large class of memoryless sources [3], [4]. Gish and Pierce showed that the optimal ECSQ has uniformly spaced reconstruction levels regardless of the source probability density function [4]. Berger described the necessary conditions for the optimality of ECSQ under squared error distortion measure [5], [6]. Chou presented an algorithm for the Entropy Constrained Vector Quantization (ECVQ) using a Lagrangian formulation with an iterative design algorithm similar to generalized Lloyd algorithm [7].

All the above methods have an inherent problem in the presence of channel noise. Due to the sequential decoding nature of such codes, channel errors can lead to a loss of synchronization resulting in error propagation. In practice, errors may propagate for a considerable period, with a loss of many words, before synchronization is re-established. Conventionally, variable length bit streams are made channel robust through packetization and standard forward-error correction (FEC). However, FEC may be bandwidth inefficient when channel conditions are fairly mild. Reference [9] explored ways in which Huffman codes might be improved in respect of synchronization. The mechanism they use to explain self-synchronization, further developed in [10], relies on the existence of universal synchronizing codewords. However, in an optimally efficient code structure, such synchronizing codewords are not likely to occur frequently, and will contribute in only a minor way to the overall synchronization performance. Another problem with these methods is that they require an output buffer to convert their variable-rate output to a fixed-rate bit stream for transmission over a channel. In any practical situation, the buffer has a finite size and hence undesirable buffer overflow/under flows can occur.

This work was financially supported by the Communications and Information Technology Ontario (CITO), and by the Natural Sciences and Engineering Research Council of Canada (NSERC).

The nature of variable-rate systems greatly complicates the estimation problem at the decoder. Recently, several different approaches for this purpose are proposed. In [11], a computationally complex exact MAP decoding method and an efficient approximation for it are studied. In [12], a different approximate method is independently proposed. In [13], another method is proposed, specifically for memoryless sources. Reference [14] proposes a MAP decoding method which does not include a constraint on the length of the decoded symbol sequences, and consequently, is best suited for the case where the number of transmitted symbols is not known.

To take advantage of entropy coding, while avoiding the dis-advantages associated with conventional methods based on using variable rate codes (including error propagation and buffering problems), one can use Fixed-rate Entropy-coded Vector Quantization (FEVQ).

The Permutation Coder (PC) is a fixed-rate encoder which can be thought of as a vector quantizer with a special structure (the codebook consists of code-vectors that are permutations of one another) [5], [6]. It has been proved that the average distortion of the permutation coder is bounded below by that of the optimal ECSQ at the same rate. As block length increases, the optimal PC becomes equivalent to optimal ECSQ. However, the block length of the PC required to achieve a performance better than LMQ is very large, resulting in a large delay.

The pyramid vector quantizer (PVQ), introduced by Fischer [15], is also a fixed-rate VQ in which the code-vectors are located on the intersection of a cubic lattice and a pyramid in an N-dimensional space. For Laplacian sources this quantizer is asymptotically optimal and achieves the performance of ECSQ, but for other sources it does not approach the ECSQ performance, even for a large N.

A popular approach to FEVQ (assuming a memoryless source) is based on considering the N-fold Cartesian product of a scalar quantizer and selecting the N-fold symbols with the lowest additive self-information (typical set). This is a vector quantizer derived from a variable-length scalar quantizer. In this case, the selected subset has a high degree of structure which can be used to substantially reduce the complexity. A method for exploiting this structure based on using a dynamic programming approach with the states corresponding to the length of the code-words is used by Laroia and Farvardin [16]. The core idea in the schemes of [16] is to use a state diagram with the transitions corresponding to the one-D symbols. This results in a trellis composed of N stages, where N is the space dimensionality. The states s and $s+c$ in two successive stages are connected by a link corresponding to the one-D symbol(s) of entropy c. Quantization is achieved by using the Viterbi algorithm to find the path with the minimum overall additive distortion through the trellis.

