An Optimized Transmitter Precoding Scheme for Synchronous DS-CDMA

Erik S. Hons, Amir K. Khandani, and W. Tong

***Abstract*—A technique is presented to reduce the multiple-access interference in the forward link of a direct-sequence code-division multiple-access system. For each symbol period, an energy-constrained nonlinear transformation is applied at the transmitter output to minimize the mean-squared error at the receiver, subject to a constraint on the peak transmitted energy. The proposed algorithm can be implemented with existing optimization techniques that solve the quadratic trust-region problem. It is shown that in the presence of forward error-correction codes, the proposed method results in a significant gain over earlier known techniques at the cost of a modest increase in computational complexity at the base station. Another advantage of the proposed precoder is that it caps the peak energy (while earlier methods cap the average energy), requiring less sophisticated power amplifiers.**

***Index Terms*—Code-division multiple access (CDMA), interference cancellation, multiple-access channels, quadratic optimization, transmitter precoding, trust region.**

I. Introduction

THE conventional direct-sequence code-division multiple-access (DS-CDMA) detector assumes an independent additive Gaussian noise model for the multiple-access interference (MAI). In reality, the MAI term is highly structured, which makes this model invalid. Multiuser detection exploits the structure of the MAI to improve performance at the cost of additional processing overhead at the receiver. This overhead is especially problematic in the forward link of a cellular mobile network, where the receiver is usually a highly resource-constrained mobile unit. Recently, approaches which transfer some or all of the processing burden from the receiver to the transmitter have been proposed [2]–[7]. These methods apply a linear transformation (linear precoder) at the transmitter side, which forces the MAI to zero at the cost of an increase in the average energy. This letter proposes a more general method based on the use of a nonlinear transformation where the MAI is minimized, subject to a constraint on the available energy per transmitted symbol. We follow a setup similar to [2], which is used as the baseline for comparison.

It is shown in [2] that linear precoding results in complete decorrelation of the transmitted symbols. In a noiseless channel, this is equivalent to the well-known decorrelating detector [8]. However, by applying the decorrelation operation at the transmitter before noise is added, transmitter precoding avoids the problem of noise enhancement at the cost of an increase in the average energy. Two methods are considered in [2] to handle the corresponding increase in the transmitted energy: *unconstrained precoding* and *constrained precoding*. The unconstrained method simply scales the transmitter output to maintain an appropriate energy level. In contrast, the constrained method imposes a constraint on the average energy in the selection of the precoding transformation. Reference [2] studies both additive white Gaussian noise (AWGN) and frequency-selective channels, where the *constrained precoding* is applied only in the AWGN case. For this reason, we limit our studies to the AWGN case to show the relative gain of the proposed optimization method.

In this letter, we revisit constrained transmitter precoding for synchronous DS-CDMA with a new formulation, in which the transmitted energy is capped for each symbol period. The MAI will be minimized in the minimum mean-square error (MMSE) sense, subject to this constraint. Thus, we refer to this technique as the " *optimizing precoder*." The focus will be on a channel with forward error-correction codes (FEC), where it is demonstrated that the constrained transmitter precoding performs significantly better in the coded case and, in fact, outperforms unconstrained transmitter precoding (this is different from the conclusion reached in [2] for the uncoded case). It will also be shown that the optimizing precoder outperforms the constrained precoder reported in [2]. The proposed algorithm can be implemented efficiently with existing nonlinear optimization techniques that solve the quadratic trust-region problem.

It should be noted that the proposed method can be applied to a variety of scenarios encountered in cancelling known interference through precoding. An example is the case of cancelling known interference in the context of a broadcast channel with multiple transmit antennas that has recently gained significant attention [9]–[11].

We first present a model for the system in Section II and the problem formulation in Section III. Solution algorithms for the optimizing precoders are presented in Section IV. Finally, numerical results and conclusions are presented in Sections V and VI, respectively.

II. System Model

The transmitted signal $x(t)$ in the forward link of a symbol- and chip-synchronous DS-CDMA system with K active users and symbol duration T_b is

$$x(t) = \sum_{n=1}^{K} A_n s_n(t) b_n, \quad 0 \le t \le T_b \tag{1}$$

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E. S. Hons and A. K. Khandani are with the Coding and Signal Transmission (CST) Laboratory, Electrical and Computer Engineering Department, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: khandani@uwaterloo.ca).

