LLL Reduction Achieves The Receive Diversity In MIMO Decoding

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Abstract

Diversity order is an important measure for the performance of communication systems over MIMO fading channels. In this paper, we prove that in MIMO multiple access systems (or MIMO point-to-point systems with V-BLAST transmission), lattice-reduction-aided decoding achieves the maximum receive diversity (which is equal to the number of receive antennas). Also, we prove that the naive lattice decoding (which discards the out-of-region decoded points) achieves the maximum diversity.

I. Introduction

In the recent years, MIMO communications over multiple-antenna channels has attracted the attention of many researchers. In [1], a transmission technique called V-BLAST is introduced for high-rate communications over point-to-point MIMO fading channels. V-BLAST sends independent symbols over different transmit antennas. Therefore, it can also be used for MIMO multi-access systems. Most of the sub-optimum decoding methods for BLAST (such as nulling and cancelling, zero forcing and GDFE-type methods) can not achieve the maximum receive diversity which is equal to the number of receive antennas. In [2], a lattice decoder is proposed for the decoding of BLAST which (according to the simulation results) achieves the maximum diversity. However, its complexity is exponential in terms of the number of antennas. In [3], [4], and [5], an approximation of lattice decoding, using lattice-basis reduction, is introduced which has a polynomial complexity and the simulation results show that it achieves the receive diversity. In this paper, we give a mathematical proof for achieving the receive diversity by the lattice-reduction-aided decoding. Also, a similar proof shows that the naive lattice decoding (which discards the out-of-region decoded points) achieves the receive diversity.

II. System Model

We consider a multiple-antenna system with *M* transmit antennas and *N* receive antennas. In a multiple-access system, we consider different transmit antennas as different users. If we consider $\mathbf{y} = [y_1, ..., y_N]^T$, $\mathbf{x} = [x_1, ..., x_M]^T$, $\mathbf{w} = [w_1, ..., w_N]^T$ and the $N \times M$ matrix $\mathbf{H}$, as the received signal, the transmitted signal, the noise vector and the channel matrix, respectively, we have the following matrix equation:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}. \tag{1}$$

The channel is assumed to be Raleigh and the noise is Gaussian, i.e. the elements of $\mathbf{H}$ are i.i.d with the zero-mean unit-variance complex Gaussian distribution. Also, we have the power constraint on the transmitted signal, $\mathrm{E}\|\mathbf{x}\|^2 = 1$. The power of the additive noise is σ^2 per antenna, i.e. $\mathrm{E}\|\mathbf{w}\|^2 = N\sigma^2$. Therefore, the signal to noise ratio (SNR) is defined as $\rho = \frac{1}{\sigma^2}$.

In a MIMO multiple-access system or a MIMO point-to-point system with V-BLAST transmission, we send the transmitted vector $\mathbf{x}$ with independent entries from $\mathbb{Z}^2$. At the receiver, we can perform two slightly different types of LLL-aided decoding:

Type I) We find $\tilde{\mathbf{x}}$ as the closest integer point to $\mathbf{B}^*\mathbf{y}$ where $\mathbf{B}$ is the reduced version of $\mathbf{H}^{*-1}$, i.e. $\mathbf{B} = \mathbf{H}^{*-1}\mathbf{U}$, where $\mathbf{U}$ is a unimodular matrix (when $M < N$, we use the pseudo-inverse instead of the inverse). The transmitted vector is decoded as,

$$\widehat{\mathbf{x}} = \mathbf{U}^{*-1}\tilde{\mathbf{x}}.$$

Type II) We find $\tilde{\mathbf{x}}$ as the closest integer point to ${\mathbf{H}_{red}}^{-1}\mathbf{y}$ where $\mathbf{H}_{red}$ is the reduced version of $\mathbf{H}$ i.e. $\mathbf{H}_{red} = \mathbf{H}\mathbf{U}$. The transmitted vector is decoded as,

$$\widehat{\mathbf{x}} = \mathbf{U}\tilde{\mathbf{x}}.$$

In the previous works [3] [4] [5], the LLL-aided decoding type II has been used. We show that the type I method is more appropriate to reduce the effective noise, and indeed, has a better performance. In the next section, we present the details of the proof of our main result for the first method and show that a similar proof is valid for the second method.

