Integer-Based Constellation-Shaping Method for PAPR Reduction in OFDM Systems

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***Abstract*—In this paper, the problem of reducing the peak-to-average-power ratio (PAPR) in an orthogonal frequency-division multiplexing system is considered. We design a cubic constellation, called the Hadamard constellation, whose boundary is along the bases defined by the Hadamard matrix in the transform domain. Then, we further reduce the PAPR by applying the selective-mapping technique. The encoding method, following the method introduced in the work of Kwok, is derived from a decomposition known as the Smith normal form. This new technique offers a PAPR that is significantly lower than those of the best-known techniques without any loss in terms of energy and/or spectral efficiency, and without any side information being transmitted. Moreover, it has a low computational complexity.**

***Index Terms*—Hadamard constellation, orthogonal frequency-division multiplexing (OFDM), peak-to-average-power ratio (PAPR), selective mapping (SLM), Smith normal form (SNF).**

I. Introduction

ORTHOGONAL frequency-division multiplexing (OFDM) is a multicarrier transmission technique which is widely adopted in different communication applications. OFDM prevents intersymbol interference by inserting a guard interval, and mitigates the frequency selectivity of a multipath channel by using a simple equalizer. This simplifies the design of the receiver and leads to inexpensive hardware implementations. Also, OFDM offers some advantages in higher order modulations and in networking operations. These advantages position OFDM as the technique of choice for the next generation of wireless networks. However, OFDM systems suffer from a large peak-to-average-power ratio (PAPR) of the transmitted signals, requiring power amplifiers with a large linear range.

Fig. 1 shows a basic block diagram of an OFDM transmitter and its receiver. Let $\mathbf{x} = [x_0, x_1, \ldots, x_{N-1}]^T$ denote a vector of $2N$-dimensional ($2N$-D) constellation points. This vector is selected from a set of N identical 2-D subconstellations $\{s_1, \ldots, s_K\}$, and it is transmitted by using one OFDM vector of size N; namely, $\mathbf{y}$.

Paper approved by Y. Li, the Editor for Wireless Communications Theory of the IEEE Communications Society. Manuscript received July 9, 2004; revised June 15, 2005. This work was supported in part by Communications and Information Technology Ontario (CITO), in part by Nortel Networks, and in part by the Natural Sciences and Engineering Research Council of Canada (NSERC). This paper was presented in part at the IEEE Vehicular Technology Conference, Los Angeles, CA, September 2004, and in part at the 22nd Biennial Symposium on Communications, Kingston, ON, Canada, May/June 2004.

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Digital Object Identifier 10.1109/TCOMM.2005.861657

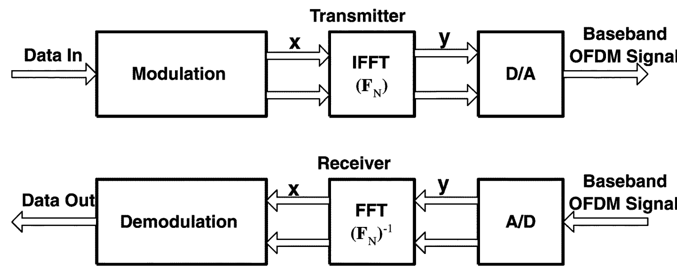

Fig. 1. Basic OFDM transmitter and receiver.

The discrete-time samples of the OFDM signal can be expressed as

$$y_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{j2\pi(nk/N)}. \tag{1}$$

The matrix representation of this signal is

$$\mathbf{y} = \mathbf{F}_N \mathbf{x} \tag{2}$$

where $\mathbf{y} = [y_0 \ldots y_{N-1}]^T$, $\mathbf{x} = [x_0 \ldots x_{N-1}]^T$, and $\mathbf{F}_N$ is the inverse fast Fourier transform (IFFT) matrix

$$\mathbf{F}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \ldots & 1 & \ldots & 1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 1 & \ldots & e^{j2\pi(nk/N)} & \ldots & e^{j2\pi(n(N-1)/N)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 1 & \ldots & e^{j2\pi(k(N-1)/N)} & \ldots & e^{j2\pi((N-1)^2/N)} \end{bmatrix}. \tag{3}$$

The 2-D constellation points $\{x_0, x_1, \ldots, x_{N-1}\}$ may add constructively and produce a time-domain signal with a large amplitude. Thus, the output signal $\mathbf{y}$ may have high output levels, which leads to the requirement of an expensive analog front end.

Usually, the level of the amplitude fluctuation of the discrete-time OFDM signal is measured in terms of the ratio of the peak power to the average envelope power of the signal as

$$\text{PAPR}(\mathbf{y}) = \frac{\|\mathbf{y}\|_\infty^2}{E_y\left[\frac{1}{N}\|\mathbf{y}\|^2\right]}. \tag{4}$$

The continuous-time PAPR is typically estimated by the discrete-time PAPR by employing the IFFT of length LN for the zero-padded sequence of length LN derived from the sequence $\{x_0, x_1, \ldots, x_{N-1}\}$ in (1) [1]–[3]. Therefore

$$y_n = \frac{\sqrt{L}}{\sqrt{LN}} \sum_{k=0}^{LN-1} x'_k e^{j2\pi(nk/LN)} \tag{5}$$

where

$$x'_k = \begin{cases} x_k, & \text{for } k < N \\ 0, & \text{for } k \geq N \end{cases} \tag{6}$$

and L is the oversampling factor.

In the following, we concentrate on matrices and equations with real entries, and complex equations like (2) are represented by real matrices as

$$\begin{bmatrix} \Re(\mathbf{y}) \\ \Im(\mathbf{y}) \end{bmatrix} = \begin{bmatrix} \Re(\mathbf{F}_N) & -\Im(\mathbf{F}_N) \\ \Im(\mathbf{F}_N) & \Re(\mathbf{F}_N) \end{bmatrix} \begin{bmatrix} \Re(\mathbf{x}) \\ \Im(\mathbf{x}) \end{bmatrix} \tag{7}$$

where $\Re(\cdot)$ and $\Im(\cdot)$, respectively, denote the real and the imaginary parts of a matrix or vector. In [4], this model is used for representing the OFDM signal by real matrices.

A large number of methods for the PAPR reduction have been proposed [3], [4]–[20]. In [5] and [6], coding techniques are used for PAPR reduction; however, codes offering a low PAPR can be constructed only at the cost of sacrificing the data rate. Clipping the OFDM signal before amplification is a simple and typical method for the PAPR reduction [7]–[9]. The effects of oversampling and clipping for an OFDM signal are analyzed in [3], [7], and [9]. The authors in [15] propose a new lattice-based multicarrier modulation technique for digital subscriber line (DSL) applications with a low PAPR; however, this technique is not based on a sinusoidal modulation that is usually employed for OFDM systems.

Another type of PAPR reduction methods are the probabilistic schemes. These schemes are classified in two known groups. One is the partial transmit sequence (PTS) [11], in which each subblock of subcarriers is multiplied by a constant phase factor, and these phase factors are optimized to minimize the PAPR. The other scheme is selective mapping (SLM), in which multiple sequences are generated from the same information, and the sequence with the lowest PAPR is transmitted [12]–[14]. Typically, the receiver needs to know which sequence is selected in order to recover the data. However, the methods introduced in [11]–[14] eliminate the need for this explicit side information.