W. Tong is with Nortel Networks, Nepean, ON K2G 6J8, Canada.

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where A_n is the transmitter gain for the nth user, and b_n, $s_n(t)$ are the nth user antipodal modulated data symbol and signature waveform, respectively. We assume that the data symbols are binary and equiprobable. Additionally, we assume that the signature waveforms are linearly independent, zero outside the range $[0, T_b]$, and normalized to have unit energy, so that

$$\int_0^{T_b} s_n^2(t)dt = 1, \quad 1 \le n \le K. \tag{2}$$

In the AWGN channel, the received waveform is

$$r(t) = x(t) + n(t), \quad 0 \le t \le T_b \tag{3}$$

where $n(t)$ is a Gaussian process with zero mean and power spectral density of $\sigma^2 = N_0/2$. The matched-filter (MF) output for user n is the scalar y_n, where

$$y_n = \int_0^{T_b} r(t)s_n(t)dt, \quad 1 \le n \le K. \tag{4}$$

If we combine the MF outputs to form the vector $\mathbf{y} = [y_1, \ldots, y_K]^t$, then the set of outputs can be described in matrix notation as

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{5}$$

where $\mathbf{b} = [b_1, \ldots, b_K]^t$, $\mathbf{A} = \mathrm{diag}\{A_1, \ldots, A_K\}$, $\mathbf{R} = [R_{i,j}]$ is the $K \times K$ cross-correlation matrix for the set of signature waveforms composed of elements

$$R_{i,j} = \int_0^{T_b} s_i(t)s_j(t)dt \tag{6}$$

and $\mathbf{n} = [n_1, \ldots, n_K]^t$ is a Gaussian noise vector whose elements have zero mean and covariance matrix $\mathbf{R}$. The detection strategy which gives the minimum bit-error rate (BER) selects the vector $\mathbf{b}$ with maximum likelihood, given the set of observations $\mathbf{y}$.

For simplicity, we consider a binary phase-shift keyed (BPSK) system for which the signatures are composed of square waveforms with duration T_c, called "chips." With this structure, the normalized signature waveforms $s_n(t)$ can be represented as the binary code vectors $\mathbf{s}_n \in \{(-1/\sqrt{L}), (1/\sqrt{L})\}^L$, where $L = (T_b/T_c)$ is the CDMA processing gain, and the collection of codes as the matrix

$$\mathbf{M} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_K \end{bmatrix}. \tag{7}$$

Note that $\mathbf{R} = \mathbf{M}^t\mathbf{M}$, where $\mathbf{M}^t$ denotes the transpose of $\mathbf{M}$, and that $\mathbf{R}$ is symmetric and positive definite. We assume that the signatures are selected randomly.

III. Problem Formulation

The conventional approach for precoding (see [2]–[7]) is based on applying a linear transformation $\mathbf{T}$ to the transmitted vector, i.e.,

$$\mathbf{y} = \mathbf{RTAb} + \mathbf{n}. \tag{8}$$

It was shown in [2] that when the MMSE criterion is used to solve for $\mathbf{T}$ and expectation is taken with respect to $\mathbf{b}$ and $\mathbf{n}$, the result is $\mathbf{T} = \mathbf{R}^{-1}$. Using the optimal $\mathbf{T}$ completely eliminates MAI; however, a side effect of precoding is increased transmitted energy. It is natural to consider imposing an energy constraint at the transmitter, say $||\mathbf{Mb}'||^2 \le r$, where $\mathbf{b}'$ is the data vector to be computed as follows. Replacing $\mathbf{R} = \mathbf{M}^t\mathbf{M}$, we can reformulate the optimization as

$$\begin{aligned} \min_{\mathbf{b}'} \quad & ||\mathbf{Ab} - \mathbf{M}^t\mathbf{Mb}'||^2 \\ \text{subject to:} \quad & ||\mathbf{Mb}'||^2 \le r. \end{aligned} \tag{9}$$

We refer to this method as the *optimizing precoder*. By constraining the available energy per transmitted symbol, we force the system to include MAI in situations where eliminating it would require high energy levels. For this reason, the optimizing precoder can be expected to have worse raw performance than unconstrained precoding, due to having different levels of noise margin at different bit positions. The situation changes, however, when FEC is used, because FEC achieves a form of averaging of instantaneous signal-to-noise ratio (SNR) over several bit periods, compensating for the aforementioned effect of variable noise margins. This conclusion will be borne out by simulation results.

