Solutions to some sampled questions of previous finals

First exam: Problem 2: The modulating signal $m(t)=A_m\cos(2\pi f_m t)$ is used to generate the VSB signal

$$s(t)=\frac{A_m A_c}{2}\beta\cos\left[2\pi\left(f_c+f_m\right)t\right]+\frac{A_m A_c}{2}(1-\beta)\cos\left[2\pi\left(f_c-f_m\right)t\right]$$

where β is a constant less than unity representing the attenuation of the upper side frequency.

2.1. Find the in phase and the quadrature components of the VSB signal s(t).

2.2. The VSB signal plus the carrier $A_c\cos(2\pi f_c t)$ is passed through an envelope detector. Derive an expression for the output.

Solution:

2.1. It is easy to see that we can represent s(t) as the following.

$$s(t)=\frac{A_m A_c}{2}\cos(2\pi f_c t)\cos(2\pi f_m t)+\frac{A_m A_c}{2}(1-2\beta)\sin(2\pi f_c t)\sin(2\pi f_m t)$$

And easily the in phase and quadrature component will be

$$s_I(t)=\frac{A_m A_c}{2}\cos(2\pi f_m t)$$

$$s_Q(t)=\frac{A_m A_c}{2}(1-2\beta)\sin(2\pi f_m t)$$

2.2. After adding the carrier and passing it through the envelope detector the output will be

$$y(t)=\sqrt{\left(A_c+\frac{A_m A_c}{2}\cos(2\pi f_m t)\right)^2+\left(\frac{A_m A_c}{2}(1-2\beta)\sin(2\pi f_m t)\right)^2}$$

$$s(t)=\frac{A_m A_c}{2}\cos(2\pi f_c t)\cos(2\pi f_m t)+\frac{A_m A_c}{2}(1-2\beta)\sin(2\pi f_c t)\sin(2\pi f_m t)$$

Problem 3: Periodic signal s(t) shown in figure is used once to frequency modulate a carrier of frequency f_c and once to phase modulate the same carrier.

3.1. Find a relation between k_p and k_f such that the peak phase deviation of the modulated signal in both cases are equal.
3.2. If $k_p=k_f=1$ what is the maximum instantaneous frequency in each case.
3.3. In the case of PM assuming $k_p=1$, find an expression for the spectral density of the resulting modulated signal.
Solution:
3.1. We know that the phase deviation in PM modulation is $k_p s(t)$. Hence the maximum deviation is $k_p \max\{s(t)\}=k_p$.
For the case of FM modulation after integrating from the signal we reach that the peak of that is 1 and hence the peak deviation will be $2\pi k_f$. So we have to have $2\pi k_f=k_p$.
3.2. The instantaneous frequency is the differentiation of phase and hence the peak instantaneous frequency in the case of PM will be $f_c+1/(2\pi)*k_p=f_c+1/(2\pi)$ because the peak of s(t) slope is 1. However in the case of the FM, s(t) represents the instantaneous frequency and hence its peak is also $f_c+1*k_f=f_c+1$.
3.3. We can write

$$x(t)=\mathrm{Re}\left[e^{j\omega_c t}e^{js(t)}\right]$$

We know that $e^{js(t)}$ is a periodic signal with period of T=6. Then we can write its Fourier series as

