defined as $\eta = R_S[k]/R_{y''}[k]$, where $R_s[k]$ is the autocorrelation of the actual signal for the lag $k$, and $R_{y''}[k]$ the distorted autocorrelation that the conventional PAR model would exhibit, we reinforce the autocorrelation of $g[n]$ for the lag $k$ so that, after being projected, the distortion of the autocorrelation will be compensated.

*Simulation and results:* To prove the proposed scheme, two different MPEG-I encoded sequences were used. The first one corresponds to the whole film 'Star Wars'. The second sequence includes thirty minutes of the soccer World Cup 1994 final. Both traces were encoded with 24 frames per second and a GOP structure of twelve frames. The model was designed to distinguish between three different scene types ($N = 3$). Distribution functions were calculated using histograms of 40 levels. Results are presented for simulated queues with a utilisation factor of 70%. In Figs. 2 and 3, the behaviour in a queue of the real traffic is compared with those of different models: a SRD model that does not consider scene changes ($N = 1$) and the proposed model using both the conventional PAR model and the modified PAR as well as two different window sizes for the averaging filter. For the film 'Star Wars', Fig. 2 proves the accuracy of the modified PAR scheme to predict losses if the window size ($W = 500$) is correctly designed. A model with a shorter window ($W = 50$) or conventional PAR scheme and, of course, an SRD model, underestimate the losses, especially when the queue size increases. Conversely, Fig. 3 proves that the proposed model is also able to adjust the behaviour of a video trace of a sport event, improving the adjustment that a SRD model performs. In this case, a shorter window ($W = 100$) is enough for a proper tuning of the model, as long as scenes are not as long as in a film.

*Conclusions:* A two level model for VBR video traffic has been presented. To cope with the existence of LRD within the video signal, the model considers two time scales: scenes and GOPs. To model the scene changes, a Markov chain is used. For the GOP level a modification for the PAR model is proposed so that the fitting of the autocorrelation function is improved. The ability of the model to adjust the behaviour of the real traffic in a queue is proved using two MPEG sequences with different characteristics. Using this scheme, the complexity of self-similar models and fractal calculations is avoided.

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## Group structure of turbo-codes

A.K. Khandani

The author discusses the group property of turbo-codes when considered as a periodic linear system. It is shown that the interleaving procedure provides a homomorphism between different encoded sequences and thereby breaks the low weight sequences by a factor of, at most, $1/2^r$, where $r$ is the constraint length of the code.

*Introduction:* Fig. 1 shows the block diagram of a rate 1/3 turbo-code composed of two recursive convolutional codes (RCC), where $i_k$ is called the systematic bit, and $p_k^1$, $p_k^2$ are called the parity check bits [1]. The effect of interleaving is equivalent to multiplying the input sequence by a permutation matrix which corresponds to a linear operation. Note that as the RCCs and also the interleaver, have the property of linearity, the resulting code is linear, and consequently the distance invariance property holds.

The weight of the code in Fig. 1 is equal to the sum of the weights of the $i_k$, $p_k^1$ and $p_k^2$ sequences over a block. Considering that the RCCs are linear systems with an impulse response of infinite length, it is expected that the $p_k^1$ and $p_k^2$ sequences will be of a large weight. This, however, depends on the pattern of the $i_k$ sequence, and unfortunately, there exists low weight $i_k$ sequences for which the resulting $p_k^1$ sequence will also be of a low weight. The role of the interleaver is to modify the pattern of such $i_k$ sequences such that the resulting $p_k^2$ sequence will be of a large weight (and vice versa for $p_k^1$).

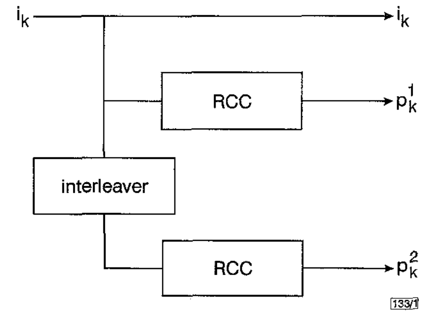


**Fig. 1** *Basic structure of turbo encoder*

*Periodicity property and group structure of turbo-code impulse response:* We assume that the RCCs are generated by the transfer function $G(d) = Q(d)/D(d)$. We know that the impulse response of $G(d)$ is periodic with period $p \leq 2^r - 1$ where $r$ is the constraint length of the code [2]. We are mainly interested in the group structure and also the periodicity property of the impulse response of $G(d)$. In this respect, without loss of generality, we limit our attention to the structure of $D(d)$.

The polynomial $D(d)$ in turbo-codes is selected to be a primitive polynomial generating generating a maximum length sequence (MLS) [3 – 5]. The intuitive reason is that we would like the period of the impulse repose of $G(d)$ to be as large as possible. It is known that if $D(d)$ is a primitive polynomial, then the resulting period is a factor of $2^r - 1$ [2]. If the period is equal to $2^r - 1$, the resulting impulse response is called an MLS. It is known that among the irreducible polynomials of degree $r - 1$, a subset of them of cardinality $\phi(2^r - 1)/r$, where $\phi(.)$ is the Euler function, results in an MLS [2]. Obviously, if $2^r - 1$ is a prime number, then any irreducible polynomial $D(d)$ results in an MLS. The rules for determining all the possible configurations of $D(d)$ in order to obtain MLS are given in [2]. It can be shown that in any period of an MLS, the number of ones is equal to $2^{r-1}$ and the number of zeros is equal to $2^{r-1} - 1$ [2].

If we look at the impulse response of $D(d)$ as a periodic sequence, we obtain $K = 2^r - 1$ non-zero sequences which are time shifts of each other. Each sequence corresponds to a specific positioning of an input impulse within the period. We refer to these sequences as different phases of the periodic signal. It can be shown that the set of phases of an MLS (plus the all-zero sequence) constitute a group under binary addition [2]. The order of each element in this group is equal to two, meaning that the sum of each phase with itself results in the all-zero sequence (denoted as the zero phase).