On the Capacity of MIMO Rician Broadcast Channels

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***Abstract*—In this paper, a downlink communication system, in which a Base Station (BS) equipped with M antennas communicates with N ($N \gg 1$) single-antenna users, in a Rician fading environment is considered. The asymptotic (in terms of the number of users) sum-rate capacity of the system, as well as the capacity-achieving strategies, are derived. The main results of the paper are as follows: i) in the region of $\mathcal{K} = o(\log N)$, where $\mathcal{K}$ denotes the *Rician factor*, the sum-rate capacity scales as $M \log(1 + \frac{P}{M}\eta)$, where P denotes the SNR and $\eta \triangleq \frac{\log N}{1+\mathcal{K}}$, which is achieved by Zero-Forcing Beam-Forming (ZFBF) along with a low-complexity user selection algorithm that considers only the scattered component of the users' channels, ii) in the region $\mathcal{K} = \omega(\log N)$, in the case of co-located transmit antennas, the capacity scales as $\log(1 + MP)$, which is achieved by Time Division Multiple Access (TDMA), iii) in the region $\mathcal{K} = \omega(\log N)$, in the case of isotropically-distributed specular components, the sum-rate capacity behaves as $M \log(1 + P)$, which is achieved by ZFBF, along with a user selection algorithm that considers only the specular component of the users' channels.**

I. Introduction

Multiple-input multiple-output (MIMO) systems have proved their ability to achieve high bit rates on a scattering wireless network [1], [2]. In a MIMO Broadcast Channel (MIMO-BC), the base station equipped with multiple antennas communicates with several users. Recently, there has been a lot of interest in characterizing the capacity region of this channel [4]–[7]. In these works, it has been demonstrated that the sum-rate capacity of MIMO broadcast channels can be achieved by applying Dirty-Paper Coding (DPC) [8] at the transmitter.

Despite the fact that the sum-rate capacity of Gaussian MIMO-BC is known, it is still interesting to study the behavior of sum-rate capacity in various scenarios. [9] compares the achievable sum-rate of MIMO-BC for DPC to that achieved by using linear precoding schemes, and characterizes the gap between the achievable sum-rates in the high SNR regime. [10] compares the achievable sum-rate of DPC to that of TDMA for a Gaussian MIMO-BC. [11] considers a MIMO-BC with a large number of users and shows that i) the sum-rate capacity of the system scales as $M \log \log N$, when N is the number of users in the network, and ii) a simple scheme of "Random Beam-Forming" asymptotically achieves the sum-rate capacity as $N \to \infty$. References [12]–[14] consider the same network set-up and prove that one can achieve the sum-rate capacity of the system by utilizing Zero-Forcing Beam-Forming at the transmitter, provided that the user selection is performed wisely. In [15] the scaling laws of the sum-rate for fading MIMO Gaussian broadcast channels using time-sharing to the strongest user, DPC and beamforming, is derived for the asymptotic case of $N \to \infty$. In all the mentioned papers ([9]-[15]), the channel model is assumed to be Rayleigh fading. Therefore, it is of interest to investigate the sum-rate capacity of MIMO-BC, assuming more general channel models.

One of the most widely-used models for the wireless channels is Rician fading. This model is suitable for wireless links when there is a line of sight (LOS) link between the transmitter and receiver. Several papers in the literature consider Rician fading in the context of point-to-point MIMO communications. In [16], the authors derive the exact capacity of MIMO Rician channel, when perfect Channel State Information (CSI) is available at the receiver, but the transmitter has neither instantaneous nor statistical CSI. Reference [17] studies the capacity of MIMO Rician channel in the high and low SNR regimes, for both coherent and non-coherenet communications. it is shown in [17] that in the low SNR regime, the specular component of the channel completely determines the form of the optimum signal whereas in the high SNR regime it has no effect on the optimum signal structure. In [18], the authors consider the min-capacity of a MIMO Rician channel with an unknown deterministic specular component. [19] analyzes the capacity of a MIMO Rician channel with isotropically random rank-one specular component, when the channel is unknown at both the transmitter and receiver sides.