A constrained transmitter precoding scheme is also presented in [2], which is based on: 1) linear precoding and 2) complete cancellation of MAI. This letter proposes a more general method, based on the use of a nonlinear transformation, where the MAI is minimized subject to a constraint on the available energy per transmitted symbol. Another significant difference is the scope of the precoder design. In (9), a new transformation is generated for each symbol period, with expectation of the mean-square error (MSE) taken with respect to channel noise only. In [2], expectation of MSE is taken with respect to $\mathbf{b}$ as well as $\mathbf{n}$, so that the current data vector is not a distinct influence on the design of the precoding transformation. These features give higher flexibility to our proposed method, accounting for a significant improvement as supported through simulation results. Furthermore, (9) caps the energy per transmitted symbol, while in [2], *average* energy is constrained. The latter method allows for undesirable spikes in the instantaneous power, requiring more sophisticated power amplifiers.

IV. Solution Algorithm

Numerical optimization techniques are required to solve (9). The problem specifies minimization of a convex quadratic function (because $\mathbf{MM}^t$ is positive semidefinite) over an ellipsoid (a convex region) whose surface is the set of signal vectors which have a transmitted power level of r. To simplify the problem, we apply the change of variable $\mathbf{x} = \mathbf{Mb}'$ which transforms (9) into minimization over a sphere, i.e.,

$$\begin{aligned} \min_{\mathbf{x}} \quad & ||\mathbf{Ab} - \mathbf{M}^t\mathbf{x}||^2 \\ \text{subject to:} \quad & ||\mathbf{x}||^2 \le r. \end{aligned} \tag{10}$$

Expanding the new objective function results in

$$\mathbf{b}^t\mathbf{A}^t\mathbf{Ab} - \mathbf{2x^tMAb} + \mathbf{x^tMM^tx}.$$

Let $\mathbf{Q} = 2\mathbf{MM^t}$, $\mathbf{c} = -2\mathbf{MAb}$, and delete the constant term to obtain

$$\begin{aligned} \min_{\mathbf{x}} \quad & \frac{1}{2}\mathbf{x}^t\mathbf{Qx} + \mathbf{x^tc} \\ \text{subject to:} \quad & ||\mathbf{x}||^2 \le r. \end{aligned} \tag{11}$$

This type of problem occurs frequently in nonlinear optimization as the *trust-region problem*. Its solution has a standard treatment, which is given in [12]. A more convenient solution algorithm is given in [13]. A detailed explanation of this algorithm tailored for the current problem is given in [1]. This algorithm is based on an iterative approach. Simulation shows that two iterations are enough to obtain solutions with near-optimal performance. Each of these iterations requires the Cholesky factorization of an $L \times L$ matrix. Thus, complexity depends on the system chip rate, but not on the number of active users.

Alternatively, another solution algorithm has been proposed in [14] and [15], tailored for large-scale systems for which forming factorizations is infeasible. The basis of this technique is the Lanczos method, which can approximate eigenvalues using only matrix-vector products. On specialized processors, this technique may provide a performance gain over the method described above.

V. Simulation Results

In this section, simulation results will be compared with those from conventional systems, as well as with both the constrained and unconstrained transmitter precoding methods of [2] (the constrained precoding method of [7] gives the same results as [2] for the sake of our comparison).[1] It is shown that in the presence of FEC, the proposed method results in a significant gain over earlier known techniques, with a modest increase in computational complexity at the base station.

Both coded and uncoded systems are studied with coded systems using the rate-1/3 convolutional code from the IS-95 standard. Receivers use a standard Viterbi decoder, which is supplied with soft outputs from the MF detector. The simulator generates independent pseudorandom data streams for each user which are independently coded when FEC is used. Another independent pseudorandom bit stream is used to generate the spreading codes. The system chip rate is set to $L = 32$ in all cases.