III. Diversity of LLL-aided decoding

For MIMO systems, diversity is defined as $\lim_{\rho\to\infty} \dfrac{-\log P_e}{\log \rho}$. When there is no joint processing among the transmit antennas, the maximum achievable diversity is equal to N, the number of receive antennas [6]. To prove that LLL-aided decoding achieves a diversity order of N, we use a bound on the orthogonality defect[1] of the LLL reduction.

Theorem 1: Let Λ be an M-dimensional real lattice and $\mathbf{B} = [\mathbf{b}_1 ... \mathbf{b}_M]$ be a reduced basis of Λ. If δ is the orthogonality defect of $\mathbf{B}$, then [7],

$$\sqrt{\delta} \leq 2^{M(M-1)/4}. \tag{2}$$

In the rest of this section, in the lemmas 1-3, we bound the error probability by the probability of an inequality on $d_{\mathbf{H}}$ (the minimum distance of the lattice generated by $\mathbf{H}$) and the length of the noise vector being valid. In lemma 4, we bound the probability that $d_{\mathbf{H}}$ is too small. Finally, in theorem 2, we prove the main result by combining the bounds on the probability that $d_{\mathbf{H}}$ is too small, and the probability that the noise vector is too large.

Lemma 1: Consider $\mathbf{B} = [\mathbf{b}_1 ... \mathbf{b}_M]$ as an $N \times M$ matrix, with the orthogonality defect δ, and $(\mathbf{B}^{-1})^* = [\mathbf{a}_1 ... \mathbf{a}_M]$ as the Hermitian of its inverse (or its pseudo-inverse if $M < N$). Then[2],

$$\max\{\|\mathbf{b}_1\|, ..., \|\mathbf{b}_M\|\} \leq \frac{\sqrt{\delta}}{\min\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\}} \tag{3}$$

[1]Orthogonality defect is defined as $\delta = \frac{(\|\mathbf{b}_1\|^2 \|\mathbf{b}_2\|^2 ... \|\mathbf{b}_M\|^2)}{\det \mathbf{B}^*\mathbf{B}}$.

[2]This lemma is an extention of lemma 1 in [8].

and

$$\max\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\} \leq \frac{\sqrt{\delta}}{\min\{\|\mathbf{b}_1\|, ..., \|\mathbf{b}_M\|\}}. \tag{4}$$

Proof: Consider $\mathbf{b}_i$ as an arbitrary column of $\mathbf{B}$. The vector $\mathbf{b}_i$ can be written as $\mathbf{b}'_i + \sum_{i \neq j} c_{i,j}\mathbf{b}_j$, where $\mathbf{b}'_i$ is orthogonal to $\mathbf{b}_j$ for $i \neq j$. Now, $[\mathbf{b}_1...\mathbf{b}_{i-1}\mathbf{b}'_i\mathbf{b}_{i+1}...\mathbf{b}_M]$ can be written as $\mathbf{BP}$ where $\mathbf{P}$ is a unit-determinant $M \times M$ matrix (a column operation matrix):

$$\|\mathbf{b}_1\|^2...\|\mathbf{b}_{i-1}\|^2.\|\mathbf{b}_i\|^2.\|\mathbf{b}_{i+1}\|^2...\|\mathbf{b}_M\|^2 \tag{5}$$

$$= \delta \det \mathbf{B}^*\mathbf{B} = \delta \det \mathbf{P}^*\mathbf{B}^*\mathbf{BP} \tag{6}$$

$$= \delta \det\left([\mathbf{b}_1...\mathbf{b}_{i-1}\mathbf{b}'_i\mathbf{b}_{i+1}...\mathbf{b}_M]^*[\mathbf{b}_1...\mathbf{b}_{i-1}\mathbf{b}'_i\mathbf{b}_{i+1}...\mathbf{b}_M]\right). \tag{7}$$