In this paper, we consider a Rician MIMO-BC, in which a transmitter equipped with M antennas communicates with N ($N \gg 1$) single-antenna users. The channels are assumed to be perfectly known at both the transmitter and receiver sides. The asymptotic (in terms of the number of users) sum-rate capacity of the system, as well as the capacity-achieving strategies, are derived. The main results of the paper are as follows: i) in the region of $\mathcal{K} = o(\log N)$, where $\mathcal{K}$ denotes the *Rician factor*, the sum-rate capacity scales as $M \log(1 + \frac{P}{M}\eta)$, where P denotes the SNR and $\eta \triangleq \frac{\log N}{1+\mathcal{K}}$, which is achieved by ZFBF along with a low-complexity user selection algorithm that considers only the scattered component of the users' channels, ii) in the region $\mathcal{K} = \omega(\log N)$, in the case of co-located transmit antennas, the capacity scales as $\log(1+MP)$, which is achieved by TDMA, iii) in the region $\mathcal{K} = \omega(\log N)$, in the case of isotropically-distributed specular components, the sum-rate capacity behaves as $M\log(1+P)$, which is achieved by ZFBF, along with a user selection algorithm that considers only the specular component of the users' channels.

This paper is organized as follows. In section II, we introduce the system model. Section III is devoted to analyzing the sum-rate capacity of the underlying system, and section IV concludes the paper.

Throughout this paper, the norm of the vectors are denoted by $\|.\|$, the Hermitian operation is denoted by $(.)^H$, and the determinant and the trace operations are denoted by $|.|$ and $\mathrm{Tr}(.)$, respectively. $\mathbb{E}\{.\}$ represents the expectation, notation "log" is used for the natural logarithm, and the rates are expressed in *nats*. For any given functions $f(N)$ and $g(N)$, $f(N) = O(g(N))$ is equivalent to $\lim_{N\to\infty}\left|\frac{f(N)}{g(N)}\right| < \infty$, $f(N) = o(g(N))$ is equivalent to $\lim_{N\to\infty}\left|\frac{f(N)}{g(N)}\right| = 0$, $f(N) = \Omega(g(N))$ is equivalent to $\lim_{N\to\infty}\frac{f(N)}{g(N)} > 0$, $f(N) = \omega(g(N))$ is equivalent to $\lim_{N\to\infty}\frac{f(N)}{g(N)} = \infty$, and $f(N) = \Theta(g(N))$ is equivalent to $\lim_{N\to\infty}\frac{f(N)}{g(N)} = c$, where $0 < c < \infty$, and $f(N) \sim g(N)$ is equivalent to $\lim_{N\to\infty}\frac{f(N)}{g(N)} = 1$.

II. System Model

In this work, a MIMO-BC in which a base station equipped with M antennas communicates with N users, each equipped with a single antenna, is considered. The received signal by user *k* can be written as

$$y_k = \mathbf{H}_k\mathbf{x} + n_k, \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the transmitted signal, $\mathbf{H}_k \in \mathbb{C}^{1\times M}$ is the channel vector from the transmitter to the kth user, which is assumed to be perfectly known at the receiver side and provided to the BS via a noiseless feedback channel [1], and $n_k \sim \mathcal{CN}(0,1)$ is the AWGN at this receiver.

Under Rician channel model, $\mathbf{H}_k$ can be written as

$$\mathbf{H}_k = \sqrt{1-r_k}\mathbf{G}_k + \sqrt{r_k M}\mathbf{b}_k, \tag{2}$$

where $\mathbf{G}_k$ is a circularly symmetric zero mean unit variance Gaussian vector, reflecting the scattered component and $\mathbf{b}_k$ is a unit-norm vector representing the specular component of the channel, and r_k is a constant related to the Rician factor $\mathcal{K}_k$ [2] via $r_k = \frac{\mathcal{K}_k}{\mathcal{K}_k+1}$. We consider two scenarios for $\mathbf{b}_k$: (i) The entries of $\mathbf{H}_k$ are i.i.d Gaussian with mean b_k and variance $1-|b_k|^2$, where b_k is a complex number satisfying $|b_k|^2 = r_k$. In this case, it is easy to observe that $\mathbf{b}_k = \frac{e^{j\theta_k}}{\sqrt{M}}\mathbf{1}$, where $\mathbf{1}$ is the vector of all ones. This model corresponds to the case that the transmit antennas are co-located, and consequently, the specular components from all transmit antennas to each of the users are equal [3] . ii) The vector $\mathbf{b}_k$ is isotropically distibuted in the unit sphere. This model has been used in [19]. It is assumed that r_k is fixed for all the users during the whole transmission period and is equal to a constant r, i.e., $r_1 = r_2 = \cdots = r_N = r$.

