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Technical Report UW-E&CE#2007-35
November 9 2007

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Abstract

This paper studies the setup of a multiple relays network in which K half-duplex single-antenna relays assist the single-antenna transmitter(s) and the single-antenna receiver. Each pair of nodes are assumed to be either connected together through a quasi-static fading channels or be disconnected. However, it is assumed that there is no direct link between the transmitter(s) and the receiver. We prove that a modified version of the sequential SAF scheme ([1]) performs optimal in the sense that it achieves the optimum diversity-multiplexing tradeoff. However, for the single relay scenario, it reduces to the naive amplify-and-forward scheme and can not follow the optimum diversity-multiplexing tradeoff curve, while the DDF scheme ([2]) performs optimum in this scenario.

I. Introduction

A. Motivation

In recent years, relay networks has gained more and more attention to combat against the existing wireless networks difficulties such as the fading effect, the coverage shortage, and interference coexistence. The main idea is to employ some extra nodes in the network to aid the transmitter/receiver in sending/receiving the signal to/from the other end. In this way, the supplementary nodes act as spatially distributed antennas assisting the signal transmission and

Financial supports provided by Nortel, and the corresponding matching funds by the Federal government: Natural Sciences and Engineering Research Council of Canada (NSERC) and Province of Ontario: Ontario Centres of Excellence (OCE) are gratefully acknowledged.

reception. Recently, cooperative diversity techniques have been proposed as candidates to exploit the potential spatial diversity exists in the relay network (for example, see [2]–[5]). A fundamental measure to evaluate the performance of the existing cooperative diversity schemes is the diversity-multiplexing tradeoff (DMT) introduced by Zheng and Tse which was firstly proposed for the MIMO point-to-point fading channels ([6]). Vaguely saying, the diversity-multiplexing tradeoff identifies the optimal compromise between transmission reliability and data rate of a system in high-SNR regime.

However, none of the existing cooperative diversity scheme is proved to achieve the DMT in the relay networks. Yet, the problem is open for the half-duplex single relay single source-destination SISO setup. Indeed, the only existing DMT achieving scheme for the single relay channel ([4]) works at the expense of having the CSI (channel state information) of all the network channels at the relay node.

In this paper, we study a new modified version of the sequential SAF (slotted amplify-and-forward) scheme ([1]), and prove that it achieves the optimum diversity-multiplexing tradeoff in the multiple-access multiple relays network under no direct transmitter-receiver link assumption.

B. Related Works

The DMT of relay systems was first studied in [3] for half-duplex relays. In this work, the authors prove that the DMT of a network consisted of half-duplex single antenna single source-destination assisted with K single antenna relay nodes, is upper-bounded by

$$d(r) = (K+1)(1-r)^{+}. \tag{1}$$

This bound can easily be proved by applying either the multiple-access or the broadcast cut-set bound ([7]) on the achievable rate of the system. This bound is still the tightest upper-bound for the DMT of the relay network. The authors in [3] also suggested two protocols based on decode-and-forward (DF) and amplify-and-forward (AF) strategies respectively, for a single relay system with single antenna nodes. In both protocols, the relay listens to the source during the first half of the frame, and transmits during the second half. To battle against the spectral efficiency reduction, the authors propose incremental relaying protocol in which the receiver sends a 1 bit feedback to the transmitter and the relay to clarify if it has decoded the transmitter's message or it needs help from the relay side to decode the message. However, none of the proposed schemes were able to follow the DMT upper-bound.

The non-orthogonal amplify-and-forward (NAF) scheme, firstly proposed in [8], was studied by [2]. Apart from analyzing the DMT of the NAF scheme, [2] showed that NAF scheme is the best scheme in the class of AF strategy for the single antenna single relay system. The dynamic decode-and-forward (DDF) scheme was also proposed independently in [2], [9], [10] based on the DF strategy. In DDF, the relay node listens to the sender until it can decodes the message and ,following that, it re-encodes the message and sends it in the remaining time. [2] analyzed the DMT of DDF scheme and showed that the DDF scheme is optimal for low rates in the sense that it achieves (1) for the multiplexing gains $r \leq 0.5$. However, for high rates, the relay needs to listen to the transmitter for most of the time, and it can not assists the transmitter for most portion of the frame. Hence, the scheme is unable to follow the upper-bound for high multiplexing gain rates. More importantly, the generalizations of NAF and DDF for the K relay system achieve far from (1), especially for high multiplexing gain rates.

[4] applied compress-and-forward (CF) strategy and proved that CF achieves the DMT for the multiple-antennas half-duplex single relay system. However, in the proposed scheme, the relay node needs to know the CSI of all the channels in the network. This assumption is impractical in real situations in which sending CSI's back to the network nodes not only costs in terms of bandwidth and power, but suffers from the problem of channel aging.

Recently, [1] proposed a class of AF relaying called slotted amplify-and-forward (SAF) scheme for the multiple half-duplex relays ($K > 1$) single source-transmitter setup. In SAF, the frame of transmission is divided into M equal length slots. In each slot, each relay transmits a linear combination of the previous slots. [1] found an upper-bound for the DMT of SAF and showed that it is impossible to achieve the MISO upper-bound for finite values of M, even with the assumption of full-duplex relaying. However, as M goes to infinity, the upper-bound meets the MISO upper-bound. Motivated by the upper-bound, the authors in [1] proposed a half-duplex sequential SAF scheme. In sequential SAF scheme, following the first slot, in each slot, one and only one of the relays is permitted to transmit an amplified version of the signal it received in the last slot. In this way, the different parts of the transmitted signal go through different paths by different relays, protected by some kind of spatial diversity between the relays. However, [1] could only show that the sequential SAF achieves the MISO upper-bound for the setup of no-interfering relays, in which the consecutive relays (in transmission ordering) do not make interference on the input of each other.

C. Contributions

In this paper, we consider the multiple-access multiple relays network. The network consists of M transmitters aided by K half-duplex relays. Each relay is just permitted to know the CSI of its corresponding backward channel (the channel between the transmitter and the relay), and the receiver is supposed to know the equivalent channel gain from the transmitter to the receiver. Furthermore, we assume that there is no direct link between the transmitters and the receiver. This assumption is reasonable when the transmitters are far away from the receiver and the relays are designed to connect the transmitters to the receiver. However, the graph of interfering relay pairs can have any topology, i.e. any two relays can have interference on each other or not. We prove that a modified version of the sequential SAF scheme achieves the DMT for the multiple-access multiple relays setup. However, for the multiple-access single relay scenario, we show that the proposed scheme is unable to follow the optimum diversity-multiplexing tradeoff curve, while the DDF scheme achieves the curve.

The rest of the paper is organized as follows. In section II, the system model is introduced. In section III, the sequential SAF scheme and the modified version of sequential SAF is described. Section IV is dedicated to the DMT analysis of the modified sequential SAF scheme. Finally, section V concludes the paper.

II. System Model

The system , as in [3], [2], [1], and [4], consists of K relays assisting the transmitter and the receiver in the half-duplex mode, i.e. in each time, the relays can either transmit or receive. Each two node is assumed to either be connected by a quasi-static flat Rayleigh-fading, i.e. the channel gains remain constant during a block of transmission and changes independently from one block to another, or be disconnected, i.e. there is no direct link between them. However, throughout the paper, we assume that there is no direct link between the transmitter and the receiver. This assumption is reasonable when the transmitter and the receiver are far from each other and the relay nodes are employed to assist the end nodes in making the connection. As in [2] and [1], each relay is assumed to know the state of its backward channel and, moreover, the receiver is supposed to know the equivalent channel gain from the transmitter to the receiver. Hence, against the CF scheme in [4], no CSI feedback is needed in the network. All nodes have the same power constraint. Also, we assume that a capacity achieving gaussian random codebook can be generated at each node of the network. Hence, the code design problem is not considered in this paper. Figure (1) shows a realization of the multiple relays single transmitter-receiver scenario in which the relay set $\{1,2\}$ is disconnected from the relay set.$\{3,4\}$. Here, we denote the output vector at the transmitter as $\mathbf{x}$, the input and output vector at the k'th relay as $\mathbf{r}_k$ and $\mathbf{t}_k$ respectively, and the input at the receiver as $\mathbf{y}$. As an example, for the scenario shown in figure (1), we have

$$\begin{aligned} \mathbf{r}_3 &= h_3\mathbf{x} + i_{4,3}\mathbf{t}_4 + \mathbf{n}_3, \\ \mathbf{y} &= g_1\mathbf{t}_1 + g_2\mathbf{t}_2 + g_3\mathbf{t}_3 + g_4\mathbf{t}_4 + \mathbf{z}, \end{aligned}$$

where h_k is the channel gain between the transmitter and the k'th relay, g_k is the channel gain between the k'th relay and the receiver, $i_{a,b}$ is the channel gain between the a'th and b'th relay nodes, $\mathbf{n}_k$ is the noise at the k'th relay, and $\mathbf{z}$ is the noise at the receiver side.