Constellation shaping is another important technique in PAPR reduction. In the method proposed in [16], the outer constellation points are extended to minimize the PAPR of the OFDM symbol. The idea of applying the trellis-shaping technique to reduce PAPR in OFDM systems is introduced in [17]. This line of research is further investigated in [18] by exploiting the property that the autocorrelation of the data sequence in the frequency domain and the power spectrum in the time domain form a Fourier transform pair. Therefore, minimizing the sidelobe of the autocorrelation of the data sequence is equivalent to reducing the PAPR of the OFDM signal. We will later provide a comparison with [18]. In [4], [19], and [20], another constellation-shaping technique is proposed to reduce the PAPR of the OFDM signals. The encoding and decoding algorithms of this method are based on the relations and generators in a free Abelian group. Due to the large complexity of this algorithm, its practical implementation, in the case of Fourier transformation in OFDM systems, is very challenging.

In this paper, we propose a constellation-shaping method in an OFDM system with a considerable PAPR reduction. The boundary of this cubic constellation, called the Hadamard constellation, is along the bases defined by the Hadamard matrix in the transform domain. In addition, this constellation can be employed in conjunction with another PAPR-reduction method. Here, an SLM method is applied in conjunction with the proposed Hadamard constellation to further reduce the PAPR. The encoding method for this shaping technique, following the method introduced in [4], is derived from the Smith normal form (SNF) decomposition, and has a minimal complexity. This new technique offers a PAPR that is significantly lower than those of the best-known techniques reported in the literature without any loss in terms of the energy and/or spectral efficiency, and without any side information being transmitted.

The rest of the paper is organized as follows. In Section II, the constellation-shaping technique is introduced. A brief description of the work in [4] is also given. Section III describes the Hadamard constellation as a shaping method in OFDM systems. Some issues regarding the encoding and decoding algorithms are also investigated. An SLM method is applied to the Hadamard constellation in Section IV. Section V is devoted to some numerical results and a comparison of the proposed method with some recent works. The paper is concluded in Section VI.

II. Constellation Shaping

In the constellation-shaping technique, a constellation in the frequency domain must be found such that the resulting shaping region in the time domain has a low PAPR. A new constellation-shaping method is introduced in [4], [19], and [20] by Kwok and Jones. Based on the encoding algorithm introduced in [4], [19], and [20], we propose a cubic constellation, along with an SLM method to reduce the PAPR in an OFDM system.

In a PAPR-reduction problem, the peak value of the signal vector is bounded by a specified value $\|\mathbf{y}\|_\infty \leq \beta$ (without loss of generality, we assume $\beta = 1$). If the time-domain signal is related to the frequency-domain constellation point by $\mathbf{y} = \mathbf{A}\mathbf{x}$, this inequality on the time-domain boundary translates to a parallelotope[1] in the frequency domain, defined by $\mathbf{A}^{-1}$. Indeed, the constellation boundary is a parallelotope, defined by $\mathbf{Q}_N = [\alpha \mathbf{A}^{-1}]$, where $[\cdot]$ denotes rounding. The parameter α is the smallest value that guarantees the number of points in the shaped constellation is the same as the number of points in the unshaped constellation. The rounding operation is required to impose the constraint that the parallelotope corners lie in an integer lattice. The main challenge in constellation shaping is to find a unique way to map the input data to the constellation points, such that the mapping (encoding) and its inverse (decoding) can be implemented by a reasonable complexity. Kwok in [4] proved that the shaped constellation for an OFDM system is the points inside the quotient group $\mathbb{Z}^N/\Lambda(\mathbf{Q}_N)$, where $\mathbb{Z}^N$ is the N-D integer space and $\Lambda(\mathbf{Q}_N)$ is the lattice defined by $\mathbf{Q}_N$, which is based on rounding off the scaled version of the IFFT matrix. The points inside this parallelotope are used as the constellation points in transmitting the OFDM signals. Using the relations and generators in a free Abelian group, the points

[1]The parallelotope bases are defined along the columns of $\mathbf{A}^{-1}$.

inside this constellation are encoded (labeled) in [4]. The following theorem provides the mathematical tool for the encoding procedure of these points [4].

Theorem 1: Any relation matrix $\mathbf{Q}_N$ can be decomposed into $\mathbf{Q}_N = \mathbf{UDV}$, where $\mathbf{D}$ is diagonal with the entries $\{\sigma_i\}_{i=1}^N$ such that $\sigma_1 \mid \sigma_2 \mid \cdots \mid \sigma_N$, and $\mathbf{U}$ and $\mathbf{V}$ are unimodular matrices.[2]

The decomposition of the relation matrix $\mathbf{Q}_N$ is performed via column and row operations [4], which is impractical for an OFDM system.

We observe that this decomposition is known as the SNF decomposition of an integer matrix [21] in the mathematical literature, and the matrix $\mathbf{D}$ is called the SNF of the matrix $\mathbf{Q}_N$. The SNF decomposition is a diagonalization of a matrix in the integer domain. Introduced by Smith [22], this concept has been used in many applications, such as solving linear diophantine equations, finding the permutation equivalence and similarity of matrices, determining the canonical decomposition of the finitely generated Abelian groups, integer programming, computing additional normal forms, including Frobenius and Jordan normal forms, and separable computing of the discrete Fourier transform (DFT). For more historical remarks and applications of the SNF, see [23]–[25].

The major contributions to the computational complexity in [4] are the decomposition of the matrix $\mathbf{Q}_N$, the *offline* procedure, and the encoding algorithm for this constellation, the *online* procedure. The interpretation of the column and row operations as SNF of an integer matrix links the problem to a rich body of knowledge, developed in the context of SNF decomposition. Unfortunately, computing the SNF decomposition for an OFDM system is impractical, due to the rapid growth in the size of the intermediate integer values. Moreover, in [4], it is shown that the complexity of the encoding procedure is $\mathcal{O}(N^2)$, i.e., for a realistic OFDM system, the online complexity remains very high, as well.

If the SNF decomposition of the matrix $\mathbf{Q}_N$ is given, the encoding algorithm for the shaped constellation can be represented by [4]

$$\begin{aligned}\hat{\mathbf{x}} &= \mathbf{U}\boldsymbol{\lambda}\\ \boldsymbol{\gamma} &= \left\lfloor \mathbf{Q}_N^{-1}\hat{\mathbf{x}}\right\rceil\\ \mathbf{x} &= \hat{\mathbf{x}} - \mathbf{Q}_N\boldsymbol{\gamma}\end{aligned} \tag{8}$$

where $N = 2^n$, $\boldsymbol{\lambda}$ is the canonical representation of an integer I which represents the data to be sent, and $\mathbf{x}$ is the constellation point corresponding to I. The time-domain signal is computed using the IFFT operation. The canonical representation of an integer I can be calculated by the recursive modulo operation; namely

$$\begin{aligned}\lambda_1 &= I \bmod \sigma_1\\ I_1 &= \frac{I-\lambda_1}{\sigma_1}\\ \lambda_i &= I_{i-1} \bmod \sigma_i\\ I_i &= \frac{I_{i-1}-\lambda_i}{\sigma_i}\end{aligned} \tag{9}$$

where $1 \le i \le N$.

[2]The condition $\sigma_1 \mid \sigma_2 \mid \cdots \mid \sigma_N$ in *Theorem 1* is defined for finding a unique decomposition and can be ignored in the encoding procedure.