We assume that the transmitter has an average power constraint P, i.e. $\mathbb{E}\{\mathrm{Tr}(\mathbf{x}\mathbf{x}^*)\} \leq P$. The power constraint is assumed to be *per frame*. In other words, the power constraint is independent of the channel realization. The channels are assumed to be quasi-static block fading, in which each channel $\mathbf{H}_k$ is drawn randomly at the start of each transmission frame and remains constant for the whole transmission frame, and changes independently to another realization in the start of the next frame. The frame itself is assumed to be long enough to allow communication at rates close to the capacity. Defining the sum-rate capacity of the system in the channel realization $\mathcal{H} \triangleq \{\mathbf{H}_k\}_{k=1}^N$ as $\mathcal{R}_{\mathrm{Opt}}(\mathcal{H})$, the average sum-rate capacity, denoted as $\mathcal{R}_{\mathrm{Opt}}$, is defined as the average over time of $\mathcal{R}_{\mathrm{Opt}}(\mathcal{H})$, which is by the ergodicity of the channel, equal to $\mathbb{E}_{\mathcal{H}}\{\mathcal{R}_{\mathrm{Opt}}(\mathcal{H})\}$. $\mathcal{R}_{\mathrm{Opt}}$ is shown in [4] to be equal to

$$\mathcal{R}_{\mathrm{Opt}} = \mathbb{E}_{\mathcal{H}}\left\{\max_{\substack{P_k \\ \sum P_k = P}} \log\left|\mathbf{I}_M + \sum_{k=1}^{N}\mathbf{H}_k^* P_k \mathbf{H}_k\right|\right\}, \tag{3}$$

where P_k is the transmit power allocated to the kth user.

III. Asymptotic Analysis; Capacity Computation

In this section, we compute the capacity of MIMO-BC under Rician fading, in the asymptotic scenario of $N \to \infty$. To this end, we consider two cases; (i) $\mathcal{K} = o(\log N)$ and (ii) $\mathcal{K} = \omega(\log N)$. For each case, we provide a lower-bound and upper-bound for the capacity and prove that as $N \to \infty$, these bounds converge to each other.

A. $\mathcal{K} = o(\log N)$

Theorem 1 *The capacity of the underlying MIMO-BC in the case of $\mathcal{K} = o(\log N)$ equals*

$$\mathcal{R}_{\mathrm{Opt}} = M\log\left(1 + \frac{P}{M}\frac{\log N}{1+\mathcal{K}}\right) + o(1), \tag{4}$$

which is asymptotically achievable by ZFBF.

Proof - The proof is based on the upper-bound and lower-bound given as follows:

[1]In fact, the BS does not need to have the perfect CSI about all the users' channels. However, the partial CSI that the BS receives through feedback is based on the perfect CSI that the receivers have.

[2]*Rician factor* is defined as the ratio of the power of the specular component to the power of the scattered component.

[3]Note that however, the specular components from each transmit antenna to different users are not necessarily equal.

1) Upper-bound: Using (2), the upper-bound for the sum-rate capacity can be derived as [4]

$$\begin{aligned}\mathcal{R}_{\text{Opt}} &\leq M\mathbb{E}\left\{\log\left(1+\frac{P}{M}\|\mathbf{H}\|_{\max}^2\right)\right\}\\ &\leq M\log\left(1+\frac{P}{M}\frac{1}{1+\mathcal{K}}\mathbb{E}\left\{\|\mathbf{G}\|_{\max}^2\right\}\right)+\\ &\quad M\mathbb{E}\left\{\frac{2\sqrt{\mathcal{K}M}\|\mathbf{G}\|_{\max}}{\frac{M}{P}(1+\mathcal{K})+\|\mathbf{G}\|_{\max}^2}\right\}+\\ &\quad M\mathbb{E}\left\{\frac{\mathcal{K}M}{\frac{M}{P}(1+\mathcal{K})+\|\mathbf{G}\|_{\max}^2}\right\}. \qquad (5)\end{aligned}$$

In [20], it is shown that

$$\mathbb{E}\left\{\frac{2\sqrt{\mathcal{K}M}\|\mathbf{G}\|_{\max}}{A+\|\mathbf{G}\|_{\max}^2}\right\}=o(1), \qquad (6)$$

and

$$\mathbb{E}\left\{\frac{\mathcal{K}M}{\frac{M}{P}(1+\mathcal{K})+\|\mathbf{G}\|_{\max}^2}\right\}=o(1). \qquad (7)$$