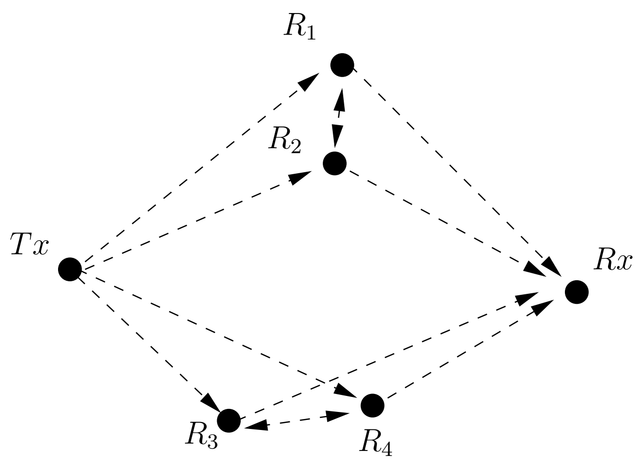

Fig. 1. An example of a multiple relays network ystem model for a single transmitter-receiver pair are assisted with 4 half-duplex relays, relay set $\{1,2\}$ are disconnected from relay set $\{3,4\}$.

III. Proposed K-Slot Switching N-sub-block Markovian Scheme (SM)

In the proposed scheme, the entire block of transmission is divided into N sub-blocks. Each sub-block consists of K slots. Each slot has T' symbols. Hence, the entire block consists of

$T = NKT'$ symbols. In order to transmit a message w, the transmitter selects the corresponding codeword of a gaussian random codebook consisting of $2^{NKT'r}$ codewords of length $\frac{NK-1}{NK}T$ and transmits the codeword during the first $NK-1$ slots. In each sub-block, each relay receives the signal in one of the slots and transmits the received signal in the next slot. So, each relay is off in $\frac{K-2}{2}$ of time. More precisely, in the k'th slot of the n'the sub-block ($1 \leq n \leq N, 1 \leq k \leq K, nk \neq NK$), the k'th relay receives the signals the transmitter is sending, and amplifies and forwards it to the receiver in the next slot. The receiver starts receiving the signal from the second slot. After receiving the last slot (NK'th slot) signal, the receiver decodes the transmitted message by using the signal of $NK-1$ slot received from K relays. It will be shown in the next section that the equivalent point-to-point channel from the transmitter to the receiver would act as a lower-triangular MIMO channel.

IV. Diversity-Multiplexing Tradeoff

In this section, we show that the proposed method achieves the optimum achievable diversity-multiplexing curve. First, according to the cut-set bound theorem [7], the point-to-point capacity of the uplink channel (the channel from the transmitter to the relays) is an upper-bound for the capacity of this system. Accordingly, the diversity-multiplexing curve of a $1 \times K$ SIMO system which is a straight line from multiplexing gain 1 to the diversity gain K is an upper-bound for the diversity-multiplexing curve of our system. In this section, we prove that the tradeoff curve of the proposed method achieves the upper-bound and thus, it is optimum. First, we prove the statement for the case that there is no link between the relays. Next, we prove the statement for the general case.

A. No Interfering Relays

Assume, the link gain between the k'th relay and the transmitter and the k'th relay and the receiver are h_k and g_k, respectively. Furthermore, assume that there is no link between the relays. Accordingly, at the k'th relay we have

$$\mathbf{r}_k = h_k \mathbf{x} + \mathbf{n}_k, \tag{2}$$

where $\mathbf{r}_k$ is the received signal vector of the k'th relay, $\mathbf{x}$ is the transmitter signal vector and $\mathbf{n}_k \sim \mathcal{N}(0, \mathbf{I}_{T'})$ is the noise vector of the channel. At the receiver side, we have

$$\mathbf{y} = \sum_{k=1}^{K} g_k \mathbf{t}_k + \mathbf{z}, \tag{3}$$

where $\mathbf{t}_k$ is the transmitted signal vector of the k'th relay, $\mathbf{y}$ is the received signal vector at the receiver side and $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I}_{T'})$ is the noise vector of the downlink channel. The output power constraint $\mathbb{E}\left\{\|\mathbf{x}\|^2\right\}, \mathbb{E}\left\{\|\mathbf{t}_k\|^2\right\} \leq T'P$ holds at the transmitter and relays side. To obtain the DM tradeoff curve of the proposed scheme, we are looking for the end-to-end probability of outage from the rate $r \log(P)$, as P goes to infinity.

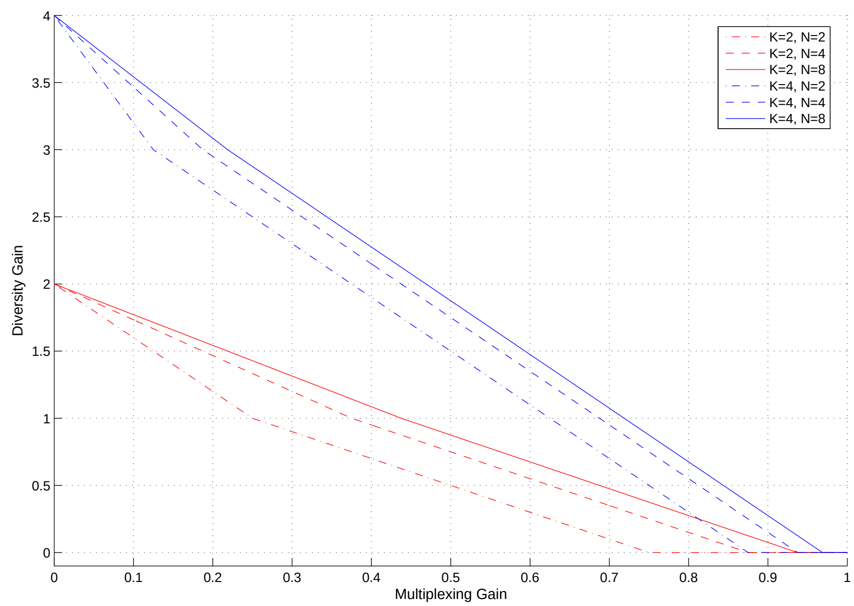

Fig. 2. DM Tradeoff for the proposed Switching Markovian Scheme and various values of (K,N), No interfering relays case

Theorem 1 *Assume a half-duplex parallel relay scenario with K no interfering relays. The proposed SM scheme achieves the diversity gain*

$$d_{SM,NI}(r) = \max\left\{0, K(1-r) - \frac{1}{N}, K(1-r) - \frac{Kr}{N-1}\right\}, \tag{4}$$

which achieves the optimum achievable DM tradeoff curve $d_{opt}(r) = K(1-r)$ as $N \to \infty$.

Proof: Let us define $\mathbf{x}_{n,k}, \mathbf{n}_{n,k}, \mathbf{r}_{n,k}, \mathbf{t}_{n,k}, \mathbf{z}_{n,k}, \mathbf{y}_{n,k}$ as the signal/noise transmitted/received by the transmitter/relay/receiver to the k'th relay/receiver in the k'th slot of the n'th sub-block. Also, let us define $(k) \equiv k-2 \mod K+1$ and $(n) \equiv n - \lfloor \frac{(k)}{K} \rfloor$. Thus, we have

$$\begin{aligned}
\mathbf{y}_{n,k} &= g_k \mathbf{t}_{n,k} + \mathbf{z}_{n,k} \\
&= g_k \alpha_{(k)} \left(h_{(k)} \mathbf{x}_{(n),(k)} + \mathbf{n}_{(n),(k)} \right) + \mathbf{z}_{n,k}, \qquad (5)
\end{aligned}$$

where $\alpha_k = \frac{P}{|h_k|^2 P + 1}$ is the amplification coefficient performed in the k'th relay. Defining the event $\mathcal{E}_k$ as the event of outage from the rate $r \log(P)$ in the k'th sub-channel consisting of the transmitter, the k'th relay, and the receiver, we have