Also, the reverse operation for finding I from the N-D vector $\mathbf{x}$ is [4]

$$\begin{aligned}&\boldsymbol{\lambda} = \mathbf{U}^{-1}\mathbf{x} = (\lambda_1, \lambda_2, \ldots, \lambda_N)^T\\ &\tilde{\lambda_i} = \lambda_i \bmod \sigma_i\\ &I = \tilde{\lambda}_1 + \sigma_1(\tilde{\lambda}_2 + \sigma_2(\ldots(\tilde{\lambda}_{N-1} + \sigma_{N-1}\tilde{\lambda}_N)\ldots)).\end{aligned} \tag{10}$$

In [19], it is shown that if the matrix $\mathbf{Q_N}$ is replaced by the Hadamard matrix, $\mathbf{H}_{2^n}$, the corresponding encoding and decoding algorithms for the constellation can be implemented by a butterfly structure that uses only bit shifting and logical AND. This simplicity is due to the following recursive formula for the Hadamard matrix:

$$\mathbf{H}_{2^n} = \begin{bmatrix}\mathbf{H}_{2^{n-1}} & \mathbf{H}_{2^{n-1}}\\ \mathbf{H}_{2^{n-1}} & -\mathbf{H}_{2^{n-1}}\end{bmatrix}, \text{ where } \mathbf{H}_1 = [1]. \tag{11}$$

The SNF decomposition of (11) can be easily computed as $\mathbf{H}_{2^n} = \mathbf{U}_{2^n}\mathbf{D}_{2^n}\mathbf{V}_{2^n}$, where

$$\begin{aligned}\mathbf{U}_{2^n} &= \begin{bmatrix}\mathbf{U}_{2^{n-1}} & 0\\ \mathbf{U}_{2^{n-1}} & \mathbf{U}_{2^{n-1}}\end{bmatrix}\\ \mathbf{D}_{2^n} &= \begin{bmatrix}\mathbf{D}_{2^{n-1}} & 0\\ 0 & 2\mathbf{D}_{2^{n-1}}\end{bmatrix}\\ \mathbf{V}_{2^n} &= \begin{bmatrix}\mathbf{V}_{2^{n-1}} & \mathbf{V}_{2^{n-1}}\\ 0 & -\mathbf{V}_{2^{n-1}}\end{bmatrix}\\ \mathbf{U}_{2^n}^{-1} &= \begin{bmatrix}\mathbf{U}_{2^{n-1}} & 0\\ -\mathbf{U}_{2^{n-1}} & \mathbf{U}_{2^{n-1}}\end{bmatrix}\end{aligned} \tag{12}$$

and $\mathbf{U}_1 = \mathbf{U}_1^{-1} = \mathbf{D}_1 = \mathbf{V}_1 = [1]$.

III. Hadamard Constellation in OFDM Systems

As mentioned in Section II, in OFDM systems, the boundary of the constellation that leads to a low PAPR is along the bases of the IFFT matrix. However, the corresponding SNF decomposition required in the encoding procedure cannot be computed. If the IFFT operation is replaced by the Hadamard operation, a simple encoding algorithm results. However, this type of multicarrier modulation is not very popular, because it does not offer the advantages of the conventional OFDM [26].

We propose to replace the conventional constellation in OFDM systems by a cubic constellation, called the Hadamard constellation, whose boundary is along the bases defined by the Hadamard matrix in the transform domain. Fig. 2 shows the boundaries of these two constellations. The solid line represents the boundary of the constellation which is based on the IFFT matrix. The dashed line shows the boundary of the Hadamard constellation. The IFFT and the Hadamard are both orthogonal matrices, and therefore, the constellation boundaries along these orthogonal bases are a rotated version of each other. As a result, it is expected that a large number of points within these boundaries will be the same, as shown in Fig. 2. Therefore, by substituting the constellation along the IFFT matrix with a constellation along the Hadamard matrix, the resulting PAPR is reduced. Moreover, the encoding of this new constellation, based on the SNF decomposition of the Hadamard matrix, is simple and practical.

Note that in this paper, the time-domain signal $\mathbf{y}$ is obtained by the IFFT transformation of the constellation point $\mathbf{x}$. This results in a traditional OFDM signal based on IFFT/fast Fourier

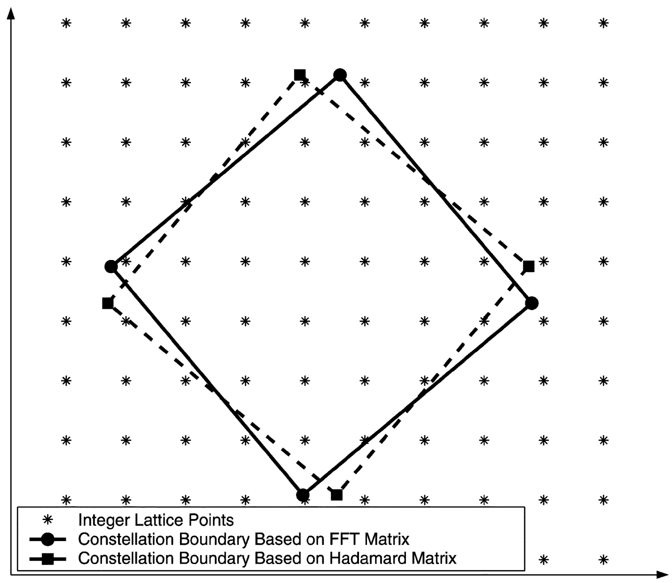

Fig. 2. N-D signal constellation for IFFT and Hadamard matrix.

transform (FFT) operation. In other words, only the constellation boundary is determined using the Hadamard matrix, i.e., $\mathbf{Q}_N = \mathbf{H}_{2^n}$ in (8).

To further reduce the PAPR, the Hadamard constellation can be concatenated with other methods for PAPR reduction. This motivates us to apply an SLM technique [27], [28] to the Hadamard constellation. In typical SLM methods [27], [28], the major PAPR reduction is achieved by the first few redundant bits. Employing more redundant bits necessitates a high level of complexity to obtain modest improvements in the PAPR. However, in the proposed SLM method, employing the Hadamard constellation causes a considerable PAPR reduction by itself. As a result, by using just one or two redundant bits in SLM, this method significantly outperforms the other PAPR-reduction techniques reported in the literature. Note that it is also possible to apply a PTS method [12] to the Hadamard constellation.

A. Complex Representation

As stated in Section I, (7) can be applied to change the complex equations of an OFDM system to real equations. This leads to the change of the constellation boundary. Generally, we can distinguish between two classes of boundaries [29], [30]: 1) the Cartesian boundary that results by viewing the real and imaginary parts of the signal as two separate real signals; and 2) the Polar boundary that considers the envelope and phase of the OFDM signal in a complex plane. The Cartesian boundary limits each component of the complex signal within a square, while the Polar boundary limits this component within a circle. In this paper, we avoid the complex representation of the OFDM signal by treating the real and the imaginary parts of the signal separately, which is equivalent to using a Cartesian boundary.

B. Encoding Procedure

The points inside the Hadamard constellation are mapped to the input data by the encoding procedure, introduced in (8)–(10). The number of these points inside the shaped constellation is determined by the determinant of the Hadamard matrix, $\det(\mathbf{H}_{2^n})$ [31].

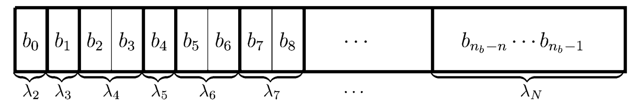

Fig. 3. Mapping between binary representation of the information and $\{\lambda_i\}$.