Substituting in (5), the upper-bound on the sum-rate capacity can be written as

$$\begin{aligned}\mathcal{R}_{\text{Opt}} &\leq M\log\left(1+\frac{P}{M}\frac{1}{1+\mathcal{K}}\mathbb{E}\left\{\|\mathbf{G}\|_{\max}^2\right\}\right)+o(1)\\ &= M\log\left(1+\frac{P\log N}{M(1+\mathcal{K})}\right)+o(1), \qquad (8)\end{aligned}$$

where the second line follows from the fact that $\mathbb{E}\left\{\|\mathbf{G}\|_{\max}^2\right\}=\log N+O(\log\log N)$ [11].

2) Achievability: Scheduling based on the scattered component: Consider the follwoing algorithm:
Algorithm 1

- Set the threshold $t=\log N+(M-3)\log\log N$
- Among the users in the following set:

  $$\mathcal{S}\triangleq\{k|\ \|\mathbf{G}_k\|^2>t\}, \qquad (9)$$

  select one user at random. Call this user s_1, and define $\mathcal{S}_1\triangleq\mathcal{S}-\{s_1\}$.
- For $m=2$ to M, repeat the following:
  - Denote the set of selected users up to the $(m-1)$th step as $\mathcal{A}_m\triangleq\{s_1,\cdots,s_{m-1}\}$. Define $\mathcal{S}_m\triangleq\mathcal{S}-\mathcal{A}_m$.
  - Define $\boldsymbol{\mathcal{P}}_m$ as the sub-space spanned by the scattered channel components of the users selected in the previous steps, i.e., $\{\mathbf{v}_{s_j}\}_{j=1}^{m-1}$, where $\mathbf{v}_k\triangleq\frac{\mathbf{G}_k}{\|\mathbf{G}_k\|}$, $k=1,\cdots,N$.
  - Let $\{\mathbf{\Phi}_j\}_{j=1}^{m-1}$ be $m-1$ orthonormal bases for $\boldsymbol{\mathcal{P}}_m$. Then,

    $$s_m=\arg\min_{k\in\mathcal{S}_m}\sum_{j=1}^{m-1}\left|\mathbf{v}_k\mathbf{\Phi}_j^H\right|. \qquad (10)$$

[4]For the details of the proofs, the reader is referred to [20].

In the above algorithm, the user selection is solely performed based on the scattered component of the channel. First, the users with scattered channel gains above the threshold t are candidated. After that, the algorithm tries to find a set of semi-orthogonal channel vectors out of the candidate users. To this end, at each step of the algorithm, the user whose channel vector is the most orthogonal to the sub-space spanned by the previously selected users is chosen. After selecting the users, the BS performs ZFBF on the (whole) channel vectors of the selected users. Defining $\mathbb{H}\triangleq\left[\mathbf{H}_{s_1}^T|\cdots|\mathbf{H}_{s_M}^T\right]^T$ and $\mathbf{u}=[u_1,\cdots,u_M]^T$ as the information vector for the selected users, we have

$$\mathbf{x}=\mathbb{H}^{-1}\mathbf{u}. \qquad (11)$$

Therefore, the achievable sum-rate of this scheme can be written as

$$\mathcal{R}=M\mathbb{E}_{\mathbb{H}}\left\{\log\left(1+\frac{P}{\text{Tr}\left\{\left[\mathbb{H}^H\mathbb{H}\right]^{-1}\right\}}\right)\right\}. \qquad (12)$$

Defining $\mathfrak{B}$ as the event that $L\triangleq|\mathcal{S}|>\log N$, $\mathfrak{C}$ as $\delta(\mathbb{G}^H)>1+2M(\log N)^{-\frac{1}{2(M-1)}}$, in which $\mathbb{G}\triangleq\left[\mathbf{G}_{s_1}^T|\cdots|\mathbf{G}_{s_M}^T\right]^T$ and $\delta(\mathbf{A})$ denotes the orthogonality defect [21] of $\mathbf{A}$, and $\mathfrak{D}$ as the event that $\|\mathbf{G}\|_{\max}^2\leq t^+$, where $t^+\triangleq\log N+M\log\log N$ and $\|\mathbf{G}\|_{\max}^2\triangleq\max_k\|\mathbf{G}_k\|^2$, we have

$$\begin{aligned}\mathcal{R} &\geq M\mathbb{E}_{\mathbb{H}}\left\{\log\left(1+\frac{P}{\text{Tr}\left\{\left[\mathbb{H}^H\mathbb{H}\right]^{-1}\right\}}\right)\middle|\mathfrak{B},\mathfrak{C},\mathfrak{D}\right\}\times\\ &\quad \Pr\{\mathfrak{B},\mathfrak{C},\mathfrak{D}\}. \qquad (13)\end{aligned}$$