According to the Hadamard theorem:

$$\det\left([\mathbf{b}_1...\mathbf{b}_{i-1}\mathbf{b}'_i\mathbf{b}_{i+1}...\mathbf{b}_M]^*[\mathbf{b}_1...\mathbf{b}_{i-1}\mathbf{b}'_i\mathbf{b}_{i+1}...\mathbf{b}_M]\right) \leq \tag{8}$$

$$\|\mathbf{b}_1\|^2...\|\mathbf{b}_{i-1}\|^2.\|\mathbf{b}'_i\|^2.\|\mathbf{b}_{i+1}\|^2...\|\mathbf{b}_M\|^2. \tag{9}$$

Therefore,

$$\|\mathbf{b}_1\|^2...\|\mathbf{b}_{i-1}\|^2.\|\mathbf{b}_i\|^2.\|\mathbf{b}_{i+1}\|^2...\|\mathbf{b}_M\|^2 \leq \delta\|\mathbf{b}_1\|^2...\|\mathbf{b}_{i-1}\|^2.\|\mathbf{b}'_i\|^2.\|\mathbf{b}_{i+1}\|^2...\|\mathbf{b}_M\|^2 \tag{10}$$

$$\Longrightarrow \|\mathbf{b}_i\| \leq \sqrt{\delta}\|\mathbf{b}'_i\|. \tag{11}$$

Also, $\mathbf{B}^{-1}\mathbf{B} = \mathbf{I}$ results in $<\mathbf{a}_i, \mathbf{b}_i> = 1$ and $<\mathbf{a}_i, \mathbf{b}_j> = 0$ for $i \neq j$. Therefore,

$$1 \;=\; <\mathbf{a}_i, \mathbf{b}_i> \;=\; <\mathbf{a}_i, (\mathbf{b}'_i + \sum_{i \neq j} c_{i,j}\mathbf{b}_j)> \;=\; <\mathbf{a}_i, \mathbf{b}'_i> \tag{12}$$

Now, $\mathbf{a}_i$ and $\mathbf{b}'_i$, both are orthogonal to the $(M-1)$-dimensional subspace generated by the vectors $\mathbf{b}_j$ ($j \neq i$). Thus,

$$1 = \;<\mathbf{a}_i, \mathbf{b}'_i> \;\;=\;\; \|\mathbf{a}_i\|.\|\mathbf{b}'_i\| \geq \|\mathbf{a}_i\|.\frac{\|\mathbf{b}_i\|}{\sqrt{\delta}} \tag{13}$$

$$\Longrightarrow 1 \geq \|\mathbf{b}_i\|.\frac{\|\mathbf{a}_i\|}{\sqrt{\delta}} \tag{14}$$

$$\Longrightarrow \|\mathbf{b}_i\| \leq \frac{\sqrt{\delta}}{\|\mathbf{a}_i\|} \tag{15}$$

The above relation is valid for every i, $1 \leq i \leq M$. Without loss of generality, we can assume that $\max\{\|\mathbf{b}_1\|, ..., \|\mathbf{b}_M\|\} = \|\mathbf{b}_k\|$:

$$\max\{\|\mathbf{b}_1\|, ..., \|\mathbf{b}_M\|\} = \|\mathbf{b}_k\| \leq \frac{\sqrt{\delta}}{\|\mathbf{a}_k\|} \tag{16}$$

$$\leq \frac{\sqrt{\delta}}{\min\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\}}. \tag{17}$$

Similarly, by using (15), we can also obtain the following inequality:

$$\max\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\} \leq \frac{\sqrt{\delta}}{\min\{\|\mathbf{b}_1\|, ..., \|\mathbf{b}_M\|\}}. \tag{18}$$

■

Lemma 2: Consider $\mathbf{B} = [\mathbf{b}_1...\mathbf{b}_M]$ as a reduced basis (LLL) [9] for the lattice generated by $\mathbf{H}^{*-1}$, $\mathbf{B}^{*-1} = [\mathbf{a}_1...\mathbf{a}_M]$, and δ as the orthogonality defect of the reduction. Then, if the magnitude of the noise vector is less than $\frac{\min\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\}}{2\sqrt{M\delta}}$, the LLL-aided decoding method correctly decodes the transmitted signal.