Theorem 2: The size of the shaped constellation defined by a $2^n \times 2^n$ Hadamard matrix is $\det(\mathbf{H}_{2^n}) = 2^{n2^{n-1}}$.

Proof: Based on (12), $\det(\mathbf{H}_{2^n}) = \det(\mathbf{D}_{2^n})$, because the matrices $\mathbf{U}_{2^n}$ and $\mathbf{V}_{2^n}$ are unimodular and their determinants are one. To prove this theorem, we use induction. For a 2×2 Hadamard matrix

$$\det(\mathbf{D}_2) = \det\left(\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right) = 2 = 2^{1\times 2^{1-1}}. \tag{13}$$

It is assumed that the claim is valid for a $2^k \times 2^k$ Hadamard matrix. Based on (12), for a $2^{k+1} \times 2^{k+1}$ Hadamard matrix

$$\begin{aligned} \mathbf{D}_{2^{k+1}} &= \begin{bmatrix} \mathbf{D}_{2^k} & 0 \\ 0 & 2\mathbf{D}_{2^k} \end{bmatrix} \\ \Rightarrow \det(\mathbf{D}_{2^{k+1}}) &= \det(\mathbf{D}_{2^k}) \times 2^{2^k} \times \det(\mathbf{D}_{2^k}) \\ &= 2^{2^k} \times (\det(\mathbf{D}_{2^k}))^2 \\ &= 2^{2^k} \times \left(2^{k2^{k-1}}\right)^2 = 2^{(k+1)2^{(k+1)-1}}. \end{aligned} \tag{14}$$

■

According to the large Hadamard constellation size, in (9), the canonical representation of the large numbers should be computed. The canonical representation of the integer numbers can be simplified based on the fact that digital communication systems deal with binary input streams. Based on (10), an integer I can be represented by

$$I = \lambda_1 + \sigma_1\lambda_2 + \sigma_1\sigma_2\lambda_3 + \cdots + \sigma_1 \ldots \sigma_{N-1}\lambda_N \tag{15}$$

where $N = 2^n$, and $\{\lambda_i\}_{i=1}^N$ is the canonical representation of I, given in (9), with $\lambda_1 = 0$. According to (12), for a $2^n \times 2^n$ Hadamard matrix, all $\{\sigma_i\}_{i=1}^N$ are powers of 2, i.e.,

$$\{\sigma_i\}_{i=1}^N = \{1, 2, 2, 4, 2, 4, 4, 8, \ldots, 2^n\}. \tag{16}$$

Let $k_i = \log_2 \sigma_i$; therefore

$$\begin{aligned} I &= 2^{k_1}\lambda_2 + 2^{k_1+k_2}\lambda_3 + 2^{k_1+k_2+k_3}\lambda_4 \\ &\quad + \cdots + 2^{k_1+\cdots+k_{N-1}}\lambda_N \\ &= \lambda_2 + 2\lambda_3 + 2^2\lambda_4 + 2^4\lambda_5 + \cdots + 2^{n2^{n-1}-n}\lambda_N. \end{aligned} \tag{17}$$

The representation of $d = 2^{n2^{n-1}}$ integer numbers corresponding to the Hadamard constellation points necessitate that $n_b = \log_2(d) = n2^{n-1}$ bits represent these numbers. Thus, the binary representation of I is expressed as

$$\begin{aligned} I &= b_0 + 2b_1 + 2^2 b_2 + 2^3 b_3 + \cdots + 2^{n_b-1} b_{n_b-1} \\ &= b_0 + 2b_1 + 2^2(b_2 + 2b_3) + 2^4 b_4 + 2^5(b_5 + 2b_6) \\ &\quad + \cdots + 2^{n2^{n-1}-n}(b_{n_b-n} + \cdots + 2^{n-1} b_{n_b-1}). \end{aligned} \tag{18}$$

A comparison of (17) and (18) is depicted in Fig. 3. Each λ_i consists of $k_i = \log_2 \sigma_i$ bits of the input binary data. This representation will simplify the encoding algorithm. Moreover, the problem of using large numbers in the encoding procedure will be avoided.

Theorem 2 shows that the size of the Hadamard constellation for a $2^n \times 2^n$ Hadamard matrix is $2^{n2^{n-1}}$. Therefore, the transmission rate is related to the number of subcarriers $N = 2^n$ in the OFDM system.[3] This rate is unacceptable, not only because it depends on N, but also because it is usually higher than the required value. Therefore, a subset of the points inside the shaped constellation are selected for transmission such that they form a constellation with the desired rate. Also, the selected points should be uniformly distributed in the original Hadamard constellation in order to maintain the same peak as well as average energy values (assuming continuous approximation). Note that the Hadamard constellation is called the root constellation for the aforementioned set of the uniformly distributed points in the following.

Noting (8) and (9), there is an isomorphism between the integer set

$$\mathcal{S} = \left\{0, 1, \ldots, 2^{n2^{n-1}} - 1\right\} \tag{19}$$

and the set of the points within the Hadamard constellation. Equivalently, the set $\mathcal{S}$ can be considered as a label group for the constellation points (refer to [32] for the definition). A subgroup of the constellation points results in a uniformly distributed subset of the Hadamard constellation points. Consequently, this subgroup of constellation points is isomorphic to a subgroup in the label group $\mathcal{S}$. This subgroup can be selected such that its elements are congruent to zero modulo c, namely

$$\mathcal{P} = \{I \in \mathcal{S} \mid I = 0 \bmod c\} \tag{20}$$

where c is determined by the ratio of the size of the Hadamard constellation $2^{n2^{n-1}}$ and the size of the constellation 2^{rN}, with the desired rate r. Employing (8) and (9), the labels in the subgroup $\mathcal{P}$ determine the set of uniformly distributed points in the Hadamard constellation. By relying on the continuous approximation, such a uniform distribution affects neither the probabilistic behavior of the PAPR nor the average energy of the constellation points.

The Hadamard constellation has almost the same average energy as the constellation resulted by employing quadrature amplitude modulation (QAM) signaling in an OFDM system. It can be easily seen that the Hadamard constellation points in (8) can be represented by $\mathbf{x} = \mathbf{H}_N\mathbf{c}$, where $-(1/2) \leq \mathbf{c} < (1/2)$. Therefore, the Hadamard constellation contains all the integer points inside a hypercube whose boundary is along the columns of the Hadamard matrix. By considering $\mathbf{H}_N\mathbf{H}'_N = N\mathbf{I}_N$, while $\mathbf{F}_N\mathbf{F}'_N = \mathbf{I}_N$, the Hadamard constellation is N times smaller than a cubic constellation whose sides are the columns of the Hadamard matrix. Then, it is straightforward that the average energy per each dimension of the Hadamard constellation is

$$E_{\text{ave}}\left(\frac{n}{2}\right) = \frac{1}{12}\frac{2^{2n} - 1}{2^n}. \tag{21}$$

[3]For $N = 2^n$, the rate for each real component is $\log_2(2^{n2^{n-1}})/N = n/2$.

Note that (21) shows the average energy per dimension for the root constellation, i.e., the transmission rate is $n/2$. This energy is $(2^n + 1)/2^n$ times the average energy of the equivalent constellation in an OFDM system employing QAM signaling with the same transmission rate.

