[10] 1. Consider a single-tone modulating signal $f(t) = a\cos(\omega_m t)$ and a carrier $c(t) = A_c \cos(\omega_c t)$, where $\omega_c \gg \omega_m$. Write a mathematical expression for each of the following modulation schemes: (a) DSB-SC, (b) SSB-SC, (c) VSB-SC, (d) FM, and (e) PM.

[10] 2. Figure 1 shows a wideband FM generator. The narrowband FM signal has a modulation index (corresponding to the maximum frequency component in $f(t)$) of 0.10 radians in order to keep distortion under control.

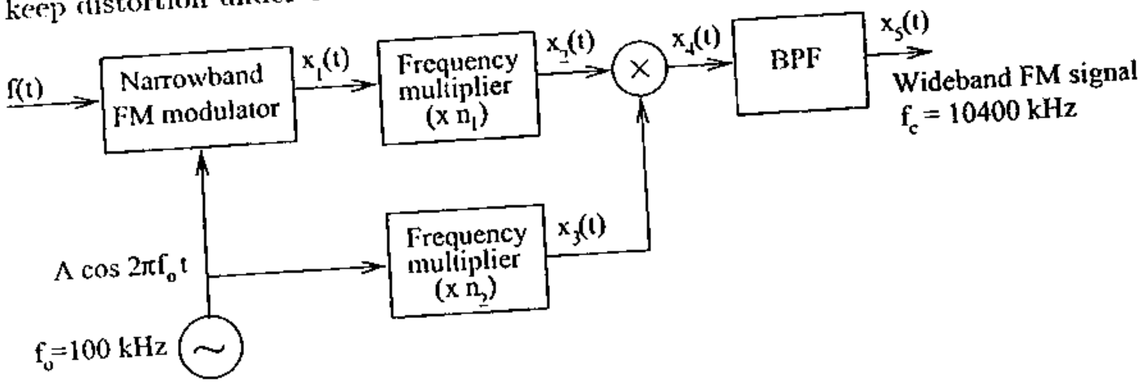

Figure 1: The wideband FM signal generator.

[6] (a) If the message signal $f(t)$ has a bandwidth of 15 kHz and the output frequency from the oscillator is 100 kHz, determine the frequency multiplication parameters n_1 and n_2 in order to generate an FM signal at a carrier frequency of $f_c = 10400$ kHz and a maximum frequency deviation of 75 kHz.

[4] (b) If the input signal $f(t)$ is a single-tone sinusoidal signal with frequency 15 kHz, determine the modulation index β and the bandwidth (based on the Carson's rule) of the wideband FM signal.

[10] 3. Figure 2 shows a block digram of the VSB-SC modulator, where $f(t)$ is the baseband message signal with bandwidth B Hz and Fourier transform $F(\omega)$. The VSB-SC signal can be represented by

$$\phi(t) = \frac{1}{2}A_c[f(t)\cos\omega_c t - f_q(t)\sin\omega_c t]$$

where $f_q(t)$ is a quadrature component at baseband, depending on $f(t)$ and $H(\omega)$.

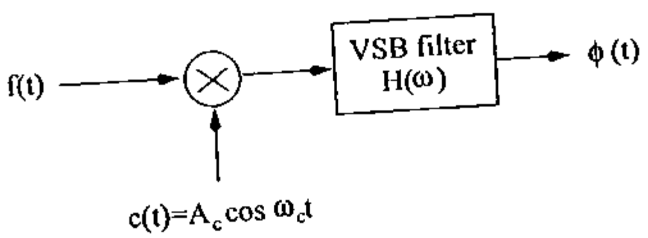

Figure 2: The VSB signal generator.

[2] (a) From Figure 2, derive $\Phi(\omega)$ of the VSB-SC signal in terms of $F(\omega)$ and $H(\omega)$.

[3] (b) In order to use an envelope detector for demodulation, the following VSB-LC signal is transmitted

$$\phi_{LC}(t) = \alpha A_c \cos\omega_c t + \frac{1}{2}A_c[f(t)\cos\omega_c t - f_q(t)\sin\omega_c t]$$

where α (> 0) is a constant. Derive an expression for the envelope of $\phi_{LC}(t)$ and specify the condition (for the α value) under which the distortion in demodulation is negligible.

