Decentralized Cognition via Randomized Masking

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Abstract

A decentralized network of one Primary User (PU) and several Secondary Users (SU) is studied. The number of SUs is modeled by a random variable with a globally known distribution. PU is licensed to exploit the resources, while the party of SUs intend to share the resources with PU. Each SU must guarantee to not disturb the performance of PU beyond a certain level, while maintaining a satisfactory quality of service for itself. It is proposed that each secondary transmitter adopts a Randomized Masking (RM) strategy where it remains silent or transmits a symbol in its codeword independently from transmission slot to transmission slot. We consider a setup where the primary transmitter is unaware of the channel gains, the code-book of the secondary users and the number of secondary users. Although the SUs are anonymous to each other, i.e, they are unaware of each other’s code-book, however, each SU is smart in the sense that it is aware of the code-book of PU, the channel gains and the number of active SUs. Invoking the concept of $\epsilon$-outage capacity, we define the $\epsilon$-admissible region as the set of possible transmission rates for PU and possible masking probabilities for each SU such that the probability of outage for PU is maintained under a threshold $\epsilon$. Thereafter, the transmission rate of PU and the masking probability of SUs are designed through maximizing a globally known utility function of the rates of users over the $\epsilon$-admissible region. In our analysis, the primary receiver treats interference as noise, however, each secondary receiver has the option to decode interference caused by PU, while treating the signals of other SUs as noise. In another scenario, referred to as Power Allocation (PA), each SU transmits continuously (no masking applied), however, it regulates its transmission power yielding the largest utility value. It is demonstrated that the PA scheme is more appropriate in the regime that the transmission power of PU is relatively low, while the RM scheme outperforms PA for comparably larger values of the transmission power of PU.
I. INTRODUCTION

A. Related Work

Increasing demand for wireless applications on one hand, and the limited available resources on the other hand, provoke more efficient usage of such resources. Due to its significance, many researchers have addressed the problem of resource allocation in wireless networks. One major challenge in wireless networks is the destructive effect of multiuser interference, which degrades the performance when multiple users share the spectrum. As such, an efficient and low complexity resource allocation scheme that maximizes the quality of service while mitigating the impact of the multiuser interference is desirable. Due to the complexity of adapting to the network structure (e.g. number of active users), centralized resource allocation schemes are usually designed for a fixed network structure. This makes inefficient usage of resources because, in most cases, the number of active users may be considerably less than the value assumed in the design process. Therefore, it is of interest to devise an efficient and low-complexity decentralized resource allocation scheme. In decentralized schemes, decisions concerning network resources are made by individual nodes based on their local information. Most of decentralized schemes reported in the literature rely on either game-theoretic approaches or cognitive radios.

Distributed strategies based on game theoretic arguments have already attracted a great deal of attention [1]–[5]. In [1], the authors introduce a non-cooperative game theoretic framework to investigate the spectral efficiency issue when several users compete over an unlicensed band with no central controller. This approach has been applied to the case where users have incomplete information about the channel gains in the network [2]. This setup is more relevant to a fast fading environment. Reference [3] offers a brief overview of game theoretic dynamic spectrum sharing. Repeated non-cooperstive market game methods are adopted in [4] for resource allocation in a decentralized network. In [5] the interaction between multiple random access networks has been considered from a game theoretic point of view. Another scenario is sharing the spectrum by a certain number of users competing over a certain open bandwidth [6]–[9]. In [8], through an asynchronous distributed pricing scheme, users exchange signals that indicate the negative effect of interference at the receivers. In [9], users affected by the mobility event, self-organize into bargaining groups and adapt their spectrum assignment to approximate a new optimal assignment.

Cognitive radios [11] are flexible devices that have the ability to sense the unoccupied portion of the available spectrum and use this information in resource allocation. In its prime definition, a cognitive transmitter was assumed to only transmit over the white spaces, i.e., the cognitive transmitter remains
silent until the so-called PUs become silent. The model suggested in [11] is a landmark in the literature on cognitive radios where it is proposed that in order to start its transmission, a cognitive transmitter need not to wait for the PU to become idle. In fact, the cognitive transmitter can transmit simultaneously with the PU as far as it adopts an elegant transmission scheme and designs its code-book properly. In [11] the network of one PU and one cognitive user is modeled by a two-user Interference Channel (IC) such that the cognitive user is aware of the message of the PU. Two scenarios are considered, referred to as non-causal and causal cognitive networks:

1- In the non-causal case, the cognitive transmitter is aware of the PU’s message non-causally through the aid of a ”genie”. In this case, a combined signaling strategy is proposed by invoking Gel’fand-Pinker coding [12] (or its counterpart in the additive Gaussian noise channel called Dirty Paper Coding (DPC) [13]) and the Han-Kobayashi scheme [14]. In fact, the cognitive transmitter utilizes its knowledge of the PU’s message to mitigate the interference on its receiver and relaying a replica of the PU’s message to the primary receiver.