Proof: When we use the LLL-aided decoding method, we find the nearest integer point to $\mathbf{By}$. We should show that this point is the same as the transmitted vector; or in other words, all the elements of $\mathbf{Bw}$ are in the interval $(-\frac{1}{2}, \frac{1}{2})$. To prove this, we show that $\|\mathbf{Bw}\| < \frac{1}{2}$. It is easy to show that,

$$\|\mathbf{Bw}\| \leq \sqrt{M}\|\mathbf{b}_{max}\|.\|\mathbf{w}\| \tag{19}$$

Now, according to (3),

$$\max\{\|\mathbf{b}_1\|, ..., \|\mathbf{b}_M\|\} \leq \frac{\sqrt{\delta}}{\min\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\}} \tag{20}$$

Therefore,

$$\|\mathbf{Bw}\| \leq \frac{\sqrt{M\delta}.\|\mathbf{w}\|}{\min\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\}} \tag{21}$$

By using the assumption of the lemma,

$$\|\mathbf{Bw}\| < \frac{\sqrt{M\delta}.\frac{\min\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\}}{2\sqrt{M\delta}}}{\min\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\}} \tag{22}$$

$$\Longrightarrow \|\mathbf{Bw}\| < \frac{1}{2}. \tag{23}$$

■

Lemma 3: Consider $\mathbf{B} = [\mathbf{b}_1 ... \mathbf{b}_M]$ as a reduced basis (LLL) [9] and $d_{\mathbf{H}}$ as the minimum distance of the lattice generated by $\mathbf{H}$, respectively. Then, there is a constant number c_M (independent of $\mathbf{H}$) such that the LLL-aided decoding method correctly decodes the transmitted signal, if the magnitude of the noise vector is less than $c_M d_{\mathbf{H}}$.

Proof: For an LLL reduction,

$$\sqrt{\delta} \leq 2^{M(M-1)/4}. \tag{24}$$

Therefore, if we consider $c_M = \dfrac{2^{-1-M(M-1)/4}}{\sqrt{M}}$,

$$\|\mathbf{w}\| \leq c_M d_{\mathbf{H}} \Longrightarrow \|\mathbf{w}\| \leq \frac{1}{2\sqrt{M\delta}} d_{\mathbf{H}} \tag{25}$$

The basis $\mathbf{B}$ can be written as $\mathbf{B} = (\mathbf{H}^*)^{-1}\mathbf{U}$ for some unimodular matrix $\mathbf{U}$:

$$(\mathbf{B}^{-1})^* = (((\mathbf{H}^*)^{-1}\mathbf{U})^{-1})^* = (\mathbf{U}^{-1}\mathbf{H}^*)^* = \mathbf{H}(\mathbf{U}^{-1})^* \tag{26}$$

Thus, $(\mathbf{B}^{-1})^* = [\mathbf{a}_1, ..., \mathbf{a}_M]$ is another basis for the lattice generated by $\mathbf{H}$. Therefore, $\mathbf{a}_1, ..., \mathbf{a}_M$ are vectors from the lattice generated by $\mathbf{H}$, and therefore, the length of each of them is at least $d_{\mathbf{H}}$. Therefore,

$$\|\mathbf{w}\| \leq \frac{1}{2\sqrt{M\delta}} d_{\mathbf{H}} \leq \frac{1}{2\sqrt{M\delta}} \min\{\|\mathbf{a}_1\|, ..., \|\mathbf{a}_M\|\}. \tag{27}$$