In [20], it has been shown that $\Pr\{\mathfrak{B},\mathfrak{C},\mathfrak{D}\}=1+o\left(\frac{1}{\log N}\right)$ and conditioned on $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$,

$$\text{Tr}\left\{\left[\mathbb{H}^H\mathbb{H}\right]^{-1}\right\}\leq\frac{M(\mathcal{K}+1)}{\log N}\left[1+O\left((\log N)^{-\mu}\right)\right], \qquad (14)$$

where $\mu\triangleq\frac{1}{4(M-1)}$. Substituting in (13) yields

$$\mathcal{R}\geq M\log\left(\frac{P\log N}{M(1+\mathcal{K})}\right)+O\left((\log N)^{-\mu}\right). \qquad (15)$$

Since $\mathcal{K}=o(\log K)$, it follows that $\log\left(\frac{P\log N}{M(1+\mathcal{K})}\right)=\log\left(1+\frac{P\log N}{M(1+\mathcal{K})}\right)+o(1)$. Noting this fact and comparing the above lower-bound with the upper-bound derived in (8) the proof of Theorem 1 follows.

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B. $\mathcal{K}=\omega(\log N)$

1) Co-located transmit antennas: In this scenario, the specular components from all transmit antennas to each receiver are equal. In other words, $\mathbf{b}_k=\frac{e^{i\theta_k}}{\sqrt{M}}\mathbf{1}_M$, where $\mathbf{1}_M$ is the all-one vector with size M. However, the scattered component of all users' channels follow the circularly symmetric complex Gaussian distribution. The following theorem gives the capacity of MIMO-BC in this scenario:

Theorem 2 *The capacity of MIMO-BC in the case of* $\mathcal{K} = \omega(\log N)$ *and co-located transmit antennas scales as*

$$\mathcal{R}_{\text{Opt}} = \log(1 + MP) + o(1), \tag{16}$$

which is achievable by TDMA.

Proof - Similar to the proof of Theorem 1, we first give an upper-bound on the sum-rate capacity and then, by giving an achievable rate which is asymptotically equal to the upper-bound the theorem is proved.

Upper-bound: Writing the sum-rate capacity of MIMO-BC from (3), we have

$$\begin{aligned}\mathcal{R}_{\text{Opt}} &= \mathbb{E}\left\{ \max_{\substack{P_k \\ \sum P_k = P}} \log\det\left(\mathbf{I}_M + \sum_{k=1}^{N} \mathbf{H}_k^H P_k \mathbf{H}_k\right)\right\} \\ &= \mathbb{E}\Bigg\{ \max_{\substack{P_k \\ \sum P_k = P}} \log\left|\mathbf{I}_M + rM\sum_{k=1}^{N} \mathbf{b}_k^H P_k \mathbf{b}_k\right| + \\ &\quad \log\left|\mathbf{I}_M + \mathbf{QP}\right| \Bigg\} \\ &\leq \log\left(1 + PM\right) + M\mathbb{E}\Bigg\{ \max_{\substack{P_k \\ \sum P_k = P}} \log\left|\mathbf{I}_M + \mathbf{QP}\right| \Bigg\}\end{aligned} \tag{17}$$

where $\mathbf{Q} \triangleq \sqrt{r(1-r)M}\sum_{k=1}^{N}\left[\mathbf{G}_k^H P_k \mathbf{b}_k + \mathbf{b}_k^H P_k \mathbf{G}_k\right] + (1-r)\sum_{k=1}^{N}\mathbf{b}_k^H P_k \mathbf{b}_k$, and $\mathbf{P} \triangleq \left(\mathbf{I}_M + rP\mathbf{1}_M^H\mathbf{1}_M\right)^{-1}$. Denoting the second term in the right hand side of the above equation by R_2, in [20], it has been shown that