2- In the causal case, the users go through two different phases. In phase I, the cognitive transmitter is in the ”listening mode” where it discovers the message of the PU as the PU broadcasts its message to its receiver and the cognitive transmitter. In phase II, the cognitive transmitter proceeds as the non-causal scenario.

Following the model proposed in [11], the authors in [15] derive the largest achievable rate for the cognitive user under the following conditons:

1- The primary transmitter must achieve a transmission rate as large as the capacity of its forward point to point link as if the cognitive user was inactive.

2- The PU does not perform multiuser decoding.

This is an interesting setup that is in agreement with more realistic scenarios, because one may not expect any cooperation between the primary and cognitive users. However, the assumption that the cognitive user is aware of the PU’s message non-causally demands some sort of cooperation between the two users. In some scenarios if one can justify that the message sent by the PU is delayed along its forward link, then it is reasonable to assume that the cognitive transmitter can discover the PU’s message before it is received at the primary receiver. The observation in [15] is a capacity result only in the weak interference regime where the cognitive transmitter is closer to its affiliated receiver than the primary receiver. In fact, it is shown that multiuser decoding at the primary receiver does not improve the set of achievable rates attained...
by the users. Moreover, the authors demonstrate that interference decoding at the primary receiver leads to improving the set of achievable rates in the cognitive IC in the strong interference regime.

The result in [15] is in fact derived in a more general setup in a concurrent work in [16] where the authors look at the cognitive IC as a special case of an IC with Degraded Message Sets (IC-DMS). Inner and outer bounds are developed on a discrete memoryless IC-DMS. Generalizing the results to a Gaussian IC-DMS, the capacity region in the weak interference regime is established. The capacity region of the IC-DMS in the strong interference regime is studied in [17], [18].

The authors in [19] considers a causal cognitive radio channel in the context of interference channel with conferencing where it is assumed that the cognitive transmitter receives a feedback from the channel. Coding strategies such as block Markov coding and Gel’fand Pinsker coding and a backward decoding strategy are applied to derive an achievable rate region.

Motivated by the fact that the SUs must not disturb the performance of the PUs, an interesting approach is taken in [20] where the capacity region of wireless networks are studied under constraints on the received Signal-to-Noise Ratio (SNR) rather constraints on the power at the transmitters. The results in [20] are specialized to fading environments in [21].

Cognitive interference channels with more than one cognitive transmitter have rarely been studies as most attention is given to the cognitive interference channels with only one cognitive user. [22] considers a cognitive IC with several cognitive users. Adopting a coding technique based on nested lattices [23], it is established that in the strong interference regime, it is possible for all users to achieve rates as if the other users were absent. Some restrictive assumptions are made to simplify the presentation, e.g., the channel gains are symmetric and the cognitive users do not interfere with each other. [24] studies the achievable SNR scaling of the sum rate in a multiuser interference channel where each transmitter only knows the messages of other surrounding cognitive transmitters that are located in its vicinity. The so-called clustered local processing is allowed at the receivers where each receiver has access to the signals received by the nearby users.

B. Contributions

We consider a network of one primary and several SUs modeled by an interference channel. The number of SUs is a random variable with a globally known distribution. However, we assume the number of users is fixed along the communication period of interest once it is set first. The PU is dumb in the sense that:

1- The primary transmitter is unaware of the channel gains.
2- The PU is unaware of the code-book of each SU.
3- The primary transmitter is unaware of the presence of the SUs.

On the other hand, each SU is smart in the sense that:
1- Each SU is aware of the code-book of the PU.
2- Each SU is aware of the presence of the PU.
3- Each SU is aware of its forward channel gain and the gain of channels connecting other secondary transmitters to its receiver.

Note that we only assume each SU is aware of the PU’s code-book and not its message. This is in contrast to the assumption made in the literature on non-casual cognitive radios where the secondary transmitter is aware of the PU’s message. Therefore, the secondary transmitters in our setup are unable to perform DPC, however, the secondary receivers are capable of multiuser decoding.

Our setup for the cognitive network is a decentralized setup where the secondary transmitter only sneaks into the network and has to make sure the PU’s performance is always maintained at a satisfactory level, while the quality of service for itself is convincing as well. The primary transmitter’s strategy is to continuously transmit Gaussian codewords as if the SUs were not present at all. It is proposed that each secondary transmitter follows the RM scheme where it transmits with a probability of $\theta$ or remains silent with a probability of $1 - \theta$ independently from transmission slot to transmission slot. The parameter $\theta$ is called the activity factor for each SU. This is to provide the PU with a partially interference-free reception.