In the case where the transmission rate is r, as mentioned in (20), the constellation points form a subgroup of the Hadamard constellation points (uniformly distributed subset). Therefore, the constellation has the same energy as in (21); however, the distance among the points is increased by a factor of 2^{n-2r}. Therefore

$$E_{\text{ave}}(r) = \frac{1}{2^{n-2r}}E_{\text{ave}}\left(\frac{n}{2}\right). \tag{22}$$

Note that the average energy in (22) is $(2^{(2n)} - 1)/(2^{(2n)}) \times (2^{(2r)}/(2^{(2r)} - 1))$ times the average energy of the equivalent constellation in an OFDM system employing QAM signaling with the same transmission rate. This justifies our earlier claim that the average energy remains almost constant.

C. Decoding Procedure

At the receiver end, the time-domain signal is filtered by a low-pass filter and sampled at the Nyquist rate. The samples are processed by an FFT to recover the constellation point in the frequency domain. For an additive white Gaussian noise (AWGN) channel, the received vector is given by

$$\mathbf{z} = \mathbf{y} + \mathbf{n} \tag{23}$$

where $\mathbf{y}$ is the transmitted time-domain signal in (8) and $\mathbf{n}$ is a zero-mean complex AWGN. The approximated constellation point is written as

$$\hat{\mathbf{x}} = \text{FFT}(\mathbf{z}) = \mathbf{x} + \text{FFT}(\mathbf{n}) = \mathbf{x} + \mathbf{n}' \tag{24}$$

where $\mathbf{x}$ is the transmitted constellation point, and $\mathbf{n}'$ is a zero-mean complex AWGN. The maximum-likelihood decoder simply rounds off the received constellation point $\hat{\mathbf{x}}$ in the integer domain. Then, the resulting constellation point is replaced in (10) to decode the transmitted signal.

D. Example

To further clarify the algorithm, we compute the constellation points in an OFDM system with 16 subcarriers. *Theorem 2* states that there are 2^{32} points inside the Hadamard constellation, i.e., the real and imaginary parts of the signal can be one of these points (equivalent to using a 16-QAM in the OFDM system). The Hadamard matrix and its SNF decomposition are calculated by (11) and (12). Then the input data is encoded using (8). In Table I, we have computed some of the constellation points.

The SNF decomposition of the matrix $\mathbf{Q}_N$ based on the IFFT matrix, even for this small case, is difficult.

IV. Selective Mapping

SLM is a method to reduce the PAPR in an OFDM system, which involves generating a large set of data vectors that represent the same information, where the data vector with the lowest PAPR is used for the transmission. Here, we present a method

TABLE I
AN EXAMPLE OF THE ENCODING PROCEDURE FOR THE CONSTELLATION POINTS IN THE HADAMARD CONSTELLATION IN AN OFDM SYSTEM WITH 16 SUBCARRIERS EMPLOYING 16-QAM

data	$b_0 b_1 b_2 \cdots b_{31}$	$\lambda_1 \lambda_2 \cdots \lambda_{16}$	$x_1 x_2 \cdots x_{16}$
0	00000000000000000000000000000000	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1	10000000000000000000000000000000	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0	-1 0 -1 0 -1 0 -1 0 -1 0 -1 0 -1 0 -1 0
2	01000000000000000000000000000000	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0	-1 -1 0 0 -1 -1 0 0 -1 -1 0 0 -1 -1 0 0
3	11000000000000000000000000000000	0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0	-1 0 0 1 -1 0 0 1 -1 0 0 1 -1 0 0 1
4	00100000000000000000000000000000	0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1
5	10100000000000000000000000000000	0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0	0 -1 0 0 0 -1 0 0 0 -1 0 0 0 -1 0 0
6	01100000000000000000000000000000	0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0	0 0 -1 0 0 0 -1 0 0 0 -1 0 0 0 -1 0
7	11100000000000000000000000000000	0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0	1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0
8	00010000000000000000000000000000	0 0 0 2 0 0 0 0 0 0 0 0 0 0 0 0	-2 0 0 0 -2 0 0 0 -2 0 0 0 -2 0 0 0
9	10010000000000000000000000000000	0 1 0 2 0 0 0 0 0 0 0 0 0 0 0 0	-1 0 1 0 -1 0 1 0 -1 0 1 0 -1 0 1 0
10	01010000000000000000000000000000	0 0 1 2 0 0 0 0 0 0 0 0 0 0 0 0	-1 1 0 0 -1 1 0 0 -1 1 0 0 -1 1 0 0
$\vdots$	$\ddots$	$\ddots$	$\vdots$
10^3	00010111110000000000000000000000	0 0 0 2 0 3 3 1 0 0 0 0 0 0 0 0	-1 1 1 1 -1 0 0 0 -1 1 1 1 -1 0 0 0
$\vdots$	$\ddots$	$\ddots$	$\vdots$
10^6	00000010010000101111000000000000	0 0 0 0 0 2 0 1 0 2 2 7 0 0 0 0	0 0 0 0 0 -2 0 -1 -2 0 0 1 -2 -2 0 0
$\vdots$	$\ddots$	$\ddots$	$\vdots$
$2^{32}-1$	11111111111111111111111111111111	0 1 1 3 1 3 3 7 1 3 3 7 3 7 7 15	2 -1 -1 -1 -1 -1 -1 1 -1 -1 -1 1 -1 1 1 3

to apply the SLM technique to further reduce the PAPR in the constellation developed earlier.

Assume that the data rate is r bits per block of length-N FFT symbols. Let r_s denote the number of redundant bits specified for SLM [$r_s \ll r$ and $r = \log_2$ (constellation size)]. Therefore, there are $N_s = 2^{r_s}$ constellation points representing the same information for transmission in SLM. In the proposed SLM method, the input integers I are mapped to the Hadamard constellation points, and the constellation points corresponding to the integers with the same r_s most significant bits (MSBs) are classified in the same subset. Note that the constellation points in each subset represent the same information. The time-domain signals corresponding to the frequency-domain constellation points are computed by the IFFT transformation, and the constellation point with the lowest PAPR is transmitted.

The details of this scheme are described in the following. In the first step, the input binary sequence is divided into blocks of $r - r_s$ bits. Then, r_s bits of zeros are added to each information block, and these blocks are divided into subblocks of lengths $\log_2 \sigma_i$, $i = 1, \ldots, N$ bits (refer to Fig. 3). The binary representations of these subblocks form the vector $\boldsymbol{\lambda}$, which leads to the calculation of the constellation point using (8). The other data vectors are obtained by changing the r_s MSBs of the binary information sequence. Therefore, N_s Hadamard constellation points with different values for the PAPR are calculated. Finally, the constellation point with the lowest PAPR is selected for the transmission.

The different constellation points that represent the same information have the same $r - r_s$ bits. Thus, at the receiver end, the constellation point is decoded by (10), and the r_s extra bits are discarded. Therefore, this method can be expressed as a variant of SLM in which no side information on the choice of the transmit signal needs to be transmitted. The degradation in the data rate can be ignored, since a significant PAPR reduction is obtained by using only one or two redundant bits. To be fair in viewing the potential loss in the data rate, we have to include the impact of using the SLM method on the average energy of the constellation, as well. The Hadamard constellation has a zero shaping gain[4] due to its cubic boundary (shaping gain is computed using continuous approximation [33]). Numerical results show that applying the SLM method to the resulting cubic constellation results in a reduction in the average energy, reflected in a small, but positive, shaping gain. This justifies our earlier claim that the reduction in the PAPR is achieved at no extra cost in terms of a reduction in the spectral efficiency and/or an increase in the average energy of the constellation.