Thus, according to lemma 2, LLL-aided decoding method correctly decodes the transmitted signal. ■

Lemma 4: Assume that the entries of the $N \times M$ matrix $\mathbf{H}$ has independent complex Gaussian distribution with zero mean and unit variance and consider $d_{\mathbf{H}}$ as the minimum distance of the lattice generated by $\mathbf{H}$. Then, there is a constant $\beta_{N,M}$ such that [8],

$$\Pr\{d_{\mathbf{H}} \leq \varepsilon\} \leq \begin{cases} \beta_{N,M}\varepsilon^{2N} & \text{for } M < N \\ \beta_{N,N}\varepsilon^{2N}. \max\left\{(-\ln\varepsilon)^{N+1}, 1\right\} & \text{for } M = N \end{cases}. \tag{28}$$

Theorem 2: For a MIMO multi-access system (or a point-to-point MIMO system with the V-BLAST transmission) with M transmit antennas and N receive antennas, when we use the LLL lattice-aided-decoding,

$$\lim_{\rho\to\infty} \frac{-\log P_e}{\log\rho} = N. \tag{29}$$

Proof: When $\|\mathbf{w}\| \leq c_M d_{\mathbf{H}}$, we have no decoding error. Thus,

$$P_e \leq \Pr\{\|\mathbf{w}\| > c_M d_{\mathbf{H}}\} \tag{30}$$

$$= \Pr\{c_M^2 d_{\mathbf{H}}^2 \leq \frac{1}{\rho}\}. \Pr\left\{\|\mathbf{w}\| > c_M d_{\mathbf{H}} \middle| c_M^2 d_{\mathbf{H}}^2 \leq \frac{1}{\rho}\right\}$$

$$+\Pr\{\frac{1}{\rho}<c_M^2 d_{\mathbf{H}}^2\leq\frac{2}{\rho}\}.\Pr\left\{\|\mathbf{w}\|>c_M d_{\mathbf{H}}\left|\frac{1}{\rho}<c_M^2 d_{\mathbf{H}}^2\leq\frac{2}{\rho}\right.\right\}$$

$$+\Pr\{\frac{2}{\rho}<c_M^2 d_{\mathbf{H}}^2\leq\frac{4}{\rho}\}.\Pr\left\{\|\mathbf{w}\|>c_M d_{\mathbf{H}}\left|\frac{2}{\rho}<c_M^2 d_{\mathbf{H}}^2\leq\frac{4}{\rho}\right.\right\}+... \quad (31)$$

$$\leq\Pr\{c_M^2 d_{\mathbf{H}}^2\leq\frac{1}{\rho}\}+$$

$$\Pr\{c_M^2 d_{\mathbf{H}}^2\leq\frac{2}{\rho}\}.\Pr\left\{\|\mathbf{w}\|^2\geq\frac{1}{\rho}\right\}+$$

$$\Pr\{c_M^2 d_{\mathbf{H}}^2\leq\frac{4}{\rho}\}.\Pr\left\{\|\mathbf{w}\|^2\geq\frac{2}{\rho}\right\}+... \quad (32)$$

The noise vector has complex Gaussian distribution with variance $\frac{1}{2\rho}$ per each real dimension. Thus, by using the union bound, we can bound the second part of each product term as,

$$\Pr\left\{\|\mathbf{w}\|^2\geq\frac{\gamma}{\rho}\right\}\leq\sum_{i=1}^{2N}\Pr\left\{|w_i|^2\geq\frac{\gamma}{2N\rho}\right\}\leq 2NQ\left(\sqrt{\frac{\gamma}{N}}\right)\leq 2Ne^{-\frac{\gamma}{2N}} \quad (33)$$

Also, for the first part of the product terms, we have,

$$\Pr\left\{c_M^2 d_{\mathbf{H}}^2\leq\frac{\theta}{\rho}\right\}=\Pr\left\{d_{\mathbf{H}}\leq\sqrt{\frac{\theta}{c_M^2\rho}}\right\} \quad (34)$$

By using (33) and (34), we can bound (32).