$$\begin{aligned}
\mathbb{P}\{\mathcal{E}_k\} &= \mathbb{P}\left\{ \log\left[1 + P|g_k|^2|\alpha_k|^2|h_k|^2 \left(1+|g_k|^2|\alpha_k|^2\right)^{-1} \right] \leq r\log(P) \right\} \\
&\doteq \min\left\{ \text{sign}(r), \mathbb{P}\left\{ |g_k|^2|\alpha_k|^2|h_k|^2 \left(1+|g_k|^2|\alpha_k|^2\right)^{-1} \leq P^{r-1} \right\} \right\} \\
&\overset{(a)}{\doteq} \min\left\{ \text{sign}(r), \mathbb{P}\left\{ |g_k|^2|\alpha_k|^2|h_k|^2 \min\left\{ \frac{1}{2}, \frac{1}{2|g_k|^2|\alpha_k|^2} \right\} \leq P^{r-1} \right\} \right\} \\
&\overset{(b)}{\doteq} \min\left\{ \text{sign}(r), \mathbb{P}\left\{ |h_k|^2 \leq 2P^{r-1} \right\} + \mathbb{P}\left\{ |g_k|^2|\alpha_k|^2|h_k|^2 \leq 2P^{r-1} \right\} \right\} \\
&\overset{(c)}{\doteq} \min\left\{ \text{sign}(r), P^{-(1-r)} + \mathbb{P}\left\{ |g_k|^2 \min\left\{ \frac{1}{2}, \frac{|h_k|^2 P}{2} \right\} \leq 2P^{r-1} \right\} \right\} \\
&\overset{(d)}{\doteq} \min\left\{ \text{sign}(r), P^{-(1-r)} + \mathbb{P}\left\{ |g_k|^2 \leq 4P^{r-1} \right\} + \mathbb{P}\left\{ |g_k|^2|h_k|^2 \leq 4P^{r-2} \right\} \right\} \\
&\overset{(e)}{\doteq} \min\left\{ \text{sign}(r), P^{-(1-r)} \right\}, \qquad (6)
\end{aligned}$$

where sign(r) is the sign function, i.e. sign$(r) = 1, r \geq 0,$ sign$(r) = 0, r < 0$. Here, (a) follows from the fact that $\frac{1}{1+|g_k|^2|\alpha_k|^2} \doteq \min\left\{ \frac{1}{2}, \frac{1}{2|g_k|^2|\alpha_k|^2} \right\}$, (b) and (d) follow from the union bound inequality, (c) follows from the fact that $|\alpha_k|^2|h_k|^2 \doteq \min\left\{ \frac{1}{2}, \frac{|h_k|^2 P}{2} \right\}$ and the pdf distribution of the rayleigh-fading parameter near zero, and (e) follows from the fact that the product of two independent rayleigh-fading parameters behave as a rayleigh-fading parameter near zero. (6) shows that each sub-channel's tradeoff curve performs as a single-antenna point-to-point channel.

Defining $R_k(P)$ as the random variable showing the rate of the k'th sub-channel consisting of the transmitter, the k'th relay, and the receiver in terms of P, the outage event of the entire channel from the $r\log(P)$, the event $\mathcal{E}$, is equal to

$$\mathbb{P}\{\mathcal{E}\} = \mathbb{P}\left\{N\sum_{k=1}^{K-1} R_k(P) + (N-1)R_K(P) \le NKr\log(P)\right\} \tag{7}$$

Assuming $R_k(P) = r_k \log(P)$, we have

$$\mathbb{P}\{\mathcal{E}\} \doteq \mathbb{P}\left\{N\sum_{k=1}^{K-1} r_k + (N-1)r_K \le NKr\right\} \tag{8}$$

$\mathbb{P}\{R_k(P) \le r_k \log(P)\}$ is known by (6). Defining the region $\mathcal{R}$ as

$$\mathcal{R} = \left\{(r_1, r_2, \cdots, r_K)\,|0 \le r_k \le 1, N\sum_{k=1}^{K-1} r_k + (N-1)r_K \le NKr\right\} \tag{9}$$

it is easy to check that all the vectors $(r_1, r_2, \cdots, r_K)$ that result in the outage event almost surely lie in $\mathcal{R}$. In fact, according to (6), for all k we know $r_k \ge 0$. Also, for $r_k > 1$, $\mathbb{P}\{R_k(P) \ge r_k \log(P)\} \le e^{-P^{r-1}}$ which is exponential in terms of P. Hence, $r_k > 1$ can be disregarded for the outage region. As a result, $\mathbb{P}\{\mathcal{E}\} \doteq \mathbb{P}\{\mathbf{r} \in \mathcal{R}\}$.

On the other hand, by (6) and the fact that r_k's are independent, we have

$$\mathbb{P}\left\{r_1 \le r_1^0, r_2 \le r_2^0, \cdots, r_K \le r_K^0\right\} \doteq P^{-\left(K-\sum_{k=1}^{K} r_k^0\right)} \tag{10}$$

Now, we show that $\mathbb{P}\{\mathcal{E}\} \doteq P^{-\min_{\mathbf{r}\in\mathcal{R}} K-\mathbf{1}\cdot\mathbf{r}}$. First of all, by taking derivative of (10) with respect to $r_1, r_2, \cdots, r_K$, it is easy to see that the probability density function of $\mathbf{r}$ behaves the same as the probability function in (10), i.e. $f_r(\mathbf{r}) \doteq P^{-(K-\mathbf{1}\cdot\mathbf{r})}$. Hence, the outage probability is equal to

$$\begin{aligned}\mathbb{P}\{\mathcal{E}\} &\doteq \int_{\mathbf{r}\in\mathcal{R}} f_r(\mathbf{r})d\mathbf{r} \\ &\dot{\le} vol(\mathcal{R})P^{-\min_{\mathbf{r}\in\mathcal{R}} K-\mathbf{1}\cdot\mathbf{r}} \\ &\overset{(a)}{\doteq} P^{-\min_{\mathbf{r}\in\mathcal{R}} K-\mathbf{1}\cdot\mathbf{r}}\end{aligned} \tag{11}$$

Here, (a) follows from the fact that $\mathcal{R}$ is a fixed bounded region whose volume is independent of P. On the other hand, by continuity of $P^{-(K-\mathbf{1}\cdot\mathbf{r})}$ over $\mathbf{r}$, we have $\mathbb{P}\{\mathcal{E}\} \dot{\ge} P^{-\min_{\mathbf{r}\in\mathcal{R}} K-\mathbf{1}\cdot\mathbf{r}}$ which combining with (11), results into $\mathbb{P}\{\mathcal{E}\} \doteq P^{-\min_{\mathbf{r}\in\mathcal{R}} K-\mathbf{1}\cdot\mathbf{r}}$. Defining $l(\mathbf{r}) = K - \mathbf{1}\cdot\mathbf{r}$, we have to solve the following linear programming optimization problem $\min_{\mathbf{r}\in\mathcal{R}} l(\mathbf{r})$. Notice that the region $\mathcal{R}$ is defined by a set of linear inequality constraints. To solve the problem, we have

$$\begin{aligned} l(\mathbf{r}) &\overset{(a)}{\geq} \max\left\{0, K - \frac{NKr + r_K}{N}, K - \frac{NKr - \sum_{k=1}^{K-1} r_k}{N-1}\right\} \\ &\overset{(b)}{\geq} \max\left\{0, K(1-r) - \frac{1}{N}, K(1-r) - \frac{Kr}{N-1}\right\}. \end{aligned} \tag{12}$$

Here, (a) follows from the inequality constraint in (9) governing $\mathcal{R}$, and (b) follows from the fact that $r_K \leq 1$ and $\forall k < K : r_k \geq 0$. Now, we partition the range $0 \leq r \leq 1$ into three intervals. First, in the case that $r > 1 - \frac{1}{NK}$, the feasible point $\mathbf{r} = \mathbf{1}$ achieves the lower bound 0. Second, in the case that $r < \frac{1}{K} - \frac{1}{NK}$, the feasible point $\mathbf{r} = \left(0, 0, \cdots, 0, \frac{NKr}{N-1}\right)$, achieves the lower bound $K(1-r) - \frac{Kr}{N-1}$. Finally, in the case that $\frac{1}{K} - \frac{1}{NK} \leq r \leq 1 - \frac{1}{NK}$, The lower bound $K(1-r) - \frac{1}{N}$ is achievable by the feasible point $\mathbf{r}, \forall k < K: \ r_k = \frac{NKr - N + 1}{N(K-1)}, r_K = 1$. Hence, we have $\min_{\mathbf{r} \in \mathcal{R}} l(\mathbf{r}) = \max\left\{0, K(1-r) - \frac{1}{N}, K(1-r) - \frac{Kr}{N-1}\right\}$. This completes the proof. ■

Remark - It is worth noting that as long as the graph $G(V, E)$ whose vertices are the relay nodes and edges are the non interfering relay node pairs includes a hamiltonian cycle [1], the result of this subsection remains valid.