As the primary transmitter is unaware of the channel gains and the presence of SUs, outage probability is an appropriate measure of quality of service for this user. As such, we propose that the transmission rate $R_1$ of the PU and the activity factor $\theta$ of each SU be such that the probability of outage for the PU is kept less than a threshold $\varepsilon$. The $\varepsilon$-admissible region $A_\varepsilon$ is defined by as

$$A_\varepsilon = \{(\theta, R_1) : \Pr\{\text{Outage for PU}\} < \varepsilon\}.$$ 

The primary receiver treats the interference as noise. However, the secondary receivers have two options, i.e., treating the signal of the primary user as noise or performing multiuser detection. Assuming there are $N$ secondary users denoted by $SU_2, \cdots, SU_{N+1}$ with transmission rates $R_2, \cdots, R_{N+1}$, let $\mathcal{R}_i$ be the set of all $(R_1, R_i)$ such that one of the following happens

1- $(R_1, R_i)$ is in the capacity region of the MAC consisting of the primary transmitter, the transmitter
of SU\textsubscript{i} and the receiver of SU\textsubscript{i}.

2- \( R_i \) is an achievable rate of SU\textsubscript{i} by treating the signal of PU as noise regardless of the value of \( R_1 \).

For any \( R_1 \), we define \( R_i^* \) by

\[
R_i^* = \sup \{ R_i : (R_1, R_i) \in \mathcal{R}_i \}.
\]

Thereafter, the rate \( R_1 \) and the activity factor \( \theta \) are selected based on the rule

\[
(\hat{\theta}, \hat{R}_1) = \arg \max_{(\theta, R_1) \in \mathcal{A}} \mathbb{E} \{ U(R_1, R_{2*}, \cdots, R_{N+1*}) \}
\]

where the expectation is with respect to the channel gains. This design procedure is completely distributed and the resulting \( \hat{\theta} \) and \( \hat{R}_1 \) are computable by all users. Hence, the PU regulates its transmission rate at \( \hat{R}_1 \) and the secondary transmitter regulates its activity factor at \( \hat{\theta} \).

In the sequel, we propose a different approach called Power Allocation (PA) where each secondary transmitter regulates its transmission power in order to achieve the largest utility value. It is demonstrated that the PA scheme is more appropriate in the regime that the transmission power of PU is relatively low, while the RM scheme outperforms PA for comparably larger values of the transmission power of PU.

II. SYSTEM MODEL AND ASSUMPTIONS

We consider a wireless communication system consisting of one primary transmitter-receiver pair, referred to as the Primary User (PU). The PU is labeled as user 1. A number \( N \) of secondary transmitter-receiver pairs (referred to as the Secondary Users (SU) ) become active in the vicinity of the PU. We label the SUs as user 2 to user \( N + 1 \) and denote them by SU\textsubscript{2} to SU\textsubscript{N+1}. The number of the secondary users \( N \) is a realization of a random variable \( N \) with PMF \( q_n = \Pr \{ N = n \} \), \( n = 0, 1, 2, \cdots \). We assume that \( N \) is fixed through the whole communication period of interest once it is set first. The channel from the transmitter of user \( i \) to the receiver of user \( j \) is modeled by a static and non-frequency selective gain \( h_{i,j} \) that is a realization of the random variable \( h_{i,j} \sim \mathcal{CN}(0, 1) \) representing Rayleigh fading. The SUs are capable of synchronizing themselves with the PU at the symbol and block level. As such, the \( N + 1 \) transmitter-receiver pairs represent a slotted and synchronous interference channel where each user transmits a symbol in its codeword in a transmission slot.
Denoting the signal of user $i$ by $x_i$, the signal received at the receiver of user $i$ is

$$y_i = h_{i,i}x_i + \sum_{j=1}^{N+1} h_{j,i}x_j + z_i.$$  \hfill (1)

The random variable $z_i \sim \mathcal{CN}(0,1)$ is the ambient noise at the receiver side of user $i$. Also, the signal $x_i$ must satisfy the power constraint

$$E\{|x_i|^2\} = \begin{cases} \gamma_p & i = 1 \\ \gamma_s & 2 \leq i \leq N + 1 \end{cases}.$$  \hfill (2)

The code-book of user $i$ consists of $2^{\lceil TR_i \rceil}$ codewords where each codeword is transmitted in $T$ consecutive transmission slots, i.e., the transmission rate of user $i$ is $R_i$ as $T$ tends to infinity. The PU continuously transmits complex Gaussian signals with power $\gamma_p$, i.e., $x_1$ is a $\mathcal{CN}(0,\gamma_p)$ random variable. This is an optimal scheme in the absence of the SUs. However, each SU must design its signaling scheme to guarantee a certain quality of service for itself, while not degrading the performance of the PU beyond a particular threshold.