V. SIMULATION RESULTS

In this section, we present simulation results for a complex baseband OFDM system with $N = 128$ subchannels employing 16-QAM by using 10^7 randomly generated OFDM symbols. First, we show the PAPR performance of the Hadamard constellation. The next step is then to show the capability of the SLM technique, when it is applied to the Hadamard constellation to achieve further PAPR reduction. The simulation results are presented as the complementary cumulative density function (CCDF) of the PAPR of the OFDM signals, expressed as follows:

$$\mathrm{CCDF}\left\{\mathrm{PAPR}(\mathbf{y})\right\} = P\left\{\mathrm{PAPR}(\mathbf{y}) > \gamma\right\}. \qquad (25)$$

This equation can be interpreted as the probability that the PAPR of a symbol block exceeds some clip level γ (it is referred to as symbol-clip probability [16]).

According to (5) and (6), the continuous PAPR can be estimated by the IFFT of the zero-padded sequence of length LN. Results for the oversampling to $L = 1, 2, 4$ are shown in Fig. 4. The continuous PAPR can be approximated by an oversampling factor of $L = 4$. As mentioned in [1]–[3], further oversampling will result in minor changes. We have a PAPR reduction of more than 4 dB at 10^{-5} symbol-clip probability.

[4]Shaping gain is defined as the relative reduction in the required average energy for a given number of constellation points with respect to a cubic constellation [33].

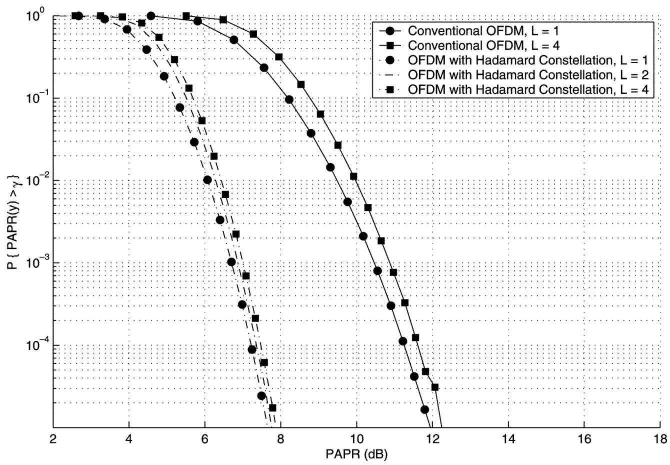

Fig. 4. CCDF of PAPR for a Hadamard constellation with different oversampling factors (128-channel OFDM system with 16-QAM constellation).

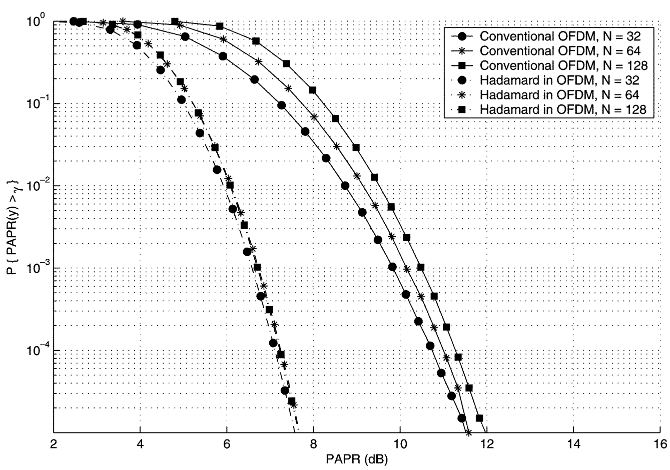

Fig. 5. CCDF of PAPR for a Hadamard constellation in an N-channel OFDM system employing 16-QAM constellation and $L = 1$.

Fig. 5 shows the PAPR of an OFDM signal using the Hadamard constellation with different numbers of block length N. The effect of the constellation size is also investigated. It is observed that the achieved PAPR is rather insensitive to the constellation size, see Fig. 6. The symbol-error rate (SER) of the proposed method and that of a conventional OFDM system are compared. As shown in Fig. 7, the gap is minimal.

Fig. 8 shows the simulation results of applying the SLM technique to the Hadamard constellation. As illustrated in Fig. 8, using only one bit of redundancy in 4×128 bits per block of a 128-FFT symbol[5] results in a 5.6-dB reduction in the PAPR. Simulation results show that by employing more redundant bits, the PAPR approaches its optimal value for a cubic constellation, namely $10\log_{10}(3)$. The PAPR of a cubic constellation is computed using continuous approximation.

A. Some Insight to the Achieved Performance

In a conventional OFDM system with N different subcarriers, the time-domain samples can be approximated by zero-mean Gaussian random variables, based on adopting the central

[5]By using 16-QAM in a 128-channel OFDM system, there are $16^{128} = 2^{4\times 128}$ constellation points.

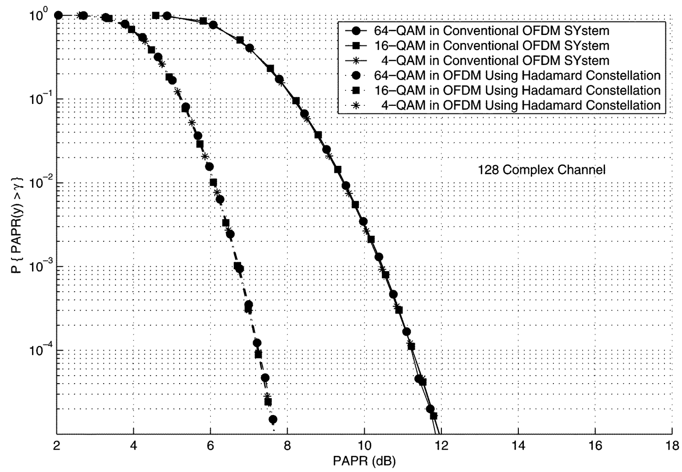

Fig. 6. CCDF of PAPR for a Hadamard constellation in a 128-channel OFDM system employing different QAM constellations and $L = 1$.

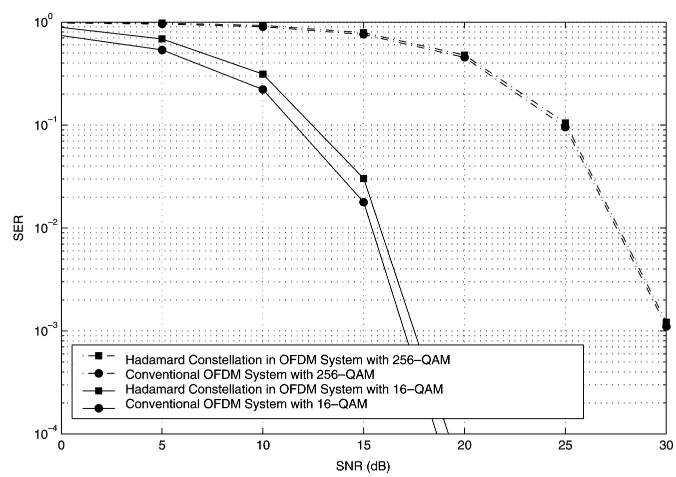

Fig. 7. SER comparison for the proposed method in a 256-channel OFDM system employing 256-QAM and 16-QAM constellation.