According to (4), we observe the SM scheme achieves the maximum multiplexing gain $1 - \frac{1}{NK}$ and the maximum diversity gain K, respectively, for the setup of non-interfering relays. Hence, it achieves the maximum diversity gain for any finite value of N. Also, assuming that the relays spend the first T' symbols out of the T symbols to initialize and listen to the transmitter's signal, we see that the SM scheme achieves the maximum multiplexing gain which is $1 - \frac{T'}{T}$.

Figure (2) shows the D-M tradeoff curve of the scheme for the case of non-interfering relays and varying number of K and N.

B. *General Case*

In the general case, an interference term due to the neighboring relay adds at the receiver antenna of each relay.

$$\mathbf{r}_k = h_k \mathbf{x} + i_{(k)} \mathbf{t}_{(k)} + \mathbf{n}_k, \tag{13}$$

where $i_{(k)}$ is the interference link gain between the k'th and (k)'th relays. Hence, the amplification coefficient is bounded as $\alpha_k \leq \frac{P}{P\left(|h_k|^2 + \left|i_{(k)}\right|^2\right) + 1}$. Here, we observe that in the case that $\alpha_k > 1$,

[1]By hamiltonian cycle, we mean a simple cycle $v_1 v_2 \cdots v_K v_1$ that goes exactly one time through each vertex of the graph.

the noise n_k at the receiving side of the k'th relay can be boosted at the receiving side of the next relay. Hence, we bound the amplification coefficient as $\alpha_k = \min\left\{1, \frac{P}{P\left(|h_k|^2+\left|i_{(k)}\right|^2\right)+1}\right\}$. In this way, it is guaranteed that the noise of relays are not boosted up through the system. This is at the expense working with the output power less than P. On the other hand, we know that almost surely [2] $|h_k|^2, \left|i_{(k)}\right|^2 \dot{\leq} 1$. Hence, almost surely we have $\alpha_k \doteq 1$. Another change we make in this part is that we assume that the entire time of transmission consists of $NK+1$ slots, and the transmitter sends the data during the first NK slots while the relays send in the last NK slots (from the second slot up to the $NK+1$'th slot). Hence, we have $T = (NK+1)T'$. This assumption makes our analysis easier and the lower bound on the diversity curve tighter. Now, we prove the main theorem of this section.

Theorem 2 *Consider a half-duplex multiple relays scenario with K interfering relays whose gains are independent rayleigh fading variables. The proposed SM scheme achieves the diversity gain*

$$d_{SM,I}(r) \geq \max\left\{0, K(1-r) - \frac{r}{N}\right\}, \tag{14}$$

which achieves the optimum achievable DM tradeoff curve $d_{opt}(r) = K(1-r)$ as $N \to \infty$.

Proof: First, we show that the entire channel matrix acts as a lower triangular matrix. At the receiver side, we have

$$\begin{aligned} \mathbf{y}_{n,k} &= g_k \mathbf{t}_{n,k} + \mathbf{z}_{n,k} \\ &= g_k \alpha_{(k)} \left(\sum_{0 < n_1, k_1, n_1(K+1)+k_1 < n(K+1)+k} p_{n-n_1,k,k_1} \left(h_{k_1} \mathbf{x}_{n_1,k_1} + \mathbf{n}_{n_1,k_1} \right) \right) + \mathbf{z}_{n,k} \end{aligned} \tag{15}$$

Here, p_{n,k,k_1} has the following recursive formula $p_{0,k,k} = 1, p_{n,k,k_1} = i_{(k)}\alpha_{(k)}p_{(n),(k),k_1}$. Defining the square $NK \times NK$ matrices as $\mathbf{G} = \mathbf{I}_N \otimes \mathit{diag}\{g_1, g_2, \cdots, g_K\}$, $\mathbf{H} = \mathbf{I}_N \otimes \mathit{diag}\{h_1, h_2, \cdots, h_K\}$, $\mathbf{\Omega} = \mathbf{I}_N \otimes \mathit{diag}\{\alpha_1, \alpha_2, \cdots, \alpha_K\}$, and

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ p_{0,2,1} & 1 & 0 & 0 & \dots \\ p_{0,3,1} & p_{0,3,2} & 1 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ p_{N-1,K,1} & p_{N-1,K,2} & \dots & p_{0,K,K-1} & 1 \end{pmatrix}, \tag{16}$$

[2] By almost surely, we mean its probability is greater than $1 - P^{-\delta}$, for all values of δ.

where $\otimes$ is the Kronecker product [11] of matrices and $\mathbf{I}_N$ is the $N \times N$ identity matrix, and the $NK \times 1$ vectors $\mathbf{x}\left(s\right) = \left[x_{1,1}(s), x_{1,2}(s), \cdots, x_{N,K}(s)\right]^T$, $\mathbf{n}\left(s\right) = \left[n_{1,1}\left(s\right), n_{1,2}(s), \cdots, n_{N,K}(s)\right]^T$, $\mathbf{z}\left(s\right) = \left[z_{1,2}(s), z_{1,3}(s), \cdots, z_{N+1,1}(s)\right]^T$, and $\mathbf{y}\left(s\right) = \left[y_{1,2}(s), y_{1,3}(s), \cdots, y_{N+1,1}(s)\right]^T$, we have

$$\mathbf{y}\left(s\right) = \mathbf{G\Omega F}\left(\mathbf{Hx}\left(s\right) + \mathbf{n}\left(s\right)\right) + \mathbf{z}\left(s\right). \tag{17}$$

Here, we observe that the matrix of the entire channel acts as a lower triangular matrix of a $NK \times NK$ MIMO channel whose noise is colored. The probability of outage of such a channel for the multiplexing gain r is defined as

$$\mathbb{P}\left\{\mathcal{E}\right\} = \mathbb{P}\left\{\log\left|\mathbf{I}_{KN} + P\mathbf{H}_T\mathbf{H}_T^H\mathbf{P}_n^{-1}\right| \leq (NK+1)r\log\left(P\right)\right\}, \tag{18}$$

where $\mathbf{P}_N = \mathbf{I}_{NK} + \mathbf{G\Omega FF}^H\mathbf{\Omega}^H\mathbf{G}^H$, and $\mathbf{H}_T = \mathbf{G\Omega FH}$. Assume $|h(k)|^2 = P^{-\mu(k)}$, $|g(k)|^2 = P^{-\nu(k)}$, $|i(k)|^2 = P^{-\omega(k)}$, and $\mathcal{R}$ as the region in $\mathbb{R}^{3K}$ that defines the outage event $\mathcal{E}$ in terms of the vector $[\mu, \nu, \omega]$, where $\mu = \left[\mu(1)\mu(2)\cdots\mu(K)\right]^T, \nu = \left[\nu(1)\nu(2)\cdots\nu(K)\right]^T, \omega = \left[\omega(1)\omega(2)\cdots\omega(K)\right]^T$. The probability distribution function (and also the inverse of cumulative distribution function) decays exponentially as $P^{-P^{-\delta}}$ for positive values of δ. Hence, the outage region $\mathcal{R}$ is almost surely equal to $\mathcal{R}_+ = \mathcal{R}\bigcap\mathbb{R}_+^{3K}$. Now, we have

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{E}\right\} &\overset{(a)}{\leq} \mathbb{P}\left\{\left|\mathbf{H}_T\right|^2\left|\mathbf{P}_n\right|^{-1} \leq P^{-NK(1-r)+r}\right\} \\
&\overset{(b)}{\leq} \mathbb{P}\left\{-N\sum_{k=1}^{K}\mu(k) + \nu(k) - \min\left\{0, \mu(k), \omega((k))\right\} + \right. \\
&\qquad \left. -\frac{NK\log(3) + \log\left|\mathbf{P}_N\right|}{\log\left(P\right)} \leq -NK(1-r) + r\right\} \\
&\overset{(c)}{\leq} \mathbb{P}\left\{-NK\frac{\log\left[3\left(N^2K^2+1\right)\right]}{\log(P)} + NK\left(1-r\right) - r \leq N\sum_{k=1}^{K}\mu(k) + \nu(k), \right. \\
&\qquad \left. \mu(k), \nu(k), \omega(k) \geq 0\right\}.
\end{aligned} \tag{19}$$

Here, (a) follows from the fact that for a positive semidefinite matrix $\mathbf{A}$ we have $|\mathbf{I} + \mathbf{A}| \geq |\mathbf{A}|$, (b) follows from the fact that

$$\alpha(k) = \min\left\{1, \frac{P}{P^{1-\mu(k)} + P^{1-\omega((k))} + 1}\right\} \geq \frac{1}{3}\min\left\{1, P, P^{\mu(k)}, P^{\omega((k))}\right\}$$

and assuming P is large enough such that $P \geq 1$, and (c) follows from the fact that $\alpha(k) \leq 1$ and accordingly, $p_{n,k,k_1} \leq 1$, and knowing that the sum of the entries of each row in $\mathbf{FF}^H$ is less than N^2K^2, we have[3] $\mathbf{F}\mathbf{F}^H \preccurlyeq N^2K^2\mathbf{I}_{NK}$, and $\mathbb{P}\left\{\mathcal{R}\right\} \doteq \mathbb{P}\left\{\mathcal{R}_+\right\}$, and conditioned on $\mathcal{R}_+$, we have $\min\left\{0, \mu(k), \omega((k))\right\} = 0$ and $\nu(k) \geq 0$ and consecutively $\mathbf{P}_N \preccurlyeq (N^2K^2+1)\mathbf{I}_{KN}$.