Case 1, $M<N$:

$$(32)\leq\beta_{N,M}\left(\frac{1}{c_M^2\rho}\right)^N+\beta_{N,M}\left(\frac{2}{c_M^2\rho}\right)^N.2N.e^{-\frac{1}{2N}}+\beta_{N,M}\left(\frac{4}{c_M^2\rho}\right)^N.2N.e^{-\frac{2}{2N}}+... \quad (35)$$

$$\Longrightarrow P_e\leq\frac{c}{\rho^N} \quad (36)$$

where c is a constant. Therefore,

$$\lim_{\rho\to\infty}\frac{-\log P_e}{\log\rho}\geq N. \quad (37)$$

Case 2, $M = N$:

$$(32) \leq \beta_{N,N} \left(\frac{1}{c_M^2 \rho}\right)^N \max\left\{\left(\frac{1}{2}\ln c_M^2\rho\right)^{N+1}, 1\right\} +$$
$$\beta_{N,N} \left(\frac{2}{c_M^2 \rho}\right)^N \max\left\{\left(\frac{1}{2}\ln \frac{c_M^2\rho}{2}\right)^{N+1}, 1\right\} .2N.e^{-\frac{1}{2N}}$$
$$+\beta_{N,N} \left(\frac{4}{c_M^2 \rho}\right)^N \max\left\{\left(\frac{1}{2}\ln \frac{c_M^2\rho}{4}\right)^{N+1}, 1\right\} .2N.e^{-\frac{2}{2N}} + \ldots \tag{38}$$

We are interested in the large values of ρ. For $\rho > {c_M}^2$ and $\ln \rho > 1$,

$$(32) \leq \beta_{N,N} \left(\frac{1}{c_M^2 \rho}\right)^N (\ln \rho)^{N+1} + \beta_{N,N} \left(\frac{2}{c_M^2 \rho}\right)^N (\ln \rho)^{N+1} .2N.e^{-\frac{1}{2N}}$$
$$+\beta_{N,N} \left(\frac{4}{c_M^2 \rho}\right)^N (\ln \rho)^{N+1} .2N.e^{-\frac{2}{2N}} + \ldots \tag{39}$$

$$\Longrightarrow P_e \leq \frac{c' (\ln \rho)^{N+1}}{\rho^N} \tag{40}$$

where c' is a constant. Therefore,

$$\lim_{\rho\to\infty} \frac{-\log P_e}{\log \rho} \geq \lim_{\rho\to\infty} \frac{\log \rho^N - (N+1)\log(\ln \rho) - \log c'}{\log \rho} = N. \tag{41}$$

■

In the above proof, we have considered the LLL-aided decoding type I. In this case, the effective noise vector is equal to $\mathbf{w}' = \mathbf{B}^* \mathbf{w}$, compared to $\mathbf{w}' = \mathbf{H}^{-1}\mathbf{w}$ in zero-forcing. In the previous works [3] [4] [5], the LLL-aided decoding type II has been used. For the type II method, the effective noise vector is equal to $\mathbf{w}' = {\mathbf{H}_{red}}^{-1}\mathbf{w}$ and the average energy of its ith component is proportional to the square norm of the ith column of $({\mathbf{H}_{red}}^*)^{-1}$. By using inequality (4) from lemma 1 (to bound the square norm of the columns of $({\mathbf{H}_{red}}^*)^{-1}$)