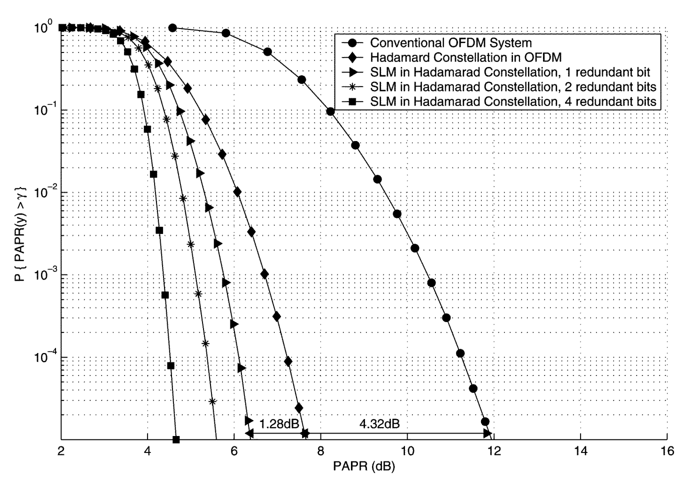

Fig. 8. CCDF of PAPR by SLM method based on Hadamard constellation in a 128-channel OFDM system employing 16-QAM constellation.

limit theorem. Therefore, the amplitude of these samples has a Rayleigh distribution, and the CCDF of the PAPR of the OFDM signal can be approximated as follows [34]:

$$P\left\{\mathrm{PAPR}(\mathbf{y}) > \gamma\right\} = 1 - (1 - e^{-\gamma})^N. \qquad (26)$$

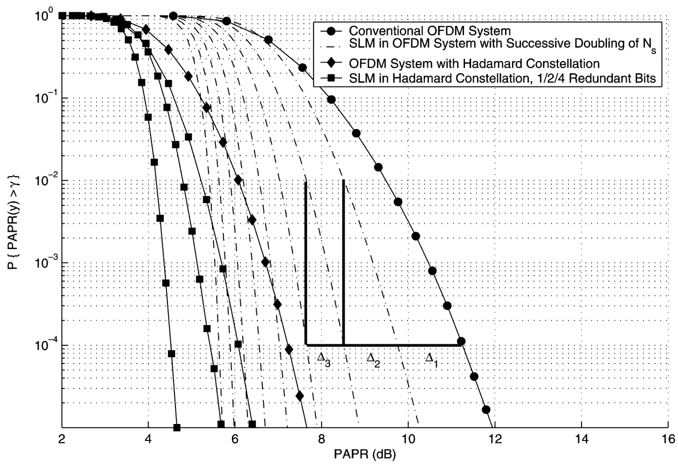

Fig. 9. CCDF of PAPR in a 128-channel OFDM system with SLM method using different number of redundant bits $L = 1$.

The use of N_s statistically independent vectors that have the same information for transmission in the SLM method changes the CCDF of the PAPR of the OFDM signal, such that

$$P\left\{\mathrm{PAPR}(\mathbf{y}) > \gamma\right\} = \left(1 - (1 - e^{-\gamma})^N\right)^{N_s}. \qquad (27)$$

Therefore, in the logarithmic CCDF versus PAPR graph, the slope of the curve is proportional to N_s (see Fig. 9). By increasing the number of vectors with the same information, the corresponding slope increases. Thus, the major PAPR reduction is achieved by the first few redundant bits, as shown in Fig. 9 ($\Delta_1 > \Delta_2 > \cdots$). In other words, we have a saturation effect on the PAPR reduction by increasing r_s. This is the reason that we have applied the SLM technique to the Hadamard constellation. As mentioned in Section IV, the method employing only the Hadamard constellation considerably reduces the PAPR. By adopting the Hadamard constellation in the proposed SLM method, not only can we lower the PAPR considerably, but also we can approximately maintain the slope of the CCDF versus PAPR curve. This results in a considerably lower PAPR by using a small number of redundant bits before reaching the saturation.

B. Comparison

In numerical simulations, we have selected the system parameters to be compatible with some recent works on PAPR reduction reported in [11], [12], [30], [34], and [35]. As a complexity measurement, the main complexity of the proposed method is due to the encoding algorithm and the multi-IFFT computations in the SLM technique. The complexity of the encoding algorithm is in the matrix multiplications of (8). As mentioned in Section III, all the elements of the Hadamard matrix and its SNF decomposition matrices are $+1$, -1, or 0, and consequently, these operations can be easily implemented using a butterfly structure. Note that in the SLM technique, for each of the N_s time-domain signals, we shall compute one IFFT.

In [12], an SLM method based on multiplying the constellation point by N_s different pseudorandom but fixed vectors is introduced. For the same system as ours, with $N_s = 4$ different vectors, a PAPR reduction of 3 dB is gained at the symbol-clip probability close to 10^{-5}. However, for the same symbol-clip rate and $N_s = 4$, we have a 6-dB reduction by using the proposed SLM method. Also, the complexity of this algorithm is comparable with the method in [12]. Note that in [12] some side information (with high sensitivity to channel error) needs to be transmitted.

Another approach, similar to [12], is introduced for the SLM in [34]. The authors have introduced this method for multiple-input multiple-output (MIMO)-OFDM systems. The simulation results in [34] are similar to [12] (the relative comparison between the proposed method and the one in [12] is explained earlier).

The tone reservation [35] is a well-known method for PAPR reduction in multicarrier systems, provided that it can quickly converge to a good solution. An efficient approximation for the tone-reservation approach with a faster convergence is developed in [30]. The complexity of [30] is comparable with ours; however, we have about 3-dB lower PAPR than that in [35] or [30] for similar system parameters. Note that in the tone-reservation method, some tones are reserved for the PAPR reduction and some of the tones are not used for data transmission, implying a loss in the data rate. Note that [30] reduces the PAPR by solving a min-max problem. This problem is solved by an interior-point method which requires a descent direction and a constraint to find the solution recursively.

Recently, we became aware of the work by Ochiai [18]. For a 256 complex channel OFDM system employing 256-QAM, a 4.5-dB reduction in the PAPR is obtained using a trellis-shaping technique. In our method, for a 128 complex channel OFDM system employing a 128-QAM, a 6-dB reduction is gained. In [18], the main complexity is in finding the path with minimum cost through a trellis diagram (this complexity is considerably higher than that of a Viterbi decoder). However, the author investigates methods to reduce this complexity by window truncation and sacrificing PAPR reduction, but still the overall complexity in [18] is significantly higher, compared with the method proposed here.

We have not provided any comparison with [4], as the method in [4] relies on using the SNF of the IFFT matrix, which is not known. Indeed, computing this SNF decomposition would be an interesting open problem. If this matrix were available, the resulting PAPR reduction in [4] would be asymptotically equal to the optimum value of $10\log_{10}(3)$. Also, as mentioned in Section II, the computational complexity of the encoding algorithm of the constellation based on the IFFT matrix is $\mathcal{O}(N^2)$, while the complexity for the encoding of the Hadamard constellation in the butterfly structure is $\mathcal{O}((3/2)\log_2(N))$ [4].

VI. Conclusion

We have proposed a constellation-shaping method that achieves a substantial reduction in the PAPR in an OFDM system with a low complexity. The boundary of the proposed constellation is along the basis defined by the Hadamard matrix in the transform domain. An SLM technique is applied to this constellation to further reduce the PAPR of the OFDM signal. The proposed scheme significantly outperforms other

PAPR-reduction techniques reported in the literature, without any loss in terms of the energy and/or spectral efficiency.