On the other hand, we know for vectors $\mu^0, \nu^0, \omega^0 \geq \mathbf{0}$, we have $\mathbb{P}\left\{\mu \geq \mu^0, \nu \geq \nu^0, \omega \geq \omega^0\right\} \doteq P^{-\mathbf{1}\cdot\left(\mu^0+\nu^0+\omega^0\right)}$. Similarly to the proof of Theorem 1, by taking derivative with respect to μ, ν we have $f_{\mu,\nu}(\mu,\nu) \doteq P^{-\mathbf{1}\cdot(\mu+\nu)}$ .Defining the lower bound l_0 as $l_0 = -\frac{\log\left[3\left(N^2K^2+1\right)\right]}{\log(P)} + (1-r) - \frac{r}{NK}$, the new region $\hat{\mathcal{R}}$ as $\hat{\mathcal{R}} = \left\{\mu, \nu \geq \mathbf{0}, \frac{1}{K}\mathbf{1}\cdot(\mu+\nu) \geq l_0\right\}$, the cube $\mathcal{I}$ as $\mathcal{I} = [0, Kl_0]^{2K}$, and for $1 \leq i \leq 2K$, $\mathcal{I}_i^c = [0,\infty)^{i-1} \times [Kl_0, \infty) \times [0,\infty)^{2K-i}$, we observe

$$
\begin{aligned}
\mathbb{P}\left\{\mathcal{E}\right\} &\overset{(a)}{\dot{\leq}} \mathbb{P}\{\hat{\mathcal{R}}\} \\
&\leq \int_{\hat{\mathcal{R}}\bigcap\mathcal{I}} f_{\mu,\nu}\left(\mu,\nu\right) d\mu d\nu + \sum_{i=1}^{2K} \mathbb{P}\left\{[\mu,\nu] \in \hat{\mathcal{R}} \cap \mathcal{I}_i^c\right\} \\
&\dot{\leq} vol(\hat{\mathcal{R}} \cap \mathcal{I}) P^{-\min_{[\mu^0,\nu^0]\in\hat{\mathcal{R}}\cap\mathcal{I}} \mathbf{1}\cdot\left(\mu^0+\nu^0\right)} + 2KP^{-Kl_0} \\
&\overset{(b)}{\doteq} P^{-Kl_0} \\
&\doteq P^{-\left[K(1-r)-\frac{r}{N}\right]}.
\end{aligned} \tag{20}
$$

Here, (a) follows from (19) and (b) follows from the fact that $\hat{\mathcal{R}}\bigcap\mathcal{I}$ is a bounded region whose volume is independent of P. (20) completes the proof. ∎

Remark - The statement in the above theorem holds for the general case in which any arbitrary set of relay pairs are non-interfering. Hence, the proposed scheme achieves the upper-bound of the tradeoff curve in the asymptotic case of $N \to \infty$ for any graph topology on the interfering relay pairs.

According to (14), we observe the maximum multiplexing gain achievable by the SM scheme is greater than or equal to $1 - \frac{1}{NK+1}$, which turns out to be tight because of the fact that the transmitter is off in 1 slot out of the $NK+1$ slots. Also, the lower-bound shows us that SM scheme achieves the multiplexing gain K, which is tight too.

Figure (3) shows the D-M tradeoff curve of the scheme for the case of interfering relays and varying number of K and N.

[3] This can be verified by the fact that every symmetric real matrix $\mathbf{A}$ which has the property that for every i, $a_{i,i} \geq \sum_{i\neq j}|a_{i,j}|$ is positive semidefinite.

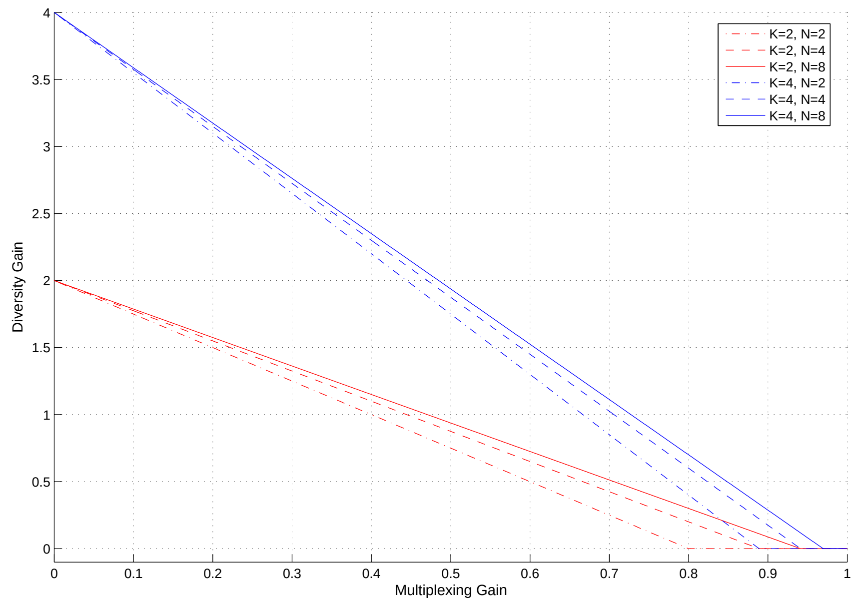

Fig. 3. DM Tradeoff for the proposed Switching Markovian Scheme and various values of (K,N), Interfering relays case

C. Multiple-Access Multiple Relays Scenario

In this section, we study the DM-T performance of the SM scheme in the multiple-access scenario aided by multiple relay nodes. Here, we assume that there is no direct link between each of the transmitters and the receiver. However, like the case of interfering relays in last subsection, the graph of interfering relay pairs can have any topology. Assuming having M transmitter, we show that for the rate sequence $r_1 \log(P), r_2 \log(P), \ldots, r_M \log(P)$, in the asymptotic case of $N \to \infty$, the SM scheme achieves the diversity gain $d_{SM,MAC}(r_1, r_2, \ldots, r_M) = K\left(1 - \sum_{m=1}^{M} r_m\right)^+$, which is shown to be optimum due to the cut-set bound on the cut between the relays and the receiver. The received signal at the k'th relay would be

$$\mathbf{r}_k = \sum_{m=1}^{M} h_{m,k}\mathbf{x}_m + i_{(k)}\mathbf{t}_{(k)} + \mathbf{n}_k, \tag{21}$$

Here, $h_{m,k}$ is the rayleigh channel coefficient between the m'th transmitter and the k'th relay and $\mathbf{x}_m$ is the transmitted vector of the m'th sender. The amplification coefficient in the k'th relay is set as

$$\alpha_k = \min\left\{1, \frac{P}{P\left(\sum_{m=1}^{M}|h_{m,k}|^2 + \left|i_{(k)}\right|^2\right)+1}\right\}. \tag{22}$$

Again, in the same way as shown in the last subsection, we can easily conclude that $\alpha_k \doteq 1$. At the receiver side, after waiting $NK+1$ slots, it decodes the transmitters' messages, $\omega_1, \omega_2, \ldots, \omega_K$, by jointly typical decoding of the received vector in the last NK slots and the transmitted signal of all senders, i.e. the same way, the jointly typical decoder works in the multiple access setup ([7]). Notice that the receiver decodes the messages based on the received vector received from all the NK slots together.

Now, we prove the main statement of this subsection.