Each SU is *smart* in the sense that:

1- It is aware of the codebook of the PU. However, it does not know the current message of the PU. Also, the SUs are anonymous to each other, i.e., they are unaware of each other’s code-book.

2- Each SU is aware of the number $N$ of active SUs in the network.

3- SU$_i$ is aware of $h_{j,i}$ and is unaware of $h_{j,k}$ for $1 \leq j \leq N + 1$ and $k \neq i$.

On the other hand, PU is *dumb* in the sense that:

1- It is not aware of the code-books of the secondary users.

2- The primary transmitter is unaware of the presence of the SUs, i.e., it is not aware of the value of $N$.

3- The primary transmitter is unaware of $h_{j,1}$ for any $1 \leq j \leq N + 1$. However, the primary receiver is aware of the realizations of these channel gains.

Throughout the paper, we will assume that there is an integer $n_{\text{max}} \geq 1$, such that $\Pr\{N > n_{\text{max}}\} = 0$. Moreover, $\vec{h}_i$ is a vector of length $N + 1$ containing the channel gains related to user $i$, i.e., $\vec{h}_i = (h_{1,i} \ldots h_{N+1,i})^t$. 
III. SIGNALING SCHEME WITHOUT LISTENING AT THE SECONDARY TRANSMITTER

Motivated by the randomized resource allocation schemes introduced in [39], SUs follow a Randomized Masking (RM) scheme where each SU transmits a Gaussian symbol in its codeword with a probability of $\theta$ and remains silent with a probability of $\bar{\theta}$ independently from transmission slot to transmission slot. The parameter $\theta$ is called the activity factor for SUs. Hence,

$$x_i = c_i s_i$$  \hspace{1cm} (3)

where $c_i$ and $s_i$ are $\text{Ber}(\theta)$ and $\mathcal{CN}(0, \frac{2\theta}{\bar{\theta}})$ random variables, respectively. We assume the receiver of SU$_i$ is aware of $c_i$, i.e., the masking pattern applied by its associated transmitter is already revealed to the receiver of SU$_i$. Moreover, each SU is unaware of the masking pattern of other SUs. Note that the only parameter to be designed at the SUs is the activity factor $\theta$.

A. Achievable Rate Region

Since PU is unaware of the code-books of SUs, multiuser decoding is not performed at the primary receiver, i.e., the primary receiver treats the signals transmitted by the secondary transmitters as additive noise. However, each secondary transmitter has the choice to perform interference cancellation as it is aware of the codebook of the PU.

As for PU,

$$y_1 = h_{1,1} x_1 + \sum_{j=2}^{N+1} h_{j,1} c_j s_j + z_1.$$  \hspace{1cm} (4)

Since multiuser decoding is not performed at the primary receiver, any rate $R_1$ in the range

$$R_1 < I(x_1; y_1)$$  \hspace{1cm} (5)

is achievable in the conventional sense. As the noise plus interference $\sum_{j=2}^{N+1} h_{j,1} c_j s_j + z_1$ at the receiver side of PU is a complex mixed Gaussian random variable, the quantity $I(x_1; y_1)$ has no closed expression. However, we are able to develop a lower bound on $I(x_1; y_1)$. The following Lemma is essential to this purpose.

**Lemma 1** Let

$$y = x + z$$  \hspace{1cm} (6)
where $x$ and $z$ are independent random variables, $x$ is a $CN(0, \sigma^2)$ random variable and $z$ is a complex mixed Gaussian random variable with density $p_z(z) = \sum_{l=1}^{L} p_l g_l(z; \sigma_l^2)$. Then,

$$h(z) \leq \log(\pi e) + \sum_{l=1}^{L} p_l \log \sigma_l^2 - \sum_{l=1}^{L} p_l \log p_l$$

and

$$I(x; z) \geq \log \left(1 + \frac{p_1 \cdots p_L \sigma^2}{\sigma_1^{2p_1} \cdots \sigma_L^{2p_L}}\right).$$

Proof: See [39].