Reference [17] uses a different approach to dynamic programming showing improvement with respect to the schemes of [16]. The key point in [17] is to decompose the underlying operations into the lower dimensional subspaces. This decomposition avoids the exponential growth of the complexity. The core of the scheme, as in any problem of dynamic programming, is a recursive relationship which is formed in a hierarchy of levels (where each level involves the Cartesian product of the lower dimensional subspaces).

Although FEVQ outperforms ECSQ in the presence of channel errors, there is still the problem of error propagation within a block which can be improved by taking advantage of the residual source redundancy caused by the imposed block structure. In this work, we propose such a joint source channel decoding method for the case of a Scalar Vector Quantizer (SVQ) [16] by taking advantage of the underlying trellis structure. The paper is organized as follows: In section II, we have a brief overview of fixed-rate entropy constrained quantizers and their formulation. Section III talks about soft decoding of FEVQ over a noisy channel. We conclude the paper in section IV by presenting some numerical results for the proposed methods.

II. Fixed-rate Entropy constrained Quantizer

Consider an N dimensional vector quantizer derived from N variable length scalar quantizers. The ith scalar quantizer consists of M_i partitions with reconstruction levels $(r_i(1), r_i(2), ..., r_i(M_i))$, where $r_i(1) < r_i(2) < ... < r_i(M_i)$. There is a variable-length, binary, prefix code associated with each quantizer, where the codeword corresponding to level $r_i(j)$ has a length of $\ell_i(j)$. The quantization operation can be formulated as follow;

$$\begin{array}{ll} \text{Minimize} & \displaystyle\sum_{i=1}^{N}(x_i - r_i(j))^2 \\ \text{Subject to:} & \left\{ \displaystyle\sum_{i=1}^{N} \ell_i(j) \leq L_{Max} \right. \end{array} \quad (1)$$

where L_{Max} is the maximum length to represent an N-D code-word. There are a few methods known to solve this problem [16], [17].

III. Soft Decoding Fixed-rate Entropy Constrained Quantizer over a Noisy Channel

We assume that each block of N source symbols are represented by L_{Max} bits. In case that the total binary length to represent the N quantized symbols is less than L_{Max}, the rest of the block is filled by zeros. To recover the transmitted codeword sequences from the received data, the straight forward approach is to decode in a sequential manner (symbol by symbol). This is achieved by reverse substitution after parsing the string into codewords. In the case that the received bits are not enough to specify all the N symbols (error has occurred), the decoder maps the rest of the symbols to a fixed value equal to the statistical average of the reconstruction levels. In this method, the receiver uses neither the fact that each block contains N symbols nor the information which is embedded in the soft data.

Knowing that each block represents N symbols, we have to compare the received block with all the possible choices of N symbols which has a total length of L_{Max} or less. One structured approach is to represent all these combinations of N symbols by a trellis diagram, with the states corresponding to the length of the codewords. In this trellis diagram, each transition corresponds to a set of one-D symbols. This results in a trellis composed of $N+1$ stages. The transition(s) from state s_j of stage $k-1$ to state $s_{j+\ell}$ of stage k correspond to the kth symbol(s) of length ℓ. In this case, since the Huffman code may consist of codewords of equal length, there may exist parallel branches between some of the states. As we will see later, this is an important point to be noted in designing a more robust code for noisy channels. The states in the nth stage, $n = 0, \ldots, N$, represent the accumulative codeword length over the set of the first n dimensions. In the final portion of the trellis, there is a transition corresponding to the sequence of terminating zero bits, ending to a common final node for all the trellis paths. This ensures that the total length of all the paths is equal to L_{Max}. The final step is to use the Viterbi algorithm to find the most likely path through the trellis.

IV. Numerical Results

In this section, we present numerical results for the performance of the proposed method. In all cases, a sequence of 600000 source samples is used to design the quantizer, and a different sequence of the same length is used to measure the corresponding performance. Input samples are from a memoryless Gaussian source. The underlying scalar quantizer is composed of 8 points, space dimension is $N = 32$, and $L_{\max} = 80$, resulting in an effective rate of 2.5 bits/sample. The quantization performance is measured in terms of the mean square distance. The channel model is Additive White Gaussian Noise (AWGN) with Binary Phase Shift Keying (BPSK) modulation.

