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**Throughput Scaling Laws for Wireless Networks
with Fading Channels**

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Abstract

A network of n communication links, operating over a shared wireless channel, is considered. Fading is assumed to be the dominant factor affecting the strength of the channels between transmitter and receiver terminals. It is assumed that each link can be active and transmit with a constant power P or remain silent. The objective is to maximize the throughput over the selection of active links. By deriving an upper bound and a lower bound, it is shown that in the case of Rayleigh fading (i) the maximum throughput scales like $\log n$ (ii) the maximum throughput is achievable in a distributed fashion. The upper bound is obtained using probabilistic methods, where the key point is to upper bound the throughput of any random set of active links by a chi-squared random variable. To obtain the lower bound, a decentralized link activation strategy is proposed and analyzed.

Index Terms

Wireless network, fading channel, throughput, scaling law, decentralized link activation.

I. INTRODUCTION

In a wireless network, a number of source nodes transmit data to their designated destination nodes through a shared wireless channel. Analysis and design of such configurations, even in

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their simplest forms, have been among the most difficult problems facing the network information theory community for many years.

Followed by the pioneering work of Gupta and Kumar [1], considerable attention has been paid to investigate how the throughput of wireless networks scales with n , the number of nodes, when n is large. This has been done assuming different network topologies, traffic patterns, protocol schemes, and channel models [1]–[10]. Most of these works consider a channel model in which the signal power decays according to a distance-based attenuation law [1]–[7]. However, in a wireless environment the presence of obstacles and scatterers adds some randomness to the received signal. This random behavior of the channel, known as fading, can drastically change the scaling laws of a network in both multihop [8]–[10] and single-hop scenarios [11, Chapter 8], [12], [13].

In this paper, we follow the model of [10], [11], where fading is assumed to be the dominant factor affecting the strength of the channels between nodes. Despite the randomness of the channel, we are only interested in events that occur asymptotically almost surely, i.e., with probability tending to one as $n \rightarrow \infty$. Such a deterministic approach to random wireless networks has been also adopted in [5], [7], where the nodes' locations are random.

We consider a single-hop scenario, i.e., a network structure in which the transmitters send data to their corresponding receivers directly and without utilizing other nodes as routers. It is assumed that each link can be active and transmit with a constant power P or remain silent. The objective is to maximize the throughput over all sets of active links. We propose a threshold-based link activation strategy in which each link is active if and only if its channel gain is above some predetermined threshold. The decision on being active can be made at the receivers, where their own channel gains are estimated and a single-bit command data is fed back to the transmitters. Hence, there is no need for the exchange of information between links. Consequently, this method can be implemented in a decentralized fashion. We analyze this method for a general fading model and show how to obtain the value of the activation threshold to maximize the throughput. As an example, we derive a closed form expression for the achievable throughput in the Rayleigh fading environment.

Using probabilistic methods, we derive an upper bound on the achievable throughput when the channel is Rayleigh fading. Interestingly, this upper bound scales the same as the lower bound achieved by the proposed strategy. This proves the asymptotic optimality of the proposed

technique among all link activation strategies.

In addition to the channel modeling, [10] is a relevant work in the sense that transmissions occur with the same power and the objective is to maximize the throughput. However, they allow multihop communication in their scheme. Their proposed scheme requires a central unit which is aware of all channel conditions and decides on active source-destination pairs and the paths between them. Despite this complexity, the achievable throughput of their method in the popular model of Rayleigh fading is by a factor of 4 less than the value obtained in this work for a more restricted configuration, i.e., single-hop networks with decentralized management¹.

The rest of the paper is organized as follows: In Section II, the network model and problem formulation are presented. By proposing a decentralized link activation strategy, a lower bound on the network throughput is derived in Section III. In Section IV, we prove the optimality of the proposed decentralized method in a Rayleigh fading environment. Finally, we conclude the paper in Section V.