$$R_2 = o(1). \tag{18}$$

Consequently,

$$\mathcal{R}_{\text{Opt}} \leq \log(1 + PM) + o(1). \tag{19}$$

Achievability - In order to show that the sum-rate given in (16) is achievable, we propose a TDMA scheme, in which the transmitter (randomly) selects one user at a time and communicates with that user. Therefore, the maximum achievable rate is equal to the capacity of a MISO Rician channel, expressed as bellow:

$$\begin{aligned}\mathcal{R} &= \mathbb{E}\left\{\log\left(1 + P\|\mathbf{H}_k\|^2\right)\right\} \\ &\geq \mathbb{E}\left\{\log\left(1 + P\left|\sqrt{rM} - \sqrt{1-r}\|\mathbf{G}_k\|\right|^2\right)\right\}\end{aligned} \tag{20}$$

Let us define $\mathfrak{E}$ as the event that $\|\mathbf{G}_k\|^2 < \log N$. $\mathcal{R}$ can be lower-bounded as

$$\begin{aligned}\mathcal{R} &\geq \mathbb{E}\left\{\log\left(1 + P\left|\sqrt{rM} - \sqrt{1-r}\|\mathbf{G}_k\|\right|^2\right)\middle|\,\mathfrak{E}\right\} \times \\ &\quad \Pr\left\{\mathfrak{E}\right\} \\ &\overset{(a)}{=} \log(1 + PM) + o(1).\end{aligned} \tag{21}$$

In the above equation, (a) follows from i) the assumption of $\mathcal{K} = \omega(\log N)$, which implies that conditioned on $\mathfrak{E}$, $\sqrt{1-r}\|\mathbf{G}_k\| = \frac{\|\mathbf{G}_k\|}{\sqrt{\mathcal{K}+1}} = o(1)$, ii) as $\|\mathbf{G}_k\|^2$ has Chi-Square distribution with $2M$ degrees of freedom, $\Pr\{\mathfrak{E}\} \sim \frac{\log^M(N)}{(M-1)!N} = o(1)$ and iii) as $r = \frac{\mathcal{K}}{\mathcal{K}+1}$ and $\mathcal{K} = \omega(\log N)$, we have $r = 1 + o\left(\frac{1}{\log N}\right)$. This completes the proof of achievability and hence, the proof of Theorem 2.

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2) Isotropic specular components: In this case, it is assumed that the specular component of all users' channels, i.e., $\mathbf{b}_k$, $k = 1, \cdots, N$, are isotropically distributed in the unit sphere. The difference between this case and the previous case is that in the case of co-located transmit anennas, there is only one available coordinate in the system (the coordinate of $\mathbf{1}_M$) for transmission, and as a result, we do not have the M-fold capacity increase. However, in this case, by wisely selecting the users one can achieve the M-fold capacity increase. The following theorem gives the capacity in this case:

Theorem 3 *The capacity of Rician MIMO-BC in the case of* $\mathcal{K} = \omega(\log N)$ *and isotropic specular components is equal to*

$$\mathcal{R}_{\text{Opt}} = M\log(1 + P) + o(1). \tag{22}$$

Proof - *Upper-bound:*

$$\begin{aligned}\mathcal{R}_{\text{Opt}} &\leq M\mathbb{E}\left\{\log\left(1 + \frac{P}{M}\|\mathbf{H}\|_{\max}^2\right)\right\} \\ &\leq M\mathbb{E}\left\{\log\left(1 + \frac{P}{M}\left|\sqrt{rM} + \sqrt{1-r}\|\mathbf{G}\|_{\max}\right|^2\right)\right\} \\ &\overset{(a)}{\leq} M\log\left(1 + \frac{P}{M}\mathbb{E}\left\{\left|\sqrt{rM} + \sqrt{1-r}\|\mathbf{G}\|_{\max}\right|^2\right\}\right) \\ &\overset{(b)}{=} M\log\left(1 + \frac{P}{M}\left|\sqrt{rM} + o(1)\right|^2\right) \\ &= M\log(1 + rP) + o(1) \\ &\overset{(c)}{=} M\log(1 + P) + o(1).\end{aligned} \tag{23}$$

In the above equation, (a) follows from the concavity of $\log$ function along with the Jensen's inequality, (b) results from the fact that $\|\mathbf{G}\|_{\max} = O(\log N)$ and since $1 - r = \frac{1}{1+\mathcal{K}} = o\left(\frac{1}{\log N}\right)$, we have $\sqrt{1-r}\|\mathbf{G}\|_{\max} = o(1)$, and (c) results from $r = 1 + o(1)$.

