1. A message signal $f(t)$ (with a bandwidth $B$) is transmitted using DSB-SC modulation; thus the transmitted waveform is given by $\phi(t) = f(t) \cos(\omega_c t)$. During transmission, the frequency and phase of the carrier signal are distorted, so that the received signal is $r(t) = f(t) \cos[(\omega_c + \Delta \omega_c) t + \psi]$. If the receiver local oscillator signal is $A_c \cos(\omega_c t)$:

(a) Show the functional block diagram of the demodulator. Find an expression for the output of the demodulator.

(b) If $\Delta \omega_c = 0$, find an expression for the total energy in the demodulator output, and plot the energy as a function of $\psi$ for $\int_{-\infty}^{\infty} f^2(t) dt = 1$.

(c) Let $\psi = 0$, and describe the effect of the erroneous frequency reference. Sketch a typical Fourier transform of the demodulator output for $|\Delta \omega_c| < B$. Assume a shape for $F(\omega) = \mathcal{F}\{f(t)\}$. 
2. A nonlinear element with the input-output relation \( v_o(t) = a v_i^2(t) + b v_i(t) \) is used in a DSB-LC modulator, where \( a > 0 \) and \( b > 0 \). The message signal is \( f(t) \) (\(|f(t)| \leq 1\)) with the Fourier transform \( F(\omega) \) (\(|F(\omega)| = 0 \) for \(|\omega| > 2\pi B_f\)). The output must have a form of \( A[1 + mf(t)] \cos(\omega_c t) \), where \( \omega_c \gg 2\pi B_f \).

(a) Draw a block diagram of the modulator with minimum configuration. Specify necessary parameters in the diagram.

(b) Describe how the modulator works by using mathematical expressions.

(c) Express \( m \) and \( A \) in terms of the parameters \( a \) and \( b \).

3. The carrier \( c(t) = 100 \cos(2\pi 10^6 t) \) volts is frequency modulated by the sinusoid signal \( f(t) = 2 \cos(2000\pi t) \) volts. The frequency sensitivity of the modulator is \( k_f = 3000 \) Hz/volt.

(a) Determine the modulation index \( \beta \).

(b) Determine the bandwidth of the FM signal using Carson’s rule.

(c) Determine the average power of the FM signal over a 1-ohm resistor.

(d) If the amplitude of \( f(t) \) is decreased by a factor of 2, how would your answers to parts (a)-(c) change?

(e) If the frequency of \( f(t) \) is increased by a factor of 2, how would your answers to parts (a)-(c) change?

4. A communication system operates in the presence of white noise with two-sided power spectral density \( S_n(f) = 0.25 \times 10^{-14} \) watts/Hz, and with total path loss of 100 dB. The input bandwidth is 15 kHz. For a 15-kHz sinusoidal input and for a 40-dB output S/N ratio, calculate the total transmitted power if the modulation is

(a) DSB-LC with \( m = 0.5 \) and with envelop detection.

(b) SSB-SC with coherent demodulation.

(c) FM with \( \Delta f = 30 \) kHz using frequency discriminator for demodulation.
5. A given preemphasis/deemphasis system is shown in Figure 1. The power spectral density of the additive noise is $S_n(f) = 2\exp(2\pi \times 10^{-4}|f|) \mu W/\text{Hz}$. The frequency transfer function of the deemphasis filter is designed to yield a white output noise spectral density over the frequency range $0 < f < 7.5 \text{ kHz}$.

(a) What is the magnitude frequency transfer function $H(f)$ of the preemphasis filter required to yield no overall net signal distortion (assuming that the input signal has a bandwidth of 7.5 kHz)?

(b) Calculate the SNR improvement (at the output of the system) obtained using this system over the frequency range $0 < f < 7.5 \text{ kHz}$ if $H(0) = 1$.

$$\text{Figure 1}$$

6. Consider a low-pass signal $g(t)$ having Fourier transform $G(f)$. It is to be sampled at the rate of $2f \leq 1/T_s \text{ Hz}$.

(a) Draw a diagram to illustrate the magnitude spectrum of the ideally sampled process

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s).$$

What is $G_\delta(f)$ and how is $g(t)$ recovered from $g_\delta(t)$?

(b) Draw a diagram to illustrate the magnitude spectrum of the flat-top sampled process

$$g_\Delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\text{rect}(\frac{t - nT_s}{\tau})$$

for $\tau = T_s/2$, where

$$\text{rect}(\frac{t}{\tau}) = \begin{cases} 
1, & |t| \leq \frac{\tau}{2} \\
0, & \text{otherwise}
\end{cases}$$

What is $G_\Delta(f)$ and how is $g(t)$ recovered from $g_\Delta(t)$?