Notation: \mathcal{N}_n represents the set of natural numbers less than or equal to n ; $\log(\cdot)$ is the natural logarithm function; $P(A)$ denotes the probability of event A ; $E(x)$ represents the expected value of the random variable x ; \approx means approximate equality; for any functions $f(n)$ and $h(n)$, $h(n) = O(f(n))$ is equivalent to $\lim_{n \rightarrow \infty} |h(n)/f(n)| < \infty$, $h(n) = o(f(n))$ is equivalent to $\lim_{n \rightarrow \infty} |h(n)/f(n)| = 0$, $h(n) = \omega(f(n))$ is equivalent to $\lim_{n \rightarrow \infty} |h(n)/f(n)| = \infty$, and $h(n) \sim f(n)$ is equivalent to $\lim_{n \rightarrow \infty} h(n)/f(n) = 1$; an event A_n holds *asymptotically almost surely* (a.a.s) if $P(A_n) \rightarrow 1$ as $n \rightarrow \infty$.

II. NETWORK MODEL AND PROBLEM FORMULATION

We consider a wireless communication network with n pairs of transmitters and receivers. These n communication links are indexed by the elements of \mathcal{N}_n . Each transmitter aims to send data to its corresponding receiver in a single-hop fashion. The transmit power of link i is denoted by p_i . It is assumed that the links follow an on-off paradigm, i.e., $p_i \in \{0, P\}$, where P is a constant. Hence, any power allocation scheme translates to a *link activation strategy (LAS)*. Any

¹Note that the method proposed in [10] is general and can be applied for any fading distribution. Here, we have made the comparison just for the Rayleigh fading model. For the comparison in other fading environments, see [14].

LAS yields a set of active links \mathcal{A} , which describes the transmission powers as

$$p_i = \begin{cases} P & \text{if } i \in \mathcal{A} \\ 0 & \text{if } i \notin \mathcal{A} \end{cases}. \quad (1)$$

The channel between transmitter j and receiver i is characterized by the coefficient g_{ji} . This means the received power from transmitter j at the receiver i equals $g_{ji}p_j$. We assume that the channel coefficients are *independent identically distributed* (i.i.d.) random variables drawn from a pdf $f(x)$ with mean μ and variance σ^2 . The channel between transmitter i and receiver i is simply referred to as the *direct channel* of link i .

We consider an additive white Gaussian noise (AWGN) with limited variance η at the receivers. The transmit *signal-to-noise ratio* (SNR) of the network is defined as

$$\rho = \frac{P}{\eta}. \quad (2)$$

The receivers are conventional linear receivers, i.e., without multiuser detection. Since the transmissions occur simultaneously within the same environment, the signal from each transmitter acts as interference for other links. Assuming Gaussian signal transmission from all links, the distribution of the interference will be Gaussian as well. Thus, according to the Shannon capacity formula, the maximum supportable rate of link $i \in \mathcal{A}$ is obtained as

$$r_i(\mathcal{A}) = \log(1 + \gamma_i(\mathcal{A})) \text{ nats/channel use}, \quad (3)$$

where

$$\gamma_i(\mathcal{A}) = \frac{g_{ii}}{1/\rho + \sum_{\substack{j \in \mathcal{A} \\ j \neq i}} g_{ji}} \quad (4)$$

is the *signal-to-interference-plus-noise ratio* (SINR) of link i .

As a measure of performance, in this paper we consider the throughput of the network, which is defined as

$$T(\mathcal{A}) = \sum_{i \in \mathcal{A}} r_i(\mathcal{A}). \quad (5)$$

Also, the average rate per active link, or simply rate-per-link, is defined as

$$\bar{r}(\mathcal{A}) = \frac{T(\mathcal{A})}{|\mathcal{A}|}. \quad (6)$$

In this paper, wherever there is no ambiguity, we drop the functionality of \mathcal{A} from the network parameters and simply refer to them as r_i , γ_i , T , or \bar{r} .

The problem of throughput maximization is described as

$$\max_{\mathcal{A} \subseteq \mathcal{N}_n} T(\mathcal{A}). \quad (7)$$

We denote the maximum value of this problem by T^* . Due to the nonconvex and integral nature of the throughput maximization problem, its solution is computationally intensive. However, in this paper we propose and analyze a decentralized LAS which leads to efficient solutions for the above problem. Indeed, we show that the proposed strategy a.a.s. achieves the optimum solution of the throughput maximization problem in Rayleigh fading environment.