Achievability; Scheduling based on specular component Consider the following algorithm:
Algorithm 2

- select one user at random. Call this user s_1, and define $\mathcal{S}_1 \triangleq \mathcal{S} - \{s_1\}$.
- For $m = 2$ to M, repeat the following:
  - Denote the set of selected users up to the $(m-1)$th step as $\mathcal{A}_m \triangleq \{s_1, \cdots, s_{m-1}\}$. Define $\mathcal{S}_m \triangleq \mathcal{S} - \mathcal{A}_m$.
  - Define $\boldsymbol{\mathcal{P}}_m$ as the sub-space spanned by the specular channel components of the users selected in the previous steps, i.e., $\{\mathbf{b}_{s_j}\}_{j=1}^{m-1}$.

– Let $\{\mathbf{\Phi}_j\}_{j=1}^{m-1}$ be $m-1$ orthonormal bases for $\boldsymbol{\mathcal{P}}_m$. Then,

$$s_m = \arg\min_{k\in\mathcal{S}_m} \sum_{j=1}^{m-1} \left|\mathbf{b}_k \mathbf{\Phi}_j^H\right|. \quad (24)$$

- After selecting the users, the BS performs zero-forcing beam-forming on the (whole) channel vectors of the selected users. Defining $\mathbb{H} \triangleq \left[\mathbf{H}_{s_1}^T | \cdots | \mathbf{H}_{s_M}^T\right]^T$ and $\mathbf{u} = [u_1, \cdots, u_M]^T$ as the information vector for the selected users, we have

$$\mathbf{x} = \mathbb{H}^{-1}\mathbf{u}. \quad (25)$$

Defining the event $\mathfrak{F} \triangleq \{\delta(\mathbb{B}^H) < 1+\epsilon\}$ and $\mathfrak{Q} \triangleq \left\{\text{Tr}\{\mathbb{G}^H\mathbb{G}\} < \log N\right\}$, where $\mathbb{B} = \left[\mathbf{b}_{s_1}^T | \cdots | \mathbf{b}_{s_M}\right]^T$, $\mathbb{G} = \left[\mathbf{G}_{s_1}^T | \cdots | \mathbf{G}_{s_M}\right]^T$, and $\epsilon \triangleq 2MN^{-\frac{1}{2(M-1)}}$, similar to (13), we have

$$\mathcal{R} \geq M\mathbb{E}_{\mathbb{H}}\left\{\log\left(1 + \frac{P}{\text{Tr}\left\{\left[\mathbb{H}^H\mathbb{H}\right]^{-1}\right\}}\right)\middle| \mathfrak{F}, \mathfrak{Q}\right\} \times \Pr\{\mathfrak{F}, \mathfrak{Q}\}. \quad (26)$$

In [20], it has been shown that $\Pr\{\mathfrak{F}, \mathfrak{Q}\} = 1 + o\left(\frac{1}{\sqrt{N}}\right)$ and conditioned on $\mathfrak{F}, \mathfrak{Q}$, $\text{Tr}\left\{\left[\mathbb{H}^H\mathbb{H}\right]^{-1}\right\} \leq 1 + o(1)$. Substituting in (26), we have

$$\mathcal{R} \geq M\log(1+P) + o(1), \quad (27)$$

which completes the proof of Theorem 3.

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Remark - Comparing the sum-rate capacity of the system in the two cases of co-located transmit antennas and isotropic specular components when $\mathcal{K} = \omega(\log N)$, it follows that in the first case, the capacity grows logarithmically with M, while in the second case it scales linearly with M. Moreover, since $(1+x)^M > 1 + Mx$, $\forall x, M$, it follows that

$$\mathcal{R}_{\text{Opt}}^{isotropic} \geq \mathcal{R}_{\text{Opt}}^{co-located}. \quad (28)$$

IV. Conclusion

In this paper, we have derived the asymptotic sum-rate capacity of MIMO-BC with large number of users in a Rician fading environment. It is observed that in the region $\mathcal{K} = o(\log N)$, the capacity achieving strategy is exactly the same as the Rayleigh fading case. In the region $\mathcal{K} = \omega(\log N)$, the sum-rate capacity depends on the distribution of the specular component; in the case of co-located transmit antennas, it is demonstrated that TDMA achieves the sum-rate capacity and the capacity grows logarithmically with the number of transmit antennas. In the case of isotropically distributed specular components, ZFBF along with a user selection strategy which selects M users with semi-orthogonal specular components is shown to be optimum. Moreover, the sum-rate capacity grows linearly with the number of transmit antennas.