By Lemma 1, it is easy to see that

$$R_1 \leq A(N, \tilde{h}_1; \theta)$$

is an inner bound to the region in (5) where

$$A(N, \tilde{h}_1; \theta) \triangleq \log \left(\frac{2^{-N \lambda'(\theta)}|h_{1,1}|^2 \gamma_p}{1 + \sum_{j=2}^{N+1} |h_{j,1}|^2 \gamma_{s_j} \theta} + 1\right).$$

Next, let us consider $SU_i$ for $2 \leq i \leq N + 1$. We have

$$y_i = h_{i,i} c_i s_i + h_{1,i} x_1 + \sum_{j=2, j \neq i}^{N+1} h_{j,i} c_j s_j + z_i.$$ 

The knowledge about the code-book of PU enables the secondary receiver to perform interference cancellation. Using the achievable rate region for a MAC [40],

$$\begin{cases}
R_1 \leq I(x_1; y_i|c_i, s_i) \\
R_i \leq I(s_i; y_i|c_i, x_1) \\
R_1 + R_i \leq I(s_i, x_1; y_i|c_i).
\end{cases}$$

The first constraint in (13) is indeed not required. In fact, it is a constraint which assures the PU’s message is decoded successfully while decoding of the message sent by the secondary transmitter is already successful. The secondary user is not interested in this case, and hence, we can remove the first constraint in (13). Therefore,

$$\begin{cases}
R_i \leq I(s_i; y_i|c_i, x_1) \\
R_1 + R_i \leq I(s_i, x_1; y_i|c_i).
\end{cases}$$
are the only required constraints as far as multiuser detection is applied by the secondary receiver \(i\).

Denoting \(I(s_i; y_i|c_i, x_1)\) and \(I(s_i, x_1; y_i|c_i)\) by \(B(N, \bar{h}_i; \theta)\) and \(C(N, \bar{h}_i; \theta)\), respectively, one can apply Lemma 1 to see that

\[
B(N, \bar{h}_i; \theta) = \theta \log \left( \frac{2^{-(N-1)\mathcal{N}(\theta)}|h_{i,i}|^2 \gamma_s}{\theta \left( 1 + \frac{\sum_{j=2, j \neq i}^{N+1} |h_{j,i}|^2 \gamma_s}{\theta} \right)^\theta + 1} \right)
\]

and

\[
C(N, \bar{h}_i; \theta) = \theta \log \left( \frac{2^{-(N-1)\mathcal{N}(\theta)}|h_{1,i}|^2 \gamma_s}{\theta \left( 1 + |h_{1,i}|^2 \gamma_p + \frac{\sum_{j=2, j \neq i}^{N+1} |h_{j,1}|^2 \gamma_s}{\theta} \right)^\theta + 1} \right) + \log \left( \frac{2^{-(N-1)\mathcal{N}(\theta)}|h_{1,i}|^2 \gamma_p}{\theta \left( 1 + \frac{\sum_{j=2, j \neq i}^{N+1} |h_{j,1}|^2 \gamma_s}{\theta} \right)^\theta + 1} \right)
\]

Another approach for the secondary receiver is to treat the signal of the PU as noise. In this case, any rate in the range

\[
R_i < I(s_i; y_i|c_i)
\]

is achievable. Denoting \(I(s_i; y_i|c_i)\) by \(D(N, \bar{h}_i; \theta)\) and applying Lemma 1 yield

\[
D(N, \bar{h}_i; \theta) = \theta \log \left( \frac{2^{-(N-1)\mathcal{N}(\theta)}|h_{1,i}|^2 \gamma_s}{\theta \left( 1 + |h_{1,i}|^2 \gamma_p + \frac{\sum_{j=2, j \neq i}^{N+1} |h_{j,1}|^2 \gamma_s}{\theta} \right)^\theta + 1} \right).
\]

By (13) and (16), the rate region for the secondary user is the set of all \((R_1, R_2)\) satisfying

\[
\begin{align*}
R_i &\leq B(N, \bar{h}_2; \theta) \quad \text{or} \quad R_i \leq D(N, \bar{h}_i; \theta) \\
R_1 + R_i &\leq C(N, \bar{h}_i; \theta)
\end{align*}
\]

We denote this region by \(\mathcal{R}(N, \bar{h}_i; \theta)\) which is sketched in fig. 1 for an arbitrary realization \(\bar{h}_i\) of \(\bar{h}_i\).