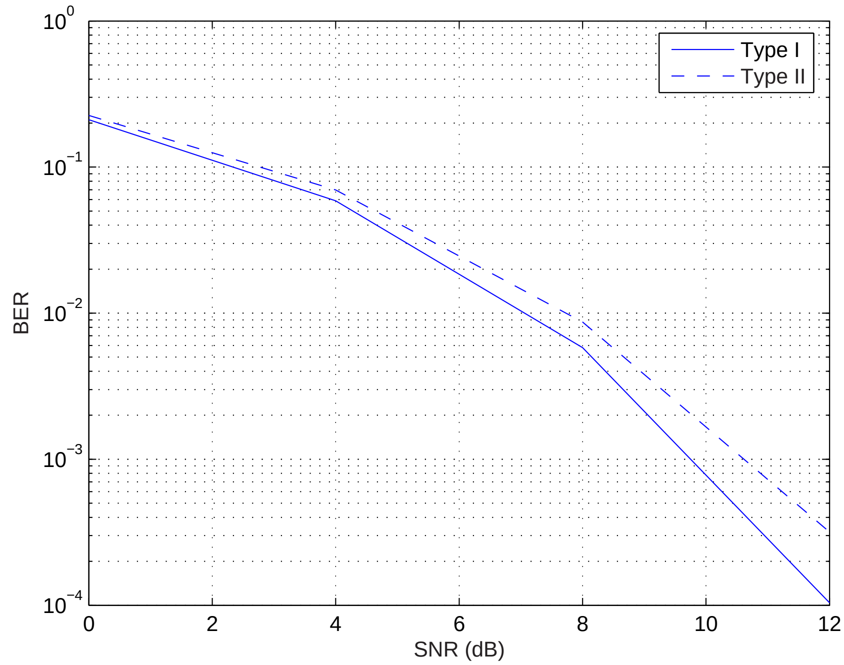

Fig. 1. Bit Error Rate of the two LLL-aided decoding methods for $M = 6$ transmit antennas and $N = 6$ receive antennas with the rate $R = 12$ bits per channel use.

and using a similar proof as lemma 2, we can show that the results of lemma 2 and theorem 2 are still valid. Therefore, both of these LLL-aided decoding methods achieve the receive diversity in V-BLAST MIMO systems (or multiple access MIMO systems). However, it is worth noting that the first method is a more natural approach to reduce the power of the entries of the effective noise vector, and has a better performance (see figure 1).

IV. Relation with the naive lattice-decoding

When we have a finite constellation, for each pair of constellation points, the pair-wise error probability can be bounded by Chernoff bound (similar to [6]). By using the union bound, we can show that the exact ML decoding achieves the diversity order of M, the number of antennas. However, when we use lattice decoding for a finite constellation and consider the out-of-region decoded lattice points as errors, achieving the maximum diversity by lattice decoding is not trivial anymore. However, by using lemma 4, we can show that this suboptimum method (called the naive lattice decoding [10]) still achieves the maximum diversity.

Theorem 3: For a MIMO multi-access system (or a point-to-point MIMO system with the V-BLAST transmission method) with M transmit antennas and N receive antennas, when we use the naive lattice decoding,

$$\lim_{\rho\to\infty} \frac{-\log P_e}{\log \rho} = N. \tag{42}$$

Proof: When $\|\mathbf{w}\| \leq \frac{1}{2} d_{\mathbf{H}}$, we have no decoding error. Thus, by using $\frac{1}{2}$ instead of c_M in the proof of theorem 2, we can bound P_e by bounding $\Pr\left\{\|\mathbf{w}\| > \frac{1}{2} d_{\mathbf{H}}\right\}$. Therefore, we can obtain the same result as theorem 2. ■

In [10], it is shown that for the naive lattice decoding, we can find a family of lattices (generating a family of space-time codes) which achieves diversity order of M ($M \leq N$ is the number of transmit antennas). The current result shows that even if we use the codes generated by the integer lattice, the naive lattice decoding achieves the maximum receive diversity of N (number of receive antennas).

V. Conclusions

We have shown that LLL reduction, which is a polynomial-time algorithm, achieves the maximum receive diversity in MIMO decoding. By using LLL reduction and the Babai approximation, the complexity of the MIMO decoding is equal to the complexity of the zero-forcing method with an additional polynomial time preprocessing. Also, it is shown that by using the naive lattice decoding, instead of ML decoding, we do not loose the diversity order.

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