References

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. on Telecommunications*, vol. 10, no. 6, pp. 585– 595, November 1999.

[2] G. J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311–335, 1998.

[3] G. Caire and S. Shamai, "On the achievable throughput of a multi-antenna Gaussian broadcast channel," *IEEE Trans. on Inform. Theory*, vol. 49, pp. 1691 1706,, July 2003.

[4] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. on Inform. Theory*, vol. 49, no. 10, pp. 2658–2668, October 2003.

[5] P. Viswanath and D. N. C. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Trans. on Inform. Theory*, vol. 49, pp. 1912–1921, Aug. 2003.

[6] Wei Yu and John Cioffi, "Sum capacity of vector Gaussian broadcast channels," *IEEE Trans. on Inform. Theory*, vol. 50, pp. 1875–1892, Sep. 2004.

[7] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian MIMO broadcast channel," in *Conference on Information Sciences and Systems*, 2004.

[8] M. Costa, "Writing on dirty paper," *IEEE Trans. on Inform. Theory*, vol. 29, pp. 439–441, May 1983.

[9] J. Lee and N. Jindal, "High SNR Analysis for MIMO Broadcast Channels: Dirty Paper Coding vs. Linear Precoding," *IEEE Trans. on Inform. Theory*, Vol. 53, No. 12, pp. 4787-4792, Dec. 2007.

[10] N. Jindal and A. Goldsmith, "Dirty paper coding vs. TDMA for mimo broadcast channels," in *IEEE Int. Conf. Commun.*, June 2004, vol. 2, pp. 682–686.

[11] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channel with partial side infonnation," *IEEE Trans. on Inform. Theory*, vol. 51, pp. 506–522, Feb. 2005.

[12] Alireza Bayesteh and Amir K. Khandani, "On the user selection for MIMO broadcast channels," *To Appear in IEEE Trans. Inform. Theory*, March 2008, Available online at http://cst.uwaterloo.ca/, Technical Report UW-ECE 2005-16.

[13] Taesang Yoo and Andrea Goldsmith, "On the optimality of Multi-Antenna Broadcast Scheduling Using Zero-Forcing Beamforming," *IEEE JSAC Special Issue on 4G Wireless Systems*, Vol. 24, No. 3, March 2006.

[14] C. Swannack, E. Uysal-Biyikoglu, and G. W. Wornell, "Low Complexity Multiuser Scheduling for Maximizing Throughput in the MIMO Broadcast Channel," *in Proc. Allerton Conf. Commun., Contr., and Computing*, Oct. 2004.

[15] M. Sharif and B. Hassibi, "A comparison of time-sharing, DPC, and beamforming for MIMO broadcast channels with many users," *IEEE Trans. on Communications*, Vol. 55, pp. 11-15, Jan. 2007.

[16] Ming Kang and Mohamed-Slim Alouini, "Capacity of MIMO Rician Channels," *Trans. on Wireless Communi.*, Vol. 50, pp. 112-122, Jan. 2006.

[17] M. Godavarti and A. O. Hero III, "Multiple antenna capacity in a deterministic Rician fading channel," *Submitted to IEEE Trans. on Information Theory*

[18] Mahesh Godavarti, Alfred O. Hero, III, and Thomas L. Marzetta, "Min-Capacity of a Multiple-Antenna Wireless Channel in a Static Ricean Fading Environment," *IEEE Trans. on Communi.*, Vol. 4, pp. 1715-1723, July 2005.

[19] Mahesh Godavarti, Thomas L. Marzetta, and Shlomo Shamai (Shitz), "Capacity of a Mobile Multiple-Antenna Wireless Link With Isotropically Random Rician Fading," *IEEE Transactions on Information Theory*, Vol. 49, pp. 3330-3334, Dec. 2003.

[20] A. Bayesteh, Kamyar Moshksar, and Amir K. Khandani, "On the Capacity of MIMO Rician Broadcast Channels," Preprint, *Also available online at www.cst.uwaterloo.ca.*

[21] Mahmoud Taherzadeh, Amin Mobasher, and Amir Keyvan Khandani, "Communication over MIMO broadcast channels using lattice-basis reduction," *IEEE Transactions on Information Theory*, Vol. 53, pp. 4567-4582, Dec. 2007.