III. ACHIEVABILITY RESULT

In this section, to derive a lower bound on the network throughput, we propose a simple heuristic LAS, which we call a *threshold-based LAS (TBLAS)*. Due to the randomness of the channel, the achievable throughput of the proposed strategy is a random variable; however, our analysis yields a deterministic lower bound which is a.a.s. achievable.

TBLAS: For a threshold Δ , choose the set of active links according to the following rule

$$i \in \mathcal{A} \quad \text{iff} \quad g_{ii} > \Delta. \quad (8)$$

If each transmitter is aware of the threshold Δ and its direct channel coefficient, it can individually determine its transmit power. Hence, TBLAS can be implemented in a *decentralized* fashion. To obtain the optimum value of Δ , we should first know the achievable throughput of TBLAS in terms of Δ .

Let $k_\Delta = |\mathcal{A}|$ denote the number of active links chosen by TBLAS with a threshold Δ . Without loss of generality, we assume that $\mathcal{A} = \{1, 2, \dots, k_\Delta\}$. By defining $I_i = \sum_{\substack{j=1 \\ j \neq i}}^{k_\Delta} g_{ji}$ and using (3), (4), and (5), the throughput can be lower bounded as

$$T > \sum_{i=1}^{k_\Delta} \log \left(1 + \frac{\Delta}{1/\rho + I_i} \right) \quad (9)$$

$$\geq k_\Delta \log \left(1 + \frac{\Delta}{1/\rho + \frac{1}{k_\Delta} \sum_{i=1}^{k_\Delta} I_i} \right), \quad (10)$$

where the first equality is based on the fact that $g_{ii} > \Delta$ for the active links and the second one is the result of applying the Jensen's inequality.

To simplify the RHS of (10), we apply the Chebyshev inequality to obtain the upper bound

$$\frac{1}{k_\Delta} \sum_{i=1}^{k_\Delta} I_i < (k_\Delta - 1)\mu + \psi, \quad (11)$$

which holds a.a.s. for any $\psi = \omega(1)$. Consequently, the lower bound (10) becomes

$$T > k_\Delta \log \left(1 + \frac{\Delta}{\mu k_\Delta + \psi} \right), \quad a.a.s. \quad (12)$$

Note that the constant $1/\rho - \mu$ is absorbed in the function ψ . Let q_Δ denote the probability of a link being active. We have $q_\Delta = 1 - F(\Delta)$, where $F(x)$ is the cumulative distribution function (cdf) of the channel gains. Due to the TBLAS, the number of active links, k_Δ , is a binomial random variable with parameters n and q_Δ . Using the Chebyshev inequality, it can be shown that k_Δ a.a.s. satisfies the lower bound

$$k_\Delta > nq_\Delta - \xi\sqrt{nq_\Delta}, \quad (13)$$

for any $\xi = \omega(1)$. Noting that for $\psi = o(k_\Delta)$, the lower bound (12) becomes an increasing function of k_Δ , and by using (13), we obtain the main result of this section, which is an achievability result on the throughput.

Theorem 1: Consider a wireless network with n links and i.i.d. random channel coefficients with pdf $f(x)$, cdf $F(x)$, and mean μ . Choose any $\Delta > 0$ and define $q_\Delta = 1 - F(\Delta)$. Then, a throughput of

$$T_a(\Delta) = (nq_\Delta - \xi\sqrt{nq_\Delta}) \log \left(1 + \frac{\Delta}{\mu(nq_\Delta - \xi\sqrt{nq_\Delta}) + \psi} \right) \quad (14)$$

is a.a.s. achievable for any $\xi = \omega(1)$ that satisfies $\xi = o(\sqrt{nq_\Delta})$ and any $\psi = \omega(1)$ that satisfies $\psi = o(nq_\Delta)$.