Theorem 3 *Consider a multiple-access channel consisting of M transmitting nodes aided by K half-duplex relays. Assume there is no direct link between the transmitters and the receiver. The proposed SM scheme achieves the diversity gain*

$$d_{SM,MAC}(r_1, r_2, \ldots, r_M) \geq \left[K\left(1-\sum_{m=1}^{M} r_m\right) - \frac{\sum_{m=1}^{M} r_m}{N}\right]^+, \tag{23}$$

where $r_1, r_2, \ldots, r_M$ are the rates corresponding to users $1, 2, \ldots, M$. Moreover, as $N \to \infty$, it achieves the optimum DM tradeoff curve which is $d_{opt,MAC}(r_1, r_2, \ldots, r_M) = K\left(1-\sum_{m=1}^{M} r_m\right)^+$.

Proof: At the receiver side, we have

$$\begin{aligned} \mathbf{y}_{n,k} &= g_k \mathbf{t}_{n,k} + \mathbf{z}_{n,k} \\ &= g_k \alpha_{(k)} \left(\sum_{0<n_1,k_1,n_1(K+1)+k_1<n(K+1)+k} p_{n-n_1,k,k_1} \left(\sum_{m=1}^{M} h_{m,k_1} \mathbf{x}_{m,n_1,k_1} + \mathbf{n}_{n_1,k_1} \right) \right) + \mathbf{z}_{n,k}, \end{aligned} \tag{24}$$

where p_{n,k,k_1} is defined in the proof of Theorem 2 and $\mathbf{x}_{m,n_1,k_1}$ represents the transmitted signal of the m'th sender in the k'th slot of the n'th sub-block. Similar to (17), we have

$$\mathbf{y}(s) = \mathbf{G}\mathbf{\Omega}\mathbf{F}\left(\sum_{m=1}^{M} \mathbf{H}_m \mathbf{x}_m(s) + \mathbf{n}(s)\right) + \mathbf{z}(s), \tag{25}$$

where $\mathbf{H}_m = \mathbf{I}_N \otimes$ *diag* $\{h_{m,1}, h_{m,2}, \cdots, h_{m,K}\}$, $\mathbf{x}_m(s) = [x_{m,1,1}(s), x_{m,1,2}(s), \cdots, x_{m,N,K}(s)]^T$, and $\mathbf{y}_s, \mathbf{n}_s, \mathbf{z}_s, \mathbf{G}, \mathbf{\Omega}, \mathbf{F}$ are defined in the proof of Theorem 2, . Again, we observe that the matrix of the entire channel from each of the transmitters to the receiver acts as a MIMO channel with a lower triangular matrix of size $NK \times NK$.

Here, the outage event occurs whenever there exists a subset $\mathcal{S} \subseteq \{1, 2, \ldots, M\}$ of the transmitters that

$$I\left(\mathbf{x}_{\mathcal{S}}(s); \mathbf{y}(s) | \mathbf{x}_{\mathcal{S}^c}(s)\right) \leq (NK+1)\left(\sum_{m\in\mathcal{S}} r_m\right)\log(P). \tag{26}$$

This event is equivalent to

$$\log\left|\mathbf{I}_{KN} + P\mathbf{H}_T\mathbf{H}_T^H\mathbf{P}_n^{-1}\right| \leq (NK+1)\left(\sum_{m\in\mathcal{S}} r_m\right)\log\left(P\right). \tag{27}$$

where $\mathbf{P}_N$ is defined in Theorem 2, $\mathbf{H}_T = \mathbf{G}\mathbf{\Omega}\mathbf{F}\mathbf{H}_{\mathcal{S}}$, and

$$\mathbf{H}_{\mathcal{S}} = \mathbf{I}_N \otimes \textit{diag}\left\{\sqrt{\sum_{m\in\mathcal{S}} |h_{m,1}|^2}, \sqrt{\sum_{m\in\mathcal{S}} |h_{m,2}|^2}, \cdots, \sqrt{\sum_{m\in\mathcal{S}} |h_{m,K}|^2}\right\}. \tag{28}$$

Defining such an event as $\mathcal{E}_{\mathcal{S}}$ and the outage event as $\mathcal{E}$, we have

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{E}\right\} &= \mathbb{P}\left\{\bigcup_{\mathcal{S}\subseteq\{1,2,\ldots,M\}} \mathcal{E}_S\right\} \\
&\leq \sum_{\mathcal{S}\subseteq\{1,2,\ldots,M\}} \mathbb{P}\left\{\mathcal{E}_S\right\} \\
&\leq (2^M - 1) \max_{\mathcal{S}\subseteq\{1,2,\ldots,M\}} \mathbb{P}\left\{\mathcal{E}_S\right\} \\
&\doteq \max_{\mathcal{S}\subseteq\{1,2,\ldots,M\}} \mathbb{P}\left\{\mathcal{E}_S\right\}.
\end{aligned} \tag{29}$$

Hence, it is sufficient to upper-bound $\mathbb{P}\left\{\mathcal{E}_S\right\}$ for all $\mathcal{S}$.

Defining $\hat{\mathbf{H}}_{\mathcal{S}} = \mathbf{I}_N \otimes \textit{diag}\left\{\max_{m\in\mathcal{S}} |h_{m,1}|, \max_{m\in\mathcal{S}} |h_{m,2}|, \cdots, \max_{m\in\mathcal{S}} |h_{m,K}|\right\}$, we have $\hat{\mathbf{H}}_{\mathcal{S}}\hat{\mathbf{H}}_{\mathcal{S}}^H \preccurlyeq \mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H$. Therefore,

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{E}_S\right\} &\leq \mathbb{P}\left\{\log\left|\mathbf{I}_{KN} + P\mathbf{G}\mathbf{\Omega}\mathbf{F}\hat{\mathbf{H}}_{\mathcal{S}}\hat{\mathbf{H}}_{\mathcal{S}}^H\mathbf{F}^H\mathbf{\Omega}^H\mathbf{G}^H\mathbf{P}_n^{-1}\right| \leq (NK+1)\left(\sum_{m\in\mathcal{S}} r_m\right)\log\left(P\right)\right\} \\
&\triangleq \mathbb{P}\left\{\hat{\mathcal{E}}_{\mathcal{S}}\right\}.
\end{aligned} \tag{30}$$

Assume $\max_{m\in\mathcal{S}} |h_{m,k}|^2 = P^{-\mu(k)}$, and $|g(k)|^2 = P^{-\nu(k)}$, $|i(k)|^2 = P^{-\omega(k)}$, and $\mathcal{R}$ as the region in $\mathbb{R}^{3K}$ that defines the outage event $\mathcal{E}$ in terms of the vector $[\mu, \nu, \omega]$. Again, since

$\mathbb{P}\left\{\mathcal{R}\bigcap\mathbb{R}_{-}^{3K}\right\} \doteq P^{-\infty}$, we can say $\mathbb{P}\left\{\mathcal{R}\right\} \doteq \mathbb{P}\left\{\mathcal{R}_{+}\right\}$ where $\mathcal{R}_{+} = \mathcal{R}\bigcap\mathbb{R}_{+}^{3K}$. Rewriting the equation series of (19), we have

$$\begin{aligned}\mathbb{P}\left\{\hat{\mathcal{E}}_{\mathcal{S}}\right\} \quad \dot{\leq} \quad \mathbb{P}\Bigg\{ & -NK\frac{\log\left[3\left(N^2K^2+1\right)\right]}{\log(P)} + NK\left(1-\sum_{m\in\mathcal{S}} r_m\right) - \sum_{m\in\mathcal{S}} r_m \leq N\sum_{k=1}^{K}\mu(k)+\nu(k), \\ & \mu(k),\nu(k),\omega(k)\geq 0\Bigg\}. \end{aligned} \tag{31}$$