The optimal value of r in (11) is determined through simulation. By holding the system parameters and SNR constant while varying r, we find a value which gives the minimum BER for each system configuration. Fortunately, the BER curve is smooth and continuous with respect to changes in r, which simplifies the search. Fig. 1 shows the effect of r on the optimizing precoder performance when the system configuration and SNR level are held constant. In all four curves, performance reaches a single global optimum with respect to r. Only 16-user results are shown. The optimum value of r was consistently lower for coded transmission than for the uncoded for all numbers of users. Moreover, the optimal r for FEC was much less affected by the number of users and changing channel conditions. Despite the effects of FEC, the optimum r was not constant for different system configurations, or even for different SNR levels. However, the set of optimum values for r need only to be calculated once and stored in a lookup table. Alternatively, r can be chosen to optimize performance at a certain SNR with only a

[1]The main difference between [2] and [7] is that in [2], a Rake receiver is used to deal with frequency selectivity of the channel, while in [7], this is replaced by a pre-Rake at the transmitter side. This means the two methods are the same over an AWGN channel, as considered in this letter.

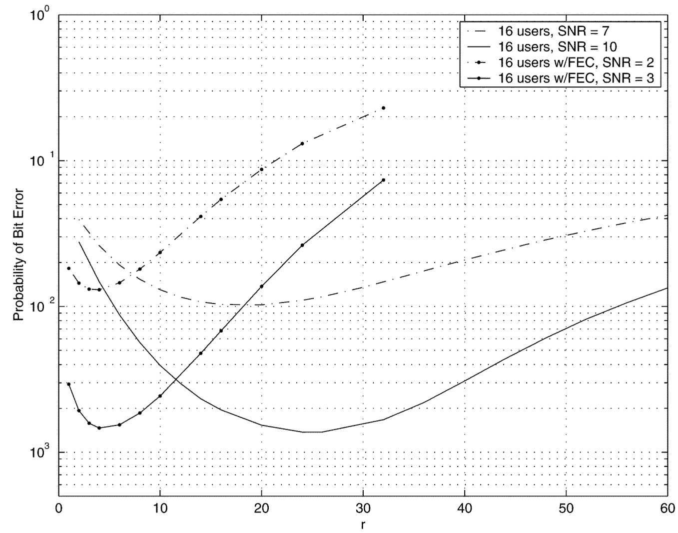

Fig. 1. BER curves (obtained through simulation) used to select the optimum r. Number of users is 16.

small loss at other SNR levels, due to the smoothness of the BER curve. Note that a conventional detector does not require knowledge of r to produce soft output metrics, and consequently, the modulator is free to adjust r as required.

Two conventional systems are simulated: a standard modulator equipped with an MF detector, and a system with a decorrelating detector. The former is labeled "Conventional System" and provides a baseline performance curve without any performance enhancing technique. The latter is labeled the "Decorrelating Detector" and gives an example of a standard receiver-based performance-enhancement technique. The two methods from [2] are labeled "Transmitter Precoding" and "Constrained Transmitter Precoding." These are appropriate generalizations of the method proposed in [2] (with FEC and power constraint added at the transmitter side). The optimizing precoder proposed in this letter is labeled "Optimizing Precoder," and is paired with a standard detector. The single-user case is also given for comparison.

As mentioned earlier, [2] studies the case of imposing a constraint on energy at the transmitter side; however, as it does not use FEC, the conclusion has been that imposing such a constraint does not improve the performance. This agrees with our results for the uncoded case. However, we have shown that imposing such a constraint indeed improves the performance for the coded case.

In Figs. 2 and 3, uncoded data is being transmitted to 8 and 16 users. In all cases, transmitter precoding gives the best performance at practical system error rates of 10^{-3} and below. As stated in [2], constrained transmitter precoding performs better for low SNR, but eventually crosses the unconstrained method as SNR rises. Our proposed optimizing precoder generally performs worse than the constrained transmitter precoder of [2] in this setting. However, this situation changes in the presence of FEC.