Note that the achievable throughput $T_a(\Delta)$ is a deterministic value. It easily follows that under the conditions described in Theorem 1, the number of active links and the achievable average rate-per-link in TBLAS scale as [14]

$$k_\Delta \sim nq_\Delta \quad a.a.s. \quad (15)$$

$$\bar{r}_\Delta \sim \log \left(1 + \frac{\Delta}{\mu(nq_\Delta - \xi\sqrt{nq_\Delta}) + \psi} \right) \quad a.a.s. \quad (16)$$

As specified in Theorem 1, the achievable throughput of TBLAS is a function of the parameter Δ . Thus, Δ can be chosen such that the achievable throughput is maximized. Let us define

$$\Delta^* = \arg \max_{\Delta} T_a(\Delta), \quad (17)$$

and

$$T_a^* = \max_{\Delta} T_a(\Delta). \quad (18)$$

In the following example, we clarify how to obtain these values for the popular Rayleigh fading model.

Example: In a Rayleigh fading channel, $f(x) = e^{-x}$, $\mu = 1$, and $q_{\Delta} = e^{-\Delta}$. By substituting q_{Δ} in (14), we obtain

$$T_a(\Delta) = \left(ne^{-\Delta} - \xi \sqrt{ne^{-\Delta}} \right) \log \left(1 + \frac{\Delta}{ne^{-\Delta} - \xi \sqrt{ne^{-\Delta}} + \psi} \right). \quad (19)$$

The result of maximizing this function over Δ is given in the following corollary.

Corollary 2: Assuming Rayleigh fading, we have

$$\Delta^* = \log n - 2 \log \log n + \log 2 + O \left(\frac{\log \log n}{\log n} \right), \quad (20)$$

$$T_a^* = \log n - 2 \log \log n + \log(2/e) + O \left(\frac{\log \log n}{\log n} \right), \quad a.a.s. \quad (21)$$

$$k_{\Delta^*} = \frac{1}{2} \log^2 n (1 + o(1)), \quad a.a.s., \quad (22)$$

$$\bar{r}_{\Delta^*} = \frac{2}{\log n} (1 + o(1)), \quad a.a.s. \quad (23)$$

Proof: see the Appendix. ■

The throughput scaling law of $\log n$ is, by a factor of 4, larger than the value obtained in [10] in a centralized and multihop scenario.

IV. OPTIMALITY RESULT

In this section, we provide an upper bound on the maximum throughput of the wireless network in a Rayleigh fading environment. First, we need the following lemma that provides a lower bound on the number of active links.

Lemma 3: In the optimum LAS, the number of active links, k^* , a.a.s. satisfies

$$k^* \geq \frac{\log n}{\log \log n} \left(1 + O \left(\frac{1}{\log \log n} \right) \right). \quad (24)$$

Proof: It can be shown that

$$g_{ii} \leq \log n + \varphi, \quad \forall i \in \mathcal{N}_n, \quad a.a.s., \quad (25)$$

for any $\varphi = \omega(1)$. In the following, we assume $\varphi = o(\log n)$. By ignoring the interference term and using the above upper bound, the maximum throughput is upper bounded as

$$T^* \leq k^* \log \left(1 + \frac{\log n + \varphi}{1/\rho} \right), \quad a.a.s. \quad (26)$$

Combining this upper bound with the lower bound in (21), we obtain

$$k^* \geq \frac{\log n + O(\log \log n)}{\log \left(1 + \frac{\log n + \varphi}{1/\rho} \right)} \quad (27)$$

$$= \frac{\log n}{\log \log n} \left(1 + O \left(\frac{1}{\log \log n} \right) \right), \quad (28)$$

where the equality is obtained by using $\varphi = o(\log n)$. ■

Theorem 4: Consider a wireless network with n links and i.i.d. random channel coefficients drawn from an exponential distribution with mean $\mu = 1$. The maximum throughput over all sets of active links is a.a.s. upper bounded as

$$T^* \leq \log n + \log \log n (1 + o(1)). \quad (29)$$

Proof: For a randomly selected set of active links \mathcal{A} with $|\mathcal{A}| = k^*$, the interference term $I_i = \sum_{\substack{j \in \mathcal{A} \\ j \neq i}} g_{ji}$ in the denominator of (4) has $\chi^2(2k^* - 2)$ distribution. Hence, we have

$$\begin{aligned} \mathbf{P}(\gamma_i > x) &= \int_0^\infty \mathbf{P}(\gamma_i > x | I_i = z) f_{I_i}(z) dz \\ &= \int_0^\infty e^{-x(1/\rho+z)} \frac{z^{k^*-2} e^{-z}}{(k^* - 2)!} dz \\ &= \frac{e^{-x/\rho}}{(1+x)^{k^*-1}}. \end{aligned} \quad (30)$$