**B. System Design**

We know that the primary transmitter has no knowledge about the channel gains and the number of active users in the network. An appropriate tool to assess the performance of this user is the so-called
outage probability. We define the outage event for the PU by

$$\mathcal{O}_1(\theta, R_1) \triangleq \left\{ \mathcal{N}, \mathcal{h}_1 : A(\mathcal{N}, \mathcal{h}_1; \theta) < R_1 \right\}. \quad (19)$$

We require the probability of outage for the PU to be less than a certain threshold $\varepsilon$, i.e., $\Pr \{ \mathcal{O}_1(\theta, R_1) \} < \varepsilon$. In this sense, we define the $\varepsilon$-admissible region for $(\theta, R_1)$ as the set $\mathcal{A}_\varepsilon$ defined by

$$\mathcal{A}_\varepsilon \triangleq \left\{ (\theta, R_1) : \Pr \{ \mathcal{O}_1(\theta, R_1) \} < \varepsilon \right\}. \quad (20)$$
We have
\[
Pr \{ \mathcal{O}_1(\theta, R_1) \} = \sum_{n=0}^{n_{\text{max}}} q_n \Pr \left\{ A(n, \tilde{h}_1; \theta) < R_1 \right\}
\]
\begin{align*}
\overset{(a)}{=} & \sum_{n=0}^{n_{\text{max}}} q_n E \left\{ \Pr \left\{ A(n, \tilde{h}_1; \theta) < R_1 \left| \{ h_{j,i} \}_{j \neq i} \right. \right\} \right\} \\
= & \sum_{n=0}^{n_{\text{max}}} q_n E \left\{ \Pr \left\{ |h_{4,1}|^2 \leq \frac{2 n \mathcal{A}(\theta) (2R_1 - 1)}{\gamma_p} \left( 1 + \frac{\sum_{j=2}^{n+1} |h_{j,1}|^2 \gamma_s}{\theta} \right) \right| \{ h_{j,i} \}_{j \neq i} \right\} \\
\overset{(b)}{=} & \sum_{n=0}^{n_{\text{max}}} q_n E \left\{ 1 - \exp \left( - \frac{2 n \mathcal{A}(\theta) (2R_1 - 1)}{\gamma_p} \left( 1 + \frac{\sum_{j=2}^{n+1} |h_{j,1}|^2 \gamma_s}{\theta} \right) \right) \right\} \\
\overset{(c)}{=} & 1 - q_1 \exp \left( - \frac{2R_1 - 1}{\gamma_p} \right) \\
+ & \sum_{n=1}^{n_{\text{max}}} \frac{q_n}{(n-1)!} \int_0^\infty t^{n-1} \exp \left( - \frac{2 n \mathcal{A}(\theta) (2R_1 - 1)}{\gamma_p} \left( 1 + \frac{\gamma_s t}{\theta} \right)^\theta - t \right) dt
\end{align*}

(21)

where (a) is by the tower property the conditional expectation [41] and (b) and (c) are due to the facts that $|h_{4,1}|^2$ and $2 \sum_{j=2}^{n+1} |h_{j,i}|$ are exponential with parameter 1 and $\chi_{2n}^2$ random variables, respectively. In the $\theta - R_1$ plane, the boundaries of the region $\mathcal{A}_\varepsilon$ are the $\theta$-axis, the $R_1$-axis and a curve $R_1 = \varphi(\theta)$ that is implicitly defined by
\[
q_1 \exp \left( - \frac{2R_1 - 1}{\gamma_p} \right) + \sum_{n=1}^{n_{\text{max}}} \frac{q_n}{(n-1)!} \int_0^\infty t^{n-1} \exp \left( - \frac{2 n \mathcal{A}(\theta) (2R_1 - 1)}{\gamma_p} \left( 1 + \frac{\gamma_s t}{\theta} \right)^\theta - t \right) dt = \varepsilon. \quad (22)
\]

Using (22), one can check that $\varphi(0) = \log (1 - \gamma_p \ln \varepsilon)$ which is the $\varepsilon$-outage capacity of a point to point channel.

For $2 \leq i \leq N + 1$, the transmitter of SU$_i$ has complete knowledge of $N$ and $\tilde{h}_i$. If this secondary user is also aware of $\tilde{R}_i$, it can regulate its transmission rate as the largest $R_i$ such that $(R_1, R_i) \in \mathcal{R}(N, \tilde{h}_i; \theta)$. We denote this value of $R_i$ by $R^*(N, \tilde{h}_i; \theta, R_1)$, i.e.,
\[
R^*(N, \tilde{h}_i; \theta, R_1) \triangleq \sup \left\{ R_i : (R_1, R_i) \in \mathcal{R}(N, \tilde{h}_i; \theta) \right\}. \quad (23)
\]