In Figs. 4 and 5, data is independently coded and then transmitted to 8 and 16 users. Clearly, the energy-constrained methods significantly outperform unconstrained methods in this setting. For 16 users, the constrained transmitter precoding

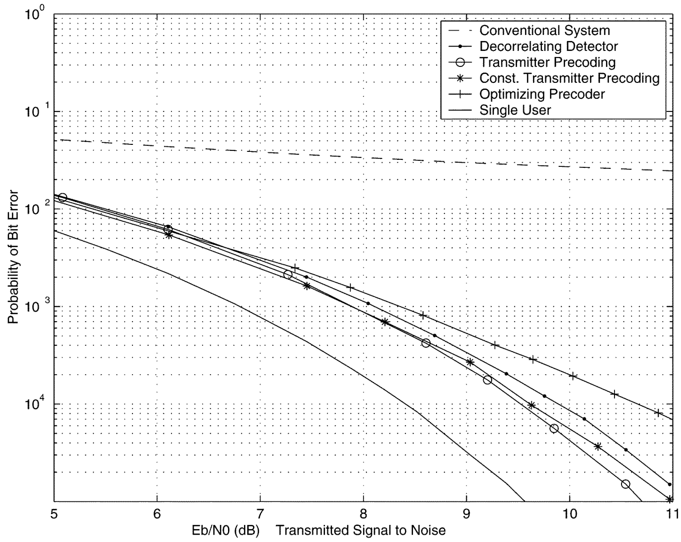

Fig. 2. BER curves for an uncoded system. Number of users is 8 in 32 chips.

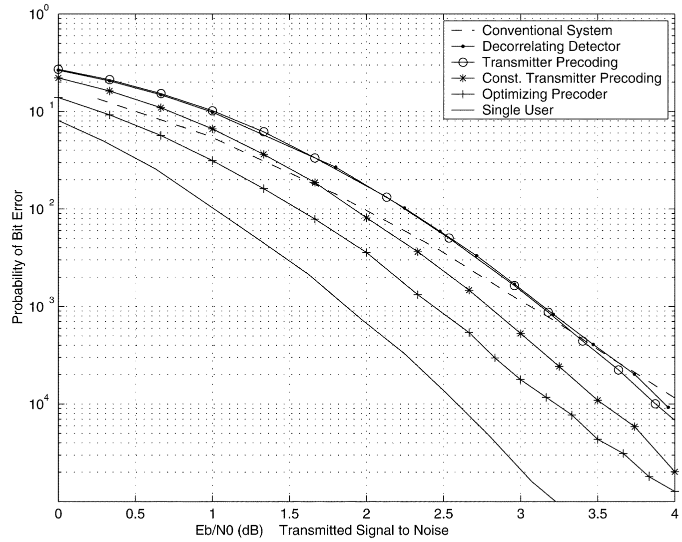

Fig. 4. BER curves for a coded system. Number of users is 8 in 32 chips.

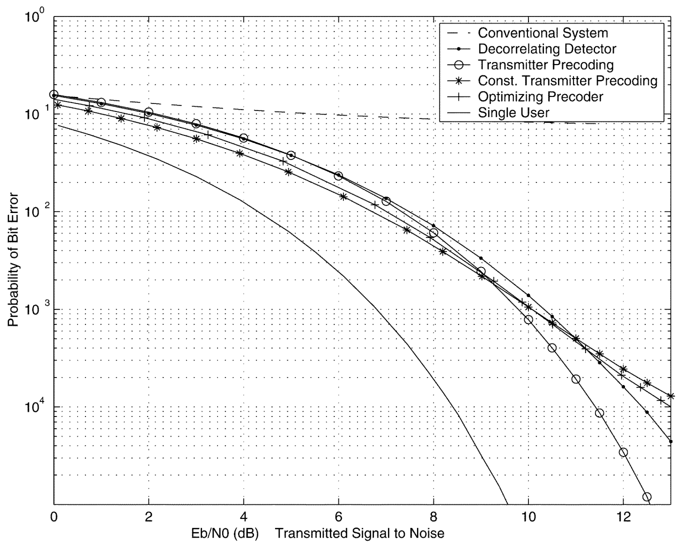

Fig. 3. BER curves for an uncoded system. Number of users is 16 in 32 chips.