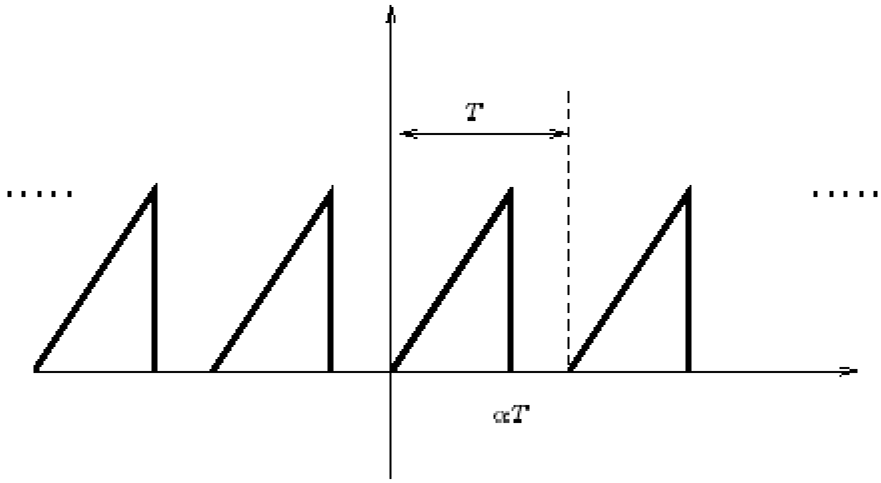
$$e^{js(t)}=\sum a_n e^{jn\omega t}$$

where a_n can be written as

$$a_n=\frac{1}{T}\int_0^T e^{js(t)}e^{-jn\omega t}dt$$

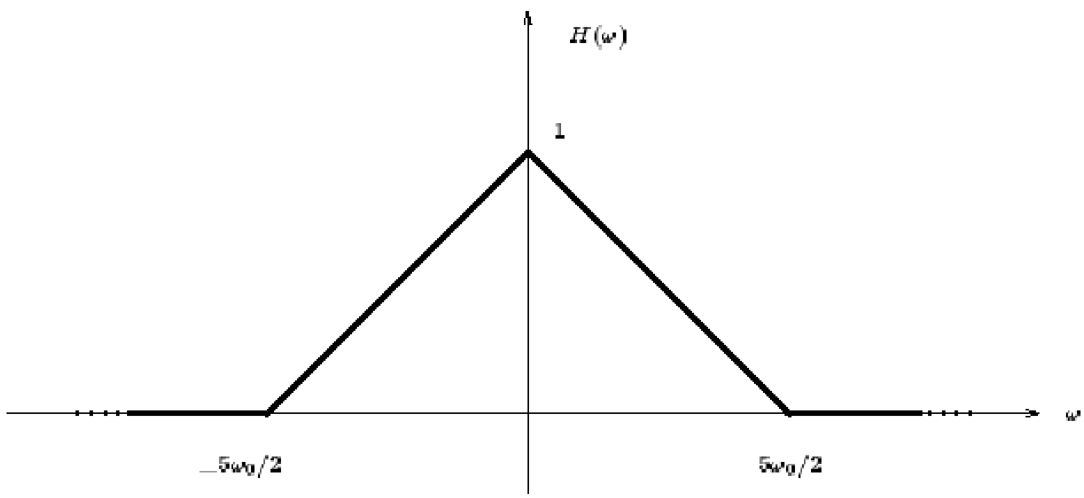
The resulting x(t) will be addition of some exponential function and hence its spectral density is easy to derive.

‘W98: Problem 5: Consider the periodic signal shown in the following Figure



5.1. Find the autocorrelation function the power spectral density and the total power of this signal

5.2. Assume that this signal is passed through a linear filter with the frequency response shown in the following figure (where $\omega_0=2\pi/T$). Compute the value of α in the periodic signal such that the power of the signal at the output of the filter is maximized.



Solution:

5.1. Suppose $\alpha>1/2$. (Similar results can be written for the other case)This signal is periodic and hence its correlation function is also periodic. So we focus on $0<\tau<T$. If $0<\tau<(1-\alpha)T$ then we have

$$\int_0^T s(t)s(t-\tau)dt = \int_\tau^{\alpha T} t(t-\tau)dt$$

And for $(1-\alpha)T<\tau<\alpha T$ we have

$$\int_0^T s(t)s(t-\tau)dt = \int_\tau^{\alpha T} t(t-\tau)dt + \int_0^{\tau - T + \alpha T} t(t+T-\tau)dt$$

And for $\alpha T < \tau < T$ we have

$$\int_0^T s(t)s(t-\tau)dt = \int_0^{\tau - T + \alpha T} t(t+T-\tau)dt$$

So the autocorrelation function can be derived easily.

For periodic signal the power spectral density can be derived from using the Fourier transform of the autocorrelation function or using the Fourier series of the signal. This signal has some coefficients in frequencies n/T. Each coefficient corresponds to one coefficient of the power spectral density (you have to square it).

For the case of energy it is the autocorrelation function at the point 0 or easily αT^2.

5.2. After passing this signal through the filter only coefficients at 0, $\pm\omega_0$ and $\pm 2\omega_0$ remain and the output power can be written as the sum of the square of those coefficients after passing it. To find of the maximum power we have to maximize that sum over all possible values of α.

W '96. Problem 1: The output (modulated) signal from an AM modulator is

$$u(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t)$$

1.1. Determine the modulating signal m(t) and the carrier signal c(t).

1.2. Determine the modulation index. Can the signal m t be recovered using an envelope detector?

1.3. Determine the ratio of the power in the side bands to the power in the carrier.

Solution:

1.1. We can rewrite the above signal as

$$u(t) = 20\cos(2000\pi t)(1 + 0.5\cos(200\pi t))$$

So m(t) will be

$$m(t) = \cos(200\pi t)$$

1.2. Using the above relation we can see that m=0.5 and we can use envelope detector because m<1.

1.3. The power of the carrier is $20^2/2=200$ and the power of each sideband is $5^2/2$ and hence the total power will be 25. So the ratio of power will be 1/8.

W ’98: Problem 2: A communication system operates in the presence of white noise with a two sided power spectral density $S_a(w)=10^{-14}$ W/Hz and with a path loss of 20dB. Calculate the minimum required band-width and the minimum required carrier power of the transmitter for a 10 KHz sinusoidal input and a 40dB output S/N ratio if the modulation is:
2.1. DSB-SC
2.2. SSB-SC
2.3. FM with Δf=10 KHz

Solution:
2.1. DSB-SC needs two times bandwidth of the original signal means 20 KHz. For DSB-SC the ratio of SNR at the input and the output are the same. So because we need SNR of 40 dB, we have to have equal SNR at the input. Hence the power at the input of the receiver must be 40 dB more than the power of noise, which is $2Sa(W)*BW=20*10^3*10^{-14}=2*10^{-10}=-97$ dB. Consider that the BW means the low pass bandwidth. On the other hand due to loss of 20 dB, the transmitter power has to be 20 dB more means, -37 dB=0.2 mWatt.
2.2. SSB-SC needs equal bandwidth of the original signal means 10 KHz. For DSB-SC the ratio of SNR at the input and the output are the same. So because we need SNR of 40 dB, we have to have equal SNR at the input. Hence the power at the input of the receiver must be 40 dB more than the power of noise, which is $2Sa(W)*BW=20*10^3*10^{-14}=4*10^{-10}=-97$ dB. On the other hand due to loss of 20 dB, the transmitter power has to be 20 dB more means, -37 dB=0.2 mWatt.
2.3. The bandwidth of FM is $(2BW+2\Delta f)$=40 kHz. On the other hand we know that the FM increases by the factor of $3DS_x$ so it increases the SNR as 1.5 and hence the required power will be that in previous ones divided by 1.5.

Please excuse me if some mistakes are happened in the solutions.
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