REFERENCES

[1] C. Tellambura, "Computation of the continuous-time PAR of an OFDM signal with BPSK subcarriers," *IEEE Commun. Lett.*, vol. 5, no. 5, pp. 185–187, May 2001.

[2] H. Yu and G. Wei, "Computation of the continuous-time PAR of an OFDM signal," in *Proc. Int. Conf. Acoust., Speech, Signal Process.*, Hong Kong, China, Apr. 2003, pp. IV-529–IV-531.

[3] H. Ochiai and H. Imai, "Performance analysis of deliberately clipped OFDM signals," *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 89–101, Jan. 2002.

[4] H. K. Kwok, "Shape Up: Peak-power reduction via constellation shaping," Ph.D. dissertation, Univ. Illinois, Urbana, IL, 2001.

[5] J. A. Davis and J. Jedwab, "Peak-to-mean power control in OFDM, Golay complementary sequences, and Reed–Muller codes," *IEEE Trans. Inf. Theory*, vol. 45, no. 11, pp. 2397–2417, Nov. 1999.

[6] K. Patterson, "Generalized Reed–Muller codes and power control in OFDM modulation," *IEEE Trans. Inf. Theory*, vol. 46, no. 1, pp. 104–120, Jan. 2000.

[7] H. Saeedi, M. Sharif, and F. Marvasti, "Clipping noise cancellation in OFDM systems using oversampled signal reconstruction," *IEEE Commun. Lett.*, vol. 6, no. 2, pp. 73–75, Feb. 2002.

[8] X. Li and L. J. Cimini, "Effects of clipping and filtering on the performance of OFDM," *IEEE Commun. Lett.*, vol. 2, no. 5, pp. 131–133, May 1998.

[9] M. Sharif, M. Gharavi-Alkhansari, and B. H. Khalaj, "On the peak-to-average power of OFDM signals based on oversampling," *IEEE Trans. Commun.*, vol. 51, no. 1, pp. 72–78, Jan. 2003.

[10] H. Ochiai and H. Imai, "On the distribution of the peak-to-average power ratio in OFDM signals," *IEEE Trans. Commun.*, vol. 49, no. 2, pp. 282–289, Feb. 2001.

[11] S. H. Muller and J. B. Hubber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Electron. Lett.*, vol. 33, no. 5, pp. 368–369, Feb. 1997.

[12] M. Breiling, S. H. Muller-Weinfurtner, and J. B. Hubber, "SLM peak-power reduction without explicit side information," *IEEE Commun. Lett.*, vol. 5, no. 6, pp. 239–241, Jun. 2001.

[13] K. Yang and S. Chang, "Peak-to-average power control in OFDM using standard arrays of linear block codes," *IEEE Commun. Lett.*, vol. 7, no. 4, pp. 174–176, Apr. 2003.

[14] N. Carson and T. A. Gulliver, "Peak-to-average power ratio reduction of OFDM using repeat-accumulate codes and selective mapping," in *Proc. IEEE Int. Symp. Inf. Theory*, Jun.–Jul. 2002, pp. 244–244.

[15] I. Collings and I. Clarkson, "A low-complexity lattice-based low-PAR transmission scheme for DSL channels," *IEEE Trans. Commun.*, vol. 52, no. 5, pp. 755–764, May 2004.

[16] B. S. Krongold and D. L. Jones, "PAR reduction in OFDM via active constellation extension," *IEEE Trans. Broadcast.*, vol. 49, no. 3, pp. 258–268, Sep. 2003.

[17] W. Henkel and B. Wagner, "Another application for trellis shaping: PAR reduction for DMT (OFDM)," *IEEE Trans. Commun.*, vol. 48, no. 9, pp. 1471–1476, Sep. 2000.

[18] H. Ochiai, "A novel trellis shaping design with both peak and average power reduction for OFDM systems," *IEEE Trans. Commun.*, submitted for publication.

[19] H. K. Kwok and D. L. Jones, "PAR reduction for hadamard transform-based OFDM," in *Proc. 34th Conf. Signals, Syst., Comput.*, Princeton, NJ, Mar. 2000, [CD-ROM].

[20] ——, "PAR reduction via constellation shaping," in *Proc. Int. Symp. Inf. Theory*, Sorrento, Italy, Jun. 2000, p. 166.

[21] H. Cohen, *Graduate Texts in Mathematics: A Course in Computational Algebraic Number Theory*. New York: Spring-Verlag, 1993, vol. 138.

[22] H. J. S. Smith, "On systems of linear indeterminate equations and congruences," *Phil. Trans. Roy. Soc. London*, vol. 151, pp. 293–326, 1861.

[23] M. Newman, "The Smith normal form," *Linear Algebra Appl.*, vol. 254, pp. 367–381, 1997.

[24] A. Storjohann and G. Labahn, "A fast Las Vegas algorithm for computing the Smith normal form of a polynomial matrix," Univ. Waterloo, Waterloo, ON, Canada, Tech. Rep. CS-94-43, 1994.

[25] R. Bernardini and R. Manduchi, "On the use of the Smith normal form theorem for driving the inverse multidimensional DFT formula," *Eur. Trans. Coomun. Related Technol.*, vol. 5, no. 3, pp. 377–380, May–Jun. 1994.

[26] P. Y. Cochet and R. Serpollet, "Digital transform for a selective channel estimation," in *Proc. IEEE Int. Conf. Commun.*, vol. 1, Jun. 1998, pp. 349–354.

[27] R. W. Bauml, R. F. H. Fischer, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," *Electron. Lett.*, vol. 32, pp. 2056–2057, 1996.

[28] J. V. Eatvelt, G. Wade, and M. Tomlinson, "Peak to average power reduction for OFDM schemes by selective scrambling," *Electron. Lett.*, vol. 32, pp. 1963–1964, 1996.

[29] H. Nikopour, A. K. Khandani, and S. H. Jamali, "Turbo-coded OFDM transmission over nonlinear channel," Univ. Waterloo, Waterloo, ON, Canada, 2004.

[30] B. S. Krongold and D. L. Jones, "An active-set approach for OFDM PAR reduction via tone reservation," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 495–509, Feb. 2004.

[31] G. D. Forney, Jr., "Multidimensional constellations—Part II: Voronoi constellation," *IEEE J. Sel. Areas Commun.*, vol. 7, no. 8, pp. 941–958, Aug. 1989.

[32] G. D. Forney, Jr. and M. D. Trott, "The dynamics of group codes: State spaces, trellis diagrams, and canonical encoders," *IEEE Trans. Inf. Theory*, vol. 39, no. 9, pp. 1491–1513, Sep. 1993.

[33] G. D. Forney, Jr. and F. Wei, "Multidimensional constellations—Part I: Introduction, figures of merit, and generalized cross constellations," *IEEE J. Sel. Areas Commun.*, vol. 7, no. 8, pp. 877–892, Aug. 1989.

[34] Y.-L. Lee, Y.-H. You, W.-G. Jeon, J.-H. Paik, and H.-K. Song, "Peak-to-average power ratio in MIMO-OFDM systems using selective mapping," *IEEE Commun. Lett.*, vol. 7, no. 12, pp. 575–577, Dec. 2003.

[35] J. Tellado, "Peak-to-average power reduction for multicarrier modulation," Ph.D. dissertation, Stanford Univ., Stanford, CA, 2000.

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