According to fig. 1,
\[
R^*(N, \tilde{h}_i; \theta, R_1) = \begin{cases} 
B(N, \tilde{h}_i; \theta) & R_1 \leq C(N, \tilde{h}_i; \theta) - B(N, \tilde{h}_i; \theta) \\
C(N, \tilde{h}_i; \theta) - R_1 & C(N, \tilde{h}_i; \theta) - B(N, \tilde{h}_i; \theta) \leq R_1 \leq C(N, \tilde{h}_i; \theta) - D(N, \tilde{h}_i; \theta) \\
D(\tilde{h}_2, \theta) & R_1 \geq C(N, \tilde{h}_i; \theta) - D(N, \tilde{h}_i; \theta)
\end{cases} \quad (24)
\]

We propose that the PU sets its transmission rate at $\tilde{R}_1$ and requires the secondary users to fix $\theta$ at $\hat{\theta}$
where
\[
(\hat{\theta}, \hat{R}_1) = \arg \sup_{(\theta, R_1) \in A} \mathbb{E}
\left\{ \mathcal{U}_N \left( R_1, R^*(N, \vec{h}_2; \theta, R_1), \cdots, R^*(N, \vec{h}_{N+1}; \theta, R_1) \right) \right\}
\]
(25)

where for each \(0 \leq n \leq n_{\max}\), the function \(\mathcal{U}_n : [0, \infty)^{n+1} \rightarrow [0, \infty)\) is a globally known utility function. For example, the utility function can represent the sum rate \(\mathcal{U}_2(R_1, R_2) = R_1 + R_2\) or the proportional fair function \(\mathcal{U}_2(R_1, R_2) = \log R_1 + \log R_2\). In this paper, we use the proportional fair function motivated by its wide applications in the game theory context [1].

Finally, SU uses its knowledge of \(N\) and \(\vec{h}_i\) to set its transmission rate at
\[
R_i = R^*(N, \vec{h}_i, \hat{\theta}, \hat{R}_1).
\]
(26)

For a realization \(N\) of \(\mathcal{N}\), let us define
\[
\delta_N^{(DP)} \triangleq \Pr \{ \text{SU decodes the message of PU} \mid \mathcal{N} = N \}
\]
(27)

and
\[
\delta_N^{(TPN)} \triangleq \Pr \{ \text{SU treats the signal of PU as noise} \mid \mathcal{N} = N \}
= 1 - \delta_N^{(DP)}.
\]
(28)

According to fig. 1 and following the same lines that led us to (21)
\[
\delta_N^{(DP)} = \Pr \left\{ C(N, \vec{h}_2; \hat{\theta}) - D(N, \vec{h}_2; \hat{\theta}) > \hat{R}_1 \right\}
= \frac{1}{(N-2)!} \int_0^\infty t^{N-2} \exp \left( - \frac{2(N-1) - 2\hat{\theta}}{\gamma_p} \left( \frac{2\hat{R}_1 - 1}{1 + \gamma_s t^{\hat{\theta}}} - t \right) \right) dt.
\]
(29)

for \(N \geq 2\). Also,
\[
\delta_1^{(DP)} = e^{-\frac{2\hat{R}_1 - 1}{\gamma_p}}
\]
(30)

Next, we examine \(\delta_N^{(DP)}\) in a network where \(\mathcal{N} \sim \text{Bi}(n_{\max}, p)\). Fixing \(\varepsilon = 0.05\), fig. 2 plots \(A_{0.05}\) for different values of \(\gamma_p, \gamma_s, n_{\max}\) and \(p\). Corresponding values of \(\hat{\theta}\) and \(\hat{R}_1\) are given in (31). It is seen that the secondary receiver decides to decode the message of PU with a high probability in terms of different realizations of the channel gains.
Fig. 2. The Region $A_{0.05}$ for different values of $\gamma_p$, $\gamma_s$, $n_{\text{max}}$ and $p$.

<table>
<thead>
<tr>
<th>$\gamma_s$</th>
<th>30dB</th>
<th>30dB</th>
<th>30dB</th>
<th>30dB</th>
</tr>
</thead>
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<tr>
<td>$\gamma_p$</td>
<td>30dB</td>
<td>20dB</td>
<td>30dB</td>
<td>20dB</td>
</tr>
<tr>
<td>$n_{\text{max}}$</td>
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<td>2</td>
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<td>5</td>
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<tr>
<td>$\hat{\theta}$</td>
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<tr>
<td>$\hat{R}_1$</td>
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<td>1.86</td>
<td>3.20</td>
<td>1.29</td>
</tr>
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<td>$\delta_1^{(\text{DP})}$</td>
<td>0.9807</td>
<td>0.9740</td>
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<td>0.9857</td>
</tr>
<tr>
<td>$\delta_2^{(\text{DP})}$</td>
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<tr>
<td>$\delta_3^{(\text{DP})}$</td>
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<td>n/a</td>
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<td>$\delta_4^{(\text{DP})}$</td>
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<td>n/a</td>
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<tr>
<td>$\delta_5^{(\text{DP})}$</td>
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<td>n/a</td>
<td>0.7898</td>
<td>0.8596</td>
</tr>
</tbody>
</table>