On the other hand, we know for vectors $\mu^0, \nu^0, \omega^0 \geq \mathbf{0}$, we have $\mathbb{P}\left\{\mu \geq \mu^0, \nu \geq \nu^0, \omega \geq \omega^0\right\} \doteq P^{-\mathbf{1}\cdot\left(|\mathcal{S}|\mu^0+\nu^0+\omega^0\right)}$. Similarly to the proof of Theorem 1, by taking derivative with respect to μ, ν we have $f_{\mu,\nu}(\mu,\nu) \doteq P^{-\mathbf{1}\cdot(|\mathcal{S}|\mu+\nu)}$ .Defining the lower bound l_0 as $l_0 = -\frac{\log\left[3\left(N^2K^2+1\right)\right]}{\log(P)} + \left(1-\sum_{m\in\mathcal{S}} r_m\right) - \frac{\sum_{m\in\mathcal{S}} r_m}{NK}$, the new region $\hat{\mathcal{R}}$ as $\hat{\mathcal{R}} = \left\{\mu, \nu \geq \mathbf{0}, \frac{1}{K}\mathbf{1}\cdot(\mu+\nu) \geq l_0\right\}$, the cube $\mathcal{I}$ as $\mathcal{I} = [0, Kl_0]^{2K}$, and for $1 \leq i \leq 2K$, $\mathcal{I}_i^c = [0,\infty)^{i-1} \times [Kl_0, \infty) \times [0,\infty)^{2K-i}$, we observe

$$\begin{aligned}\mathbb{P}\left\{\hat{\mathcal{E}}_{\mathcal{S}}\right\} & \overset{(a)}{\dot{\leq}} \mathbb{P}\{\hat{\mathcal{R}}\} \\ & \leq \int_{\hat{\mathcal{R}}\bigcap\mathcal{I}} f_{\mu,\nu}(\mu,\nu)\, d\mu d\nu + \sum_{i=1}^{2K}\mathbb{P}\left\{[\mu,\nu]\in\hat{\mathcal{R}}\cap\mathcal{I}_i^c\right\} \\ & \dot{\leq} vol(\hat{\mathcal{R}}\cap\mathcal{I})P^{-\min_{[\mu^0,\nu^0]\in\hat{\mathcal{R}}\cap\mathcal{I}}\mathbf{1}\cdot\left(|\mathcal{S}|\mu^0+\nu^0\right)} + 2KP^{-Kl_0} \\ & \overset{(b)}{\doteq} P^{-Kl_0} \\ & \doteq P^{-\left[K\left(1-\sum_{m\in\mathcal{S}} r_m\right)-\frac{\sum_{m\in\mathcal{S}} r_m}{N}\right]}. \end{aligned} \tag{32}$$

Here, (a) follows from (31) and (b) follows from the fact that $\hat{\mathcal{R}}\bigcap\mathcal{I}$ is a bounded region whose volume is independent of P. Comparing (29), (30), and (32), we observe

$$\mathbb{P}\left\{\mathcal{E}\right\}\dot{\leq}\max_{\mathcal{S}\subseteq\{1,2,\ldots,M\}}\mathbb{P}\left\{\mathcal{E}_{\mathcal{S}}\right\}\dot{\leq}\max_{\mathcal{S}\subseteq\{1,2,\ldots,M\}}\mathbb{P}\left\{\hat{\mathcal{E}}_{\mathcal{S}}\right\}\dot{\leq}P^{-\left[K\left(1-\sum_{m=1}^{M} r_m\right)-\frac{\sum_{m=1}^{M} r_m}{N}\right]}. \tag{33}$$

Now, we prove that $K\left(1-\sum_{m=1}^{M} r_m\right)^{+}$ is the upper-bound on the diversity gain of the system corresponding to the sequence rates $r_1, r_2, \ldots, r_M$. To prove, we observe

$$\mathbb{P}\left\{\mathcal{E}\right\} \geq \mathbb{P}\left\{\max_{p(\mathbf{t}_1,\mathbf{t}_2,\ldots,\mathbf{t}_K)} I\left(\mathbf{t}_1,\mathbf{t}_2,\ldots,\mathbf{t}_K;\mathbf{y}\right) \leq \left(\sum_{m=1}^{M} r_m\right)\log(P)\right\} \overset{(a)}{\doteq} P^{-K\left(1-\sum_{m=1}^{M} r_m\right)^{+}}. \tag{34}$$

Here, (a) follows from the DM tradeoff of the point-to-point MISO channel proved in [6]. This completes the proof. ∎

Remark - The statement in the above theorem holds for the general case in which any arbitrary set of the relay pairs are non-interfering. Hence, the proposed scheme achieves the upper-bound of the tradeoff curve in the asymptotic case of $N \to \infty$ for any graph topology on the interfering relay pairs.

In the *Symmetric* situation, i.e., the multiplexing gains of all the users are equal (to say r), the lower-bound function on the diversity in Theorem 3 takes a simple form. First, we observe that the maximum multiplexing gain achievable by each user is $\frac{1}{M} \cdot \frac{KN}{KN+1}$. Noticing that in the SM scheme, the receiver is receiving data in $\frac{NK}{NK+1}$ of the time, we observe the lower-bound in Theorem 3 is tight for the maximum multiplexing gain achievable by the SM scheme. Also, by , we observe that SM scheme achieves the maximum diversity gain K, which turns out to be tight too. Finally, the lower-bound on the DM curve of SM scheme is $\left[K\left(1-Mr\right)-\frac{Mr}{N}\right]^+$ for the *Symmetric* situation.

D. Multiple-Access Single Relay Scenario

As we observe, the proofs stated here sofar are valid for the scenario of having multiple relays ($K > 1$). Indeed, For the case of single relay scenario, both the sequential SAF scheme and its modified version are reduced to the simple AF scheme in which the relay listens to the transmitter for half of the frame and transmits the amplified version of the received signal in the next half of the frame. However, like the case of no interfering relays studied in [1], the statements above are no longer valid for the scenario of single relay network. Indeed, in this scenario, the DDF scheme by [2] achieves the DMT tradeoff when there is no direct link between the transmitters and the receiver.

Theorem 4 *Consider a multiple-access channel consisting of M transmitting nodes aided by a single half-duplex relay. Assume all the network nodes are equipped with single antenna and there is no direct link between the transmitters and the receiver. The amplify-and-forward scheme achieves the DMT curve which is*

$$d_{AF,MAC}(r_1, r_2, \ldots, r_M) = \left(1 - 2\sum_{m=1}^{M} r_m\right)^+. \tag{35}$$

However, the optimum DMT of the network is

$$d_{MAC}(r_1, r_2, \ldots, r_M) = \left(1 - \frac{\sum_{m=1}^{M} r_m}{1-\sum_{m=1}^{M} r_m}\right)^+, \tag{36}$$

which is achievable by the DDF scheme of [2].

Proof: First, we show that the DMT of the AF scheme follows (35). At the receiver side, we have

$$\mathbf{y} = g\alpha\left(\sum_{m=1}^{M} h_m \mathbf{x}_m + \mathbf{n}\right) + \mathbf{z}, \tag{37}$$

where h_m is the channel's gain between the m'th transmitter and the relay, g is the downlink channel's gain, and $\alpha = \sqrt{\frac{P}{P\sum_{m=1}^{M}|h_m|^2+0.5}}$ is the amplification coefficient . Defining the outage event $\mathcal{E}_{\mathcal{S}}$ for a s set $\mathcal{S} \subseteq \{1, 2, \ldots, M\}$ the same way defined in Theorem 3, we have

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{E}_{\mathcal{S}}\right\} &= \mathbb{P}\left\{ I\left(\mathbf{x}_{\mathcal{S}}; \mathbf{y}|\mathbf{x}_{\mathcal{S}^c}\right) < 2\left(\sum_{m\in\mathcal{S}} r_m\right)\log(P)\right\} \\
&= \mathbb{P}\left\{\log\left(1 + P\left(\sum_{m\in\mathcal{S}}|h_m|^2\right)|g|^2|\alpha|^2\left(0.5+0.5|g|^2|\alpha|^2\right)^{-1}\right) < \right. \\
&\quad \left. 2\left(\sum_{m\in\mathcal{S}} r_m\right)\log(P)\right\} \\
&\doteq \mathbb{P}\left\{\left(\sum_{m\in\mathcal{S}}|h_m|^2\right)|g|^2|\alpha|^2\min\left\{1, \frac{1}{|g|^2|\alpha|^2}\right\} \le P^{-\left(1-2\sum_{m\in\mathcal{S}} r_m\right)}\right\} \\
&\doteq \mathbb{P}\left\{\sum_{m\in\mathcal{S}}|h_m|^2 \le P^{-\left(1-2\sum_{m\in\mathcal{S}} r_m\right)}\right\} + \\
&\quad \mathbb{P}\left\{|g|^2\left(\sum_{m\in\mathcal{S}}|h_m|^2\right)\min\left\{P, \frac{1}{2\left(\sum_{m=1}^{M}|h_m|^2\right)}\right\} \le P^{-\left(1-2\sum_{m\in\mathcal{S}} r_m\right)}\right\} \\
&\doteq \prod_{m\in\mathcal{S}}\mathbb{P}\left\{|h_m|^2 \le P^{-\left(1-2\sum_{m\in\mathcal{S}} r_m\right)}\right\} + \\
&\quad \mathbb{P}\left\{|g|^2\left(\sum_{m\in\mathcal{S}}|h_m|^2\right) \le P^{-2\left(1-\sum_{m\in\mathcal{S}} r_m\right)}\right\} + \\
&\quad \mathbb{P}\left\{\frac{|g|^2\sum_{m\in\mathcal{S}}|h_m|^2}{\sum_{m=1}^{M}|h_m|^2} \le 2P^{-\left(1-2\sum_{m\in\mathcal{S}} r_m\right)}\right\}.
\end{aligned} \tag{38}$$