Consequently, by using (3), we obtain

$$\begin{aligned} \mathbf{P}(r_i > x) &= \mathbf{P}(\gamma_i > e^x - 1) \\ &= \frac{e^{-(e^x-1)/\rho}}{e^{(k^*-1)x}}. \end{aligned} \quad (31)$$

By defining $X_i = r_i + \frac{e^{r_i} - 1}{\rho(k^* - 1)}$ and using (31), it can be shown that X_i is exponentially distributed with mean $\frac{1}{k^*-1}$. On the other hand, from the definition of X_i it is clear that $X_i \geq r_i$.

Thus, the throughput $T(\mathcal{A}) = \sum_{i \in \mathcal{A}} r_i$ is upper bounded as

$$T(\mathcal{A}) \leq \sum_{i \in \mathcal{A}} X_i. \quad (32)$$

Consequently, we have

$$\mathbf{P}(T(\mathcal{A}) > x) \leq \mathbf{P}\left(\sum_{i \in \mathcal{A}} X_i > x\right) \quad (33)$$

$$\stackrel{(a)}{=} e^{-(k^*-1)x} \sum_{m=0}^{k^*-1} \frac{((k^*-1)x)^m}{m!} \quad (34)$$

$$\stackrel{(b)}{<} k^* e^{-(k^*-1)x} \frac{((k^*-1)x)^{k^*-1}}{(k^*-1)!} \quad (35)$$

$$\stackrel{(c)}{\approx} \sqrt{k^*} e^{-(k^*-1)(x-1)} x^{k^*-1}, \quad (36)$$

where (a) is because $\sum_{i \in \mathcal{A}} X_i$ has $\chi^2(2k^*)$ distribution, (b) is because the maximum of the summand terms occurs at $m = k - 1$ for large enough x ², and (c) is obtained by applying the Stirling's approximation for the factorial, i.e., $m! \approx \sqrt{2\pi m} m^m e^{-m}$.

Assume \mathcal{L} is the event that there exists at least one set $\mathcal{A} \subseteq \mathcal{N}_n$ with $|\mathcal{A}| = k^*$ such that $T(\mathcal{A}) > x$. We have

$$\mathbf{p}(\mathcal{L}) \leq \binom{n}{k^*} \mathbf{P}(T(\mathcal{A}) > x) \quad (37)$$

$$< \left(\frac{ne}{k^*}\right)^{k^*} \sqrt{k^*} e^{-(k^*-1)(x-1)} x^{k^*-1} \quad (38)$$

$$< \exp(\mathcal{E}(x, k^*)), \quad (39)$$

where the first inequality is due to the union bound, the second inequality is due to (36) and the Stirling's approximation, and $\mathcal{E}(x, k^*)$ is defined as

$$\mathcal{E}(x, k^*) = k^*(\log n - x - \log k^* + \log x + 2) + \frac{1}{2} \log k^* + x. \quad (40)$$

For $x = \log n + \log \log n + 2 \log \log \log n$, we have

$$\begin{aligned} \mathcal{E}(x, k^*) &\approx -k^*(2 \log \log \log n + \log k^* - 2) + \frac{1}{2} \log k^* \\ &\quad + \log n + \log \log n(1 + o(1)). \end{aligned} \quad (41)$$

Noting that the RHS of (41) is a decreasing function in k^* , we can replace k^* by its lower bound from Lemma 3 to obtain the upper bound

$$\mathcal{E}(x, k^*) \leq -\frac{\log \log \log n}{\log \log n} \log n(1 + o(1)). \quad (42)$$

²Since we are seeking an upper bound on the throughput, x is at least of order $\log n$. This value is large enough to satisfy the mentioned condition.