Figure 1 shows the comparison between three different decoding methods, namely trellis based approach using hard and soft decision, and sample by sample decoding method. The structure of the codewords is the same as case (I) in Table I. It is observed that the proposed method results in about 2 dB improvement in comparison with the other two alternatives.

A given Huffman tree can be labeled in different ways resulting in different binary Huffman codes, all with the same set of code-word lengths. Figure 2 shows a comparison of soft decoding for three such Huffman codes given in Table I. To obtain the best performance, in assigning the binary codewords to the quantization levels, we have used the rule that the quantizer points which have the same binary length and are far from each other should differ in as many bit positions as possible. This will reduce the chance of error between parallel branches in the trellis.

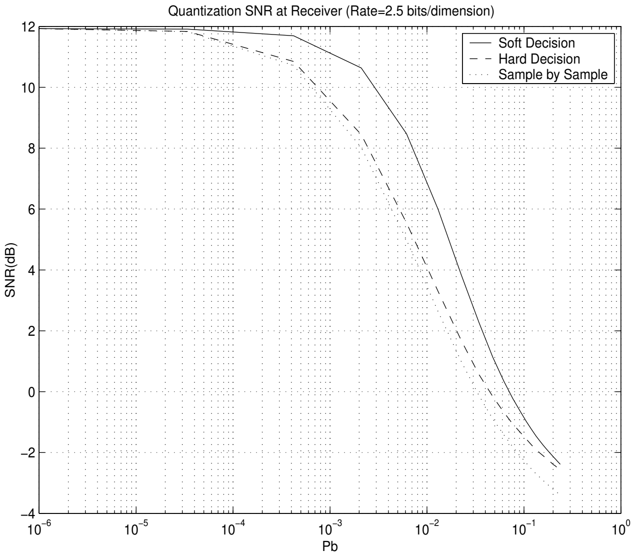

Fig. 1. Comparison of received SNR for soft, hard decoding using trellis structure and sample by sample decoding methods for different bit error probabilities of BPSK channel, P_b (Code(I) is used).

Quantizer level	Code(I)	Code(II)	Code(III)
-2.17	0000	0100	0111
-1.36	0001	0101	0110
-0.77	001	011	010
-0.25	01	00	00
$+0.25$	10	11	10
$+0.77$	110	100	110
$+1.36$	1110	1010	1110
$+2.17$	1111	1011	1111

TABLE I

BIT ASSIGNMENT FOR DIFFERENT HUFFMAN CODE STRUCTURES.

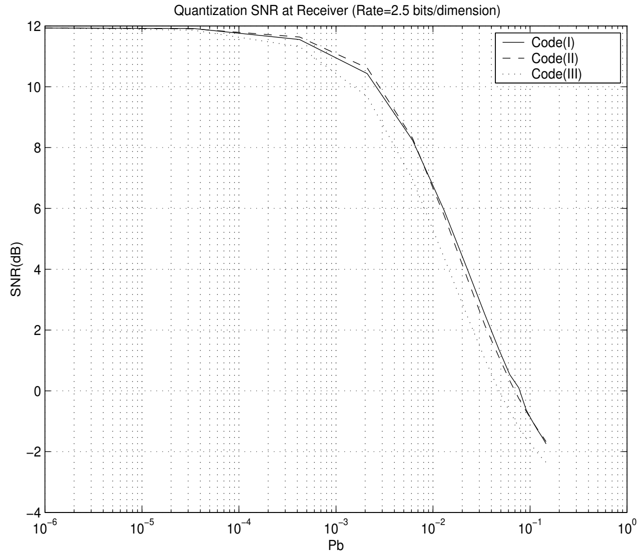

Fig. 2. Comparison of soft decoding for different Huffman codes as a function of bit error probability of BPSK channel, P_b.

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