To compute the second term in (38), we have

$$\mathbb{P}\left\{|g|^2\left(\sum_{m\in\mathcal{S}}|h_m|^2\right) \le \epsilon\right\} \le \mathbb{P}\left\{|g|^2|h_m|^2 \le \epsilon\right\} \doteq \epsilon. \tag{39}$$

On the other hand, we have

$$\mathbb{P}\left\{|g|^2\left(\sum_{m\in\mathcal{S}}|h_m|^2\right) \le \epsilon\right\} \ge \mathbb{P}\left\{|g|^2 \le \epsilon\right\}\prod_{m\in\mathcal{S}}\mathbb{P}\left\{|h_m|^2 \le \frac{1}{M}\right\} \doteq \epsilon. \tag{40}$$

Putting (39) and (40) together, we have

$$\mathbb{P}\left\{|g|^2\left(\sum_{m\in\mathcal{S}}|h_m|^2\right)\leq\epsilon\right\}\doteq\epsilon. \tag{41}$$

Now, to compute the third term in (38), we observe

$$\epsilon\doteq\mathbb{P}\left\{|g|^2\leq\epsilon\right\}\leq\mathbb{P}\left\{|g|^2\frac{\sum_{m\in\mathcal{S}}|h_m|^2}{\sum_{m=1}^{M}|h_m|^2}\leq\epsilon\right\}\overset{(a)}{\dot{\leq}}\mathbb{P}\left\{|g|^2\left(\sum_{m\in\mathcal{S}}|h_m|^2\right)\leq\epsilon\right\}\overset{(b)}{\doteq}\epsilon.$$

Here, (a) follows from the fact that with probability one (more precisely, with probability greater than $1-P^{-\delta}$ for every $\delta>0$) we have $|h_m|\dot{\leq}1$ and (b) follows from (41). As a result

$$\mathbb{P}\left\{|g|^2\frac{\sum_{m\in\mathcal{S}}|h_m|^2}{\sum_{m=1}^{M}|h_m|^2}\leq\epsilon\right\}\doteq\epsilon \tag{42}$$

From (38), (41), and (42), we have

$$\mathbb{P}\left\{\mathcal{E}_{\mathcal{S}}\right\}\doteq P^{-|\mathcal{S}|\left(1-2\sum_{m\in\mathcal{S}}r_m\right)}+P^{-2\left(1-\sum_{m\in\mathcal{S}}r_m\right)}+P^{-\left(1-2\sum_{m\in\mathcal{S}}r_m\right)}\doteq P^{-\left(1-2\sum_{m\in\mathcal{S}}r_m\right)^+}. \tag{43}$$

Observing (43) and applying the argument of (29), we have

$$\mathbb{P}\left\{\mathcal{E}\right\}\doteq\max_{\mathcal{S}\subseteq\{1,2,\ldots,M\}}\mathbb{P}\left\{\mathcal{E}_{\mathcal{S}}\right\}\doteq P^{-\left(1-2\sum_{m=1}^{M}r_m\right)^+}. \tag{44}$$

This completes the proof for the AF scheme. Now, to compute the diversity-multiplexing tradeoff for the DDF scheme, assume the relay listens to the transmitted signal for the l portion of the time. Hence, we have

$$l=\min\left\{1,\max_{\mathcal{S}\subseteq\{1,2,\ldots,M\}}\frac{\left(\sum_{m\in\mathcal{S}}r_m\right)\log(P)}{\log\left(1+\left(\sum_{m\in\mathcal{S}}|h_m|^2\right)P\right)}\right\}. \tag{45}$$

The outage event is occured whenever the relay can not transmit the reencoded information bits in the remaining portion of the time. Hence, we have

$$\mathbb{P}\left\{\mathcal{E}\right\}\doteq\mathbb{P}\left\{(1-l)\log\left(1+|g|^2P\right)<\left(\sum_{m=1}^{M}r_m\right)\log(P)\right\}. \tag{46}$$

Assuming $|h_m|^2=P^{-\mu_m}$ and $|g|^2=P^{-\nu}$, at high SNR we have

$$l\approx\min\left\{1,\max_{\mathcal{S}\subseteq\{1,2,\ldots,M\}}\frac{\sum_{m\in\mathcal{S}}r_m}{1-\min_{m\in\mathcal{S}}\mu_m}\right\} \tag{47}$$

Equivalently, an outage event is occured whenever

$$\left(1-\max_{\mathcal{S}\subseteq\{1,2,\ldots,M\}}\frac{\sum_{m\in\mathcal{S}}r_m}{1-\min_{m\in\mathcal{S}}\mu_m}\right)(1-\nu)<\sum_{m=1}^{M}r_m. \tag{48}$$

We are looking for the vector point $[\mu_1, \mu_2, \ldots, \mu_M, \nu]$ in the outage region $\mathcal{R}$, i.e. the region that satisfies (48), for which $\nu + \sum_{m=1}^{M} \mu_m$ is minimized. To find such a point, assume the subset $\mathcal{S}_0$ takes the maximum value in (48). Defining $R = \sum_{m=1}^{M} r_M$ and $\mu = \sum_{m=1}^{M} \mu_m$, we have

$$R \stackrel{(a)}{>} \left(1 - \frac{\sum_{m \in \mathcal{S}_0} r_m}{1 - \min_{m \in \mathcal{S}_0} \mu_m}\right)(1-\nu) > \left(1 - \frac{R}{1-\mu}\right)(1-\nu). \tag{49}$$

Here, (a) follows from (48). Equivalently,

$$R \stackrel{(a)}{>} \frac{(1-\mu)(1-\nu)}{(1-\mu)+(1-\nu)} > \frac{1-\mu-\nu}{(1-\mu)+(1-\nu)}. \tag{50}$$

Here, (a) follows from (49). It easily can be checked that (50) is equivalent to

$$R > (1-R)(1-\mu-\nu). \tag{51}$$

which is the equivalent condition to (48) for the vector $\left[0, 0, \ldots, 0, \nu + \sum_{m=1}^{M} \mu_m\right]$ to be in $\mathcal{R}$. Hence, having the vector $[\mu_1, \mu_2, \ldots, \mu_M, \nu]$ in the outage region, we can conclude that $\left[0, 0, \ldots, 0, \nu + \sum_{m=1}^{M} \mu_m\right] \in \mathcal{R}$. Applying the same argument as in the proof of Theorem 3, we have

$$\begin{aligned}
\mathbb{P}\{\mathcal{E}\} &\doteq \mathbb{P}\{[\mu_1, \mu_2, \ldots, \mu_M, \nu] \in \mathcal{R}\} \\
&\doteq P^{-\left(\min_{[\mu_1,\mu_2,\ldots,\mu_M,\nu]\in\mathcal{R}} \nu + \sum_{m=1}^{M} \mu_m\right)} \\
&\doteq P^{-\left(\min_{[0,0,\ldots,0,\nu]\in\mathcal{R}} \nu\right)} \\
&\doteq P^{-\left(1 - \frac{\sum_{m=1}^{M} r_m}{1 - \sum_{m=1}^{M} r_m}\right)^{+}}.
\end{aligned} \tag{52}$$

This completes the proof for the diversity-multiplexing tradeoff analysis of the DDF scheme. Now, we porve that the DDF scheme achieves the optimal diversity-multiplexing tradeoff. It easily can be seen that for any channel realization of the network, the DDF scheme achieves the capacity of the MAC single relay network with no direct transmitter-receiver link. Hence, the outage region of the DDF scheme, i.e. the region of the channel realizations that the point $(r_1, r_2, \ldots, r_M)$ is outside of the DDF scheme's achievable rate region, is a subset of any other scheme's outage region. This completes the proof of the Theorem. ■

V. Conclusion

A simple scheme based on the sequential SAF scheme is proposed and its performance is studied in the multiple-access multiple relays scenario in which there is no direct link between the transmitters and the receiver. In the case of no-interfering relays, the diversity-multiplexing tradeoff of the scheme is derived and is shown to achieve the optimum tradeoff for large values of N, the number of sub-blocks. Also, in the general scenario where the graph of interfering relays can have any topology, a lower-bound is derived for the diversity-multiplexing tradeoff of the scheme and is shown to achieve the optimum diversity-multiplexing tradeoff for asymptotic values of N. However, in the case of multiple-access channel assisted with a single relay, while it is shown that the proposed scheme is unable to follow the optimum diversity-multiplexing tradeoff, the DDF scheme of ([2]) is shown to perform optimum in this scenario.

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