[5] (c) The transfer function of the VSB filter is shown in Figure 3, where

$$H(\omega) = U(\omega - \omega_c) - H_\beta(\omega - \omega_c), \qquad \text{for } \omega > 0.$$

$U(\omega)$ is the step function and

$$H_\beta(-\omega) = -H_\beta(\omega) \qquad \text{and} \qquad H_\beta(\omega) = 0, \quad |\omega| > \beta, \beta \in (0,\ 2\pi B).$$

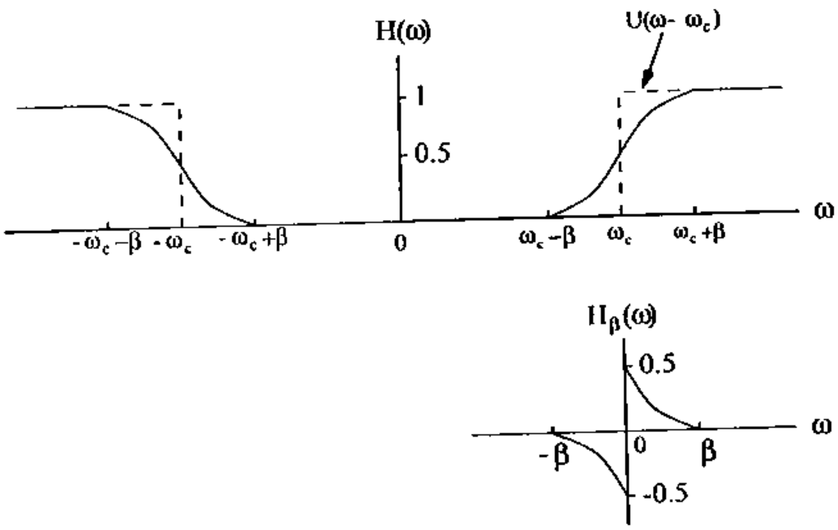

Figure 3: The transfer function of the VSB filter.

Show that

$$f_q(t) = \hat{f}(t) - \frac{1}{\pi}\int_0^{\beta}\{\mathrm{Im}[F(\omega)\exp(j\omega t)]\}H_\beta(\omega)d\omega$$

where $\hat{f}(t)$ is the Hilbert transform of $f(t)$ and $\mathrm{Im}(a + jb) = b$ for real-valued a and b.

[5] 4. (a) Consider a WSS random process $X(t)$ with

$$E[X(t)X(t+\tau)] = 1.$$

Find the power spectral density (psd) and DC power of $X(t)$.

[5] (b) Consider the filter consisting of a delay line and a summing device, as shown in Figure 4. Given that the psd of the input WSS random signal $X(t)$ is $S_X(\omega)$, derive the psd $S_Y(\omega)$ of the output signal $Y(t)$.

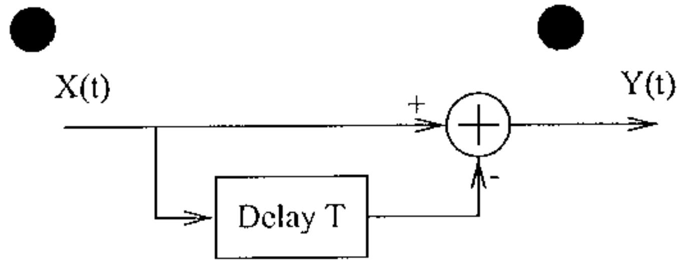

Figure 4: The filter structure in the psd calculation.

[10] 5. Figure 5 shows the noisy receiver model in AM reception. The channel introduces white noise with two-sided power spectral density $\eta/2$ W/Hz. The BPF is to let the desired signal go through without distortion and to suppress the input noise as much as possible. (S_o/N_o) and (S_i/N_i) are the signal-to-noise ratio at the demodulator output and input respectively.

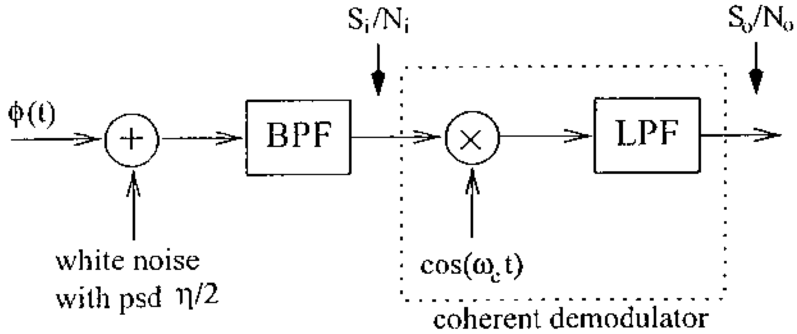

Figure 5: The noisy AM receiver model.

[5] (a) Derive the ratio $[(S_o/N_o)/(S_i/N_i)]$ for DSB-SC.

[3] (b) Derive the ratio $[(S_o/N_o)/(S_i/N_i)]$ for SSB-SC.

[2] (c) Compare the noise performance of DSB-SC with that of SSB-SC.

[10] 6. The lowpass signal $x(t)$ with a bandwidth of W Hz is sampled with a sampling interval of T_s and the signal

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)p(t - nT_s)$$

is generated, where $p(t)$ is an arbitrary shaped pulse (not necessarily time-limited to the interval $[0, T_s]$).

[6] (a) Find the Fourier transform of $x_p(t)$.

[2] (b) Find the conditions for perfect reconstruction of $x(t)$ from $x_p(t)$.

[2] (c) Determine the required reconstruction filter.