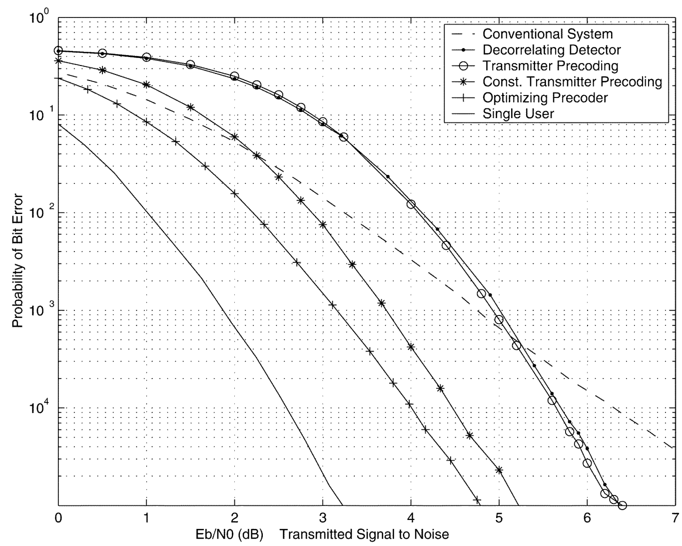

Fig. 5. BER curves for a coded system. Number of users is 16 in 32 chips.

has a gain of more than 1.0 dB over unconstrained transmitter precoding. The optimizing precoder performs even better, achieving a further gain of 0.5 dB over constrained precoding. Simulations for the eight-user case show this gain to be 0.25 dB.

The computational complexity of the standard solution algorithm for both optimizing precoders is determined by the desired accuracy of the solution. As explained in [1], accuracy translates directly into iterations of a loop which requires the Cholesky factorization of a matrix. Fig. 6 shows the result of limiting the number of iterations for the 16-user case. From this graph, it is clear that near-optimal performance is achieved as soon as the second iteration, which requires only two matrix factorizations.

In order notation, the Cholesky factorization is $O(L^3)$, while two $O(L^2)$ triangular system back-solves are required to obtain the final solution. The parameter L (chip rate) is usually fixed in practical systems, so that these costs are also fixed. These costs are not prohibitive in the forward link of a mobile cellular system because the processing is performed at the base station, where power and processing resources are readily available. The

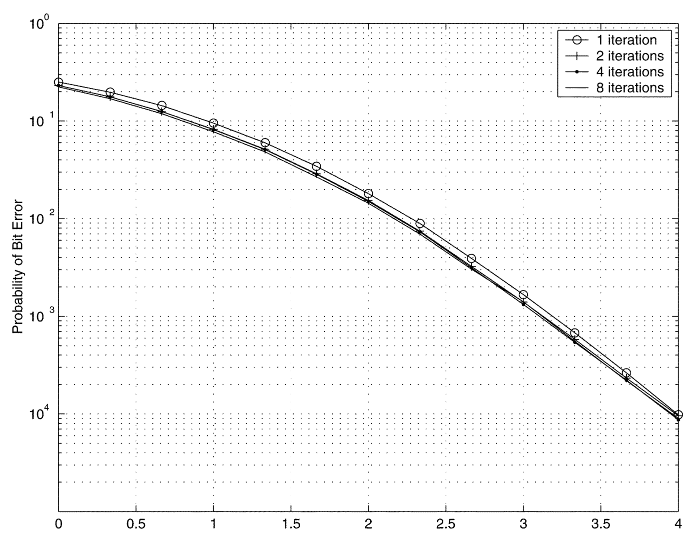

Fig. 6. Effect of the number of iterations of the optimization algorithm on the BER performance for a coded system. Number of users is 16 in 32 chips.

main benefit of this and all other precoding methods is the simplicity of the receiver, which is only an MF detector.

VI. Conclusions

A new transmitter precoding method is introduced, which, using a nonlinear transformation, minimizes the MSE at the output of a bank of MF detectors, subject to a constraint on the peak transmitted energy. It is shown that in the presence of FEC, the proposed method results in a significant gain over earlier known techniques, at the price of a modest increase in the computational complexity at the base station. Another advantage of the proposed precoder is that it caps the peak energy per transmitted symbol, and consequently, it can be realized using power amplifiers with a smaller dynamic range (as compared with the earlier known methods, which cap the average energy). Numerical results show that for typical DS-CDMA configurations, the proposed method realizes gains of: 1) 0.75–4 dB over existing unconstrained precoding methods; and 2) 0.25–0.5 dB over existing constrained precoding methods. The proposed algorithm can be implemented efficiently with existing optimization techniques that solve the quadratic trust-region problem. This requires about two fast Cholesky factorizations of an $L \times L$ matrix (L is the CDMA processing gain) to achieve those gains.

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