IV. EFFECT OF TRANSMISSION POWER AT THE SECONDARY TRANSMITTERS

A different approach is that each SU transmits Gaussian signals continuously (No masking is performed), however, the transmission power $\gamma_s$ and $R_1$ are designed such that a globally known utility function is
maximized. We call this scheme Power Allocation (PA). Due to practical constraints, we assume
\[ \gamma_s \leq \gamma_s^* \tag{32} \]
where \( \gamma_s^* \) is the maximum permitted transmission power at each secondary transmitter. As we will see in this section, this approach is more interesting in a regime where \( \gamma_p \) is relatively low.

The design approach is exactly as it was presented in section III with the difference that the quantities
\[ A(N, \bar{h}_1; \theta), B(N, \bar{h}_i; \theta), C(N, \bar{h}_i; \theta) \text{ and } D(N, \bar{h}_i; \theta) \]
must be replaced by
\[ A(N, \bar{h}_1; 1), B(N, \bar{h}_i; 1), C(N, \bar{h}_i; 1) \text{ and } D(N, \bar{h}_i; 1). \]
The new \( \varepsilon \)-admissible region is given by
\[
\tilde{A}_\varepsilon \triangleq \left\{ (\gamma_s, R_1) : \Pr\left\{ A(N, \bar{h}_1; 1) < R_1 \right\} < \varepsilon \right\}
\]
\[ = \left\{ (\gamma_s, R_1) : 1 - \exp\left( -\frac{2^{R_1} - 1}{\gamma_p} \right) \sum_{n=0}^{n_{\text{max}}} q_n \left( 1 + \frac{(2^{R_1} - 1) \gamma_s}{\gamma_p} \right)^{-n} < \varepsilon \right\} \tag{33} \]
where the last step is verified by following the same lines that led us to (21). Finally,
\[
(\hat{\gamma}_s, \hat{R}_1) = \arg \sup_{(\gamma_s, R_1) \in \tilde{A}_\varepsilon} \mathbb{E}\{ U(N, R_1, R_2^*(N, \bar{h}_2; 1, R_1), \ldots, R_2^*(N, \bar{h}_{N+1}; 1, R_1) \} \}. \tag{34} \]

For example, let us consider a network with \( N \sim \text{Bi}(2, 0.2) \). Fig. 3 (left plot) presents the maximum value of the utility function achieved under the RM and PA schemes for different values of \( \gamma_p \) where it is assumed that \( \varepsilon = 0.05 \) and \( \gamma_s^* = 30 \text{dB} \). The power optimization strategy offers a better performance for relatively smaller values of \( \gamma_p \), however, as \( \gamma_p \) increases, the RM scheme takes over and outperforms the PA scheme. Also, the optimum allocated \( \gamma_s \) at each SU is sketched in the same figure (right plot).

V. Conclusion

We addressed a decentralized network of one PU and multiple SUs. A distinct feature of the model was the randomness of the number of SUs. Each SU was entitled to not disturb the performance of PU beyond a certain level, while achieving a satisfactory quality of service for itself. It was proposed that each secondary transmitter adopts the RM strategy where it remains silent or transmits a symbol in its codeword independently from transmission slot to transmission slot. We considered a setup where the primary transmitter was unaware of the channel gains, the code-book of the secondary users and the number of secondary users. Although the SUs were anonymous to each other, however, each SU was
smart enough to be aware of the code-book of PU, the channel gains and the number of active SUs. We defined the $\varepsilon$-admissible region as the set of possible transmission rates for PU and possible masking probabilities for each SU such that the probability of outage for PU is maintained under a threshold $\varepsilon$. The transmission rate of PU and the masking probability of SUs were designed through maximizing a utility function of the rates of users over the $\varepsilon$-admissible region. In our analysis, the primary receiver treated interference as noise, however, each secondary receiver had the option to perform multiuser decoding, while treating the signals of other SUs as noise. In another scenario, referred to as Power Allocation (PA), each SU transmitted continuously (no masking applied), however, it regulated its transmission power in order to guarantee the largest utility value. It was demonstrated that the PA scheme is more appropriate in the regime that the transmission power of PU is relatively low, while the RM scheme outperforms PA for comparably larger values of the transmission power of PU.

REFERENCES


