E&CE 318: Introduction to Communication Systems
Instructor: E. Yang
Final Exam, Fall 2000, December 13, 9:00 a.m.–12:00 noon.
University of Waterloo
Dept. of E&CE

Special Instructions

- Time allowed: 3 hours.
- Closed book & notes. No crib sheet is allowed.
- Answer all questions.
- Justify your answers.
- Cheating will not be tolerated. Any instances of cheating will be handled according to university rules.

Problem 1 A modulating signal $f(t) = 20\cos 2000\pi t + 8\cos 4000\pi t$ is applied to a SSB-SC+ modulator operating at a carrier frequency of 10 kHz. Let $\phi_{SSB-SC+}(t)$ denote the resulting modulated signal.

(a) (5 points) Determine the time-domain representation of $\phi_{SSB-SC+}(t)$.
(b) (5 points) Sketch the line spectrum of $\phi_{SSB-SC+}(t)$.
(c) (5 points) Compute the average power of $\phi_{SSB-SC+}(t)$.
(d) (5 points) Repeat Parts (a) and (b) for the baseband complex envelope of $\phi_{SSB-SC+}(t)$.

Problem 2 A sinusoidal signal $f(t) = \cos 2000\pi t$ is used to frequency modulate a 500 kHz carrier signal; the resulting modulated signal is

$$\phi_{\mathrm{FM}}(t) = 100\cos(\pi \times 10^6 t + 3\sin 2000\pi t)$$

(a) (5 points) Use Carson's rule to determine the bandwidth of $\phi_{\mathrm{FM}}(t)$.
(b) (5 points) Sketch the waveform of $\phi_{\mathrm{FM}}(t)$ in the time domain.
(c) (5 points) Let $\phi_{\mathrm{PM}}(t)$ be the output of a PM modulator in response to the input $f(t)$. Suppose that the bandwidth of $\phi_{\mathrm{PM}}(t)$ is the same as that of $\phi_{\mathrm{FM}}(t)$, but the average power of $\phi_{\mathrm{PM}}(t)$ is a doubling of the average power of $\phi_{\mathrm{FM}}(t)$. Determine the time-domain representation of $\phi_{\mathrm{PM}}(t)$.
(d) (5 points) Sketch the waveform of $\phi_{\mathrm{PM}}(t)$ in the time domain.

Problem 3 A baseband signal $f(t)$ with bandwidth B Hz is used to frequency modulate a high frequency carrier signal with carrier frequency ω_c, where $\omega_c \gg 2\pi B$. The resulting modulated signal is given by

$$\phi_{\text{FM}}(t) = A\cos\left(\omega_c t + k_f\int_0^t f(\tau)d\tau\right)$$

(a) (5 points) Under what condition can $\phi_{\text{FM}}(t)$ be regarded as an NBFM waveform?
(b) (5 points) Let $\phi_{\text{DSB-SC}}(t)$ be the DSB-SC waveform corresponding to $f(t)$ with the same carrier frequency ω_c. If $\phi_{\text{FM}}(t)$ can be regarded as an NBFM waveform, does it mean that the bandwidth of $\phi_{\text{FM}}(t)$ is less than that of $\phi_{\text{DSB-SC}}(t)$? Explain why?
(c) (5 points) Sketch a diagram which can be used to generate an NBFM waveform.
(d) (8 points) Suppose that $\phi_{\text{FM}}(t)$, which could be a WBFM waveform, is transmitted over a channel which distorts the carrier amplitude but keeps the angle information unchanged. Design a receiver which can recover $f(t)$ from the received waveform. Explain how your receiver works.

Problem 4 Let $X(t)$ be a WSS process. Apply $X(t)$ to the input of a DSB-SC modulator which also introduces a random phase. The resulting output is

$$Y(t) = X(t)\cos(\omega_c t + \Theta)$$

where Θ is a random variable distributed uniformly over $(0, 2\pi)$ and independent of $X(t)$.
(a) (5 points) Is $Y(t)$ a WSS process? Explain why.
(b) (5 points) Determine the autocorrelation function of $Y(t)$ in terms of $R_X(\tau)$.
(c) (5 points) Determine the PSD of $Y(t)$ in terms of $S_X(\omega)$.

Problem 5 The DSB-LC signal

$$\phi(t) = 4\cos(10^4\pi t) + 2\cos(10^3\pi t)\cos(10^4\pi t) \quad v$$

is present with additive white noise whose power spectral density is 1 μ W/Hz. This signal plus noise is passed through an ideal low-pass filter with a bandwidth of 10 kHz. Assume that all resistance levels are 1 ohm.
(a) (5 points) Compute the signal to noise ratio at the output of the low-pass filter.
(b) (5 points) A synchronous detector is used to demodulate the above signal and any DC term is further blocked. Compute the signal to noise ratio at the out of the detector.

Problem 6 The signal $f(t) = 20\cos 100\pi t$ is sampled at the rate of $f_s = 75$ samples per second.
(a) (5 points) Determine the spectrum of the resulting sampled signal.
(b) (5 points) Determine the time domain representation of the output of the reconstruction filter with bandwidth $f_s/2$.
(c) (2 points) What is the Nyquist rate for $f(t)$?