Since the RHS of (42) goes to $-\infty$ when $n \rightarrow \infty$, from (39) we conclude that $p(\mathcal{L}) \rightarrow 0$. This means, with probability approaching 1, there does not exist any set \mathcal{A} that achieves a throughput larger than $x = \log n + \log \log n(1 + o(1))$. This completes the proof. ■

It should be noted that an upper bound of $2 \log n$ has been derived in [10]. However, this larger upper bound is obtained in a different scenario than ours; they consider a rate constraint for the active links as well as the possibility of transmitter-receiver assignment.

Comparison between the achievability result in Corollary 2 and the upper bound in Theorem 4 reveals the following result.

Theorem 5: Consider a wireless network with n links and i.i.d. random channel coefficients drawn from an exponential distribution with mean $\mu = 1$. Then, the maximum throughput a.a.s. scales like $\log n$. Moreover, this maximum throughput scaling law is a.a.s. achieved by the distributed TBLAS presented in Section III.

V. CONCLUSION

In this paper, the throughput of single-hop wireless networks with on-off strategy is investigated in a fading environment. To obtain a lower bound on the throughput, a decentralized link activation strategy is proposed and analyzed for a general fading model. It is shown that in the popular model of Rayleigh fading a throughput of order $\log n$ is achievable, which is by a factor of four larger than what was obtained in previous works with centralized methods [10]. Moreover, for the Rayleigh fading model, an upper bound of order $\log n$ is obtained that shows the optimality of the proposed link activation strategy.

APPENDIX

PROOF OF COROLLARY 2

The optimum value of the threshold, Δ^* , is the value that maximizes the achievable throughput in (19). As it is seen, $T_a(\Delta)$ is a complicated function of Δ . However, since ξ can grow as slow as desired, we can set $\xi = 0$ to obtain a more tractable form for $T_a(\Delta)$ from which a *zero order approximation* of the solution is obtained. In the next stage, we will improve the solution using this zero order approximation.

a) *Zero order approximation:* By setting $\xi = 0$, the objective function in (19) is transformed to

$$\hat{T}_a(\Delta) = ne^{-\Delta} \log \left(1 + \frac{\Delta}{ne^{-\Delta}} \right). \quad (43)$$

Using the approximation $\log(1+x) \approx x - \frac{x^2}{2}$, the above function can be approximated as

$$\hat{T}_a(\Delta) \approx \Delta - \frac{\Delta^2}{2ne^{-\Delta}}. \quad (44)$$

By taking the derivative of this function, it can be shown that the value of Δ that maximizes $\hat{T}_a(\Delta)$, satisfies the equation

$$2ne^{-\Delta} = 2\Delta + \Delta^2. \quad (45)$$

Denoting the solution of this equation by $\Delta_{(0)}^*$, it can be verified that

$$\Delta_{(0)}^* = \log n - 2 \log \log n + \log 2 + O \left(\frac{\log \log n}{\log n} \right). \quad (46)$$

b) *First order approximation:* Using $\Delta_{(0)}^*$ in (46), the term containing ξ in (19) is approximated as³

$$\xi \sqrt{ne^{-t}} = \xi \log n. \quad (47)$$

Since ψ can be chosen of order $o(\log n)$, it is negligible in comparison with $\xi \log n$. Thus, the function to be maximized takes the form

$$T_a(\Delta) = (ne^{-\Delta} - \xi \log n) \log \left(1 + \frac{\Delta}{ne^{-\Delta} - \xi \log n} \right). \quad (48)$$

Assuming $\xi = o(\log \log n)$, and taking the same approach as for obtaining $\Delta_{(0)}^*$, we obtain

$$\Delta_{(1)}^* = \log n - 2 \log \log n + \log 2 + \frac{4 \log \log n}{\log n} + O \left(\frac{\xi}{\log n} \right). \quad (49)$$

This confirms the value of Δ^* stated in Corollary 2. By substituting the value of Δ^* in (19), the achievable throughput is obtained as mentioned in the lemma. The number of active links and the rate-per-link are obtained by using the value of Δ^* in (17) and (18), respectively.

³With a little abuse of notation, we have replaced $\frac{\xi}{\sqrt{2}}$ by ξ . This is acceptable, because we are only interested in the order of the term that ξ introduces to the solution.

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