1. Consider a single-tone modulating signal $f(t) = A_c \cos(\omega_m t)$ and a carrier $c(t) = A_c \cos(\omega_c t)$, where $\omega_c \gg \omega_m$. Write a mathematical expression for each of the following modulation schemes: (a) DSB-SC, (b) SSB-SC, (c) VSB-SC, (d) FM, and (e) PM.

2. Figure 1 shows a wideband FM generator. The narrowband FM signal has a modulation index (corresponding to the maximum frequency component in $f(t)$) of 0.10 radians in order to keep distortion under control.

![Wideband FM signal generator diagram](image)

Figure 1: The wideband FM signal generator.

(a) If the message signal $f(t)$ has a bandwidth of 15 kHz and the output frequency from the oscillator is 100 kHz, determine the frequency multiplication parameters $n_1$ and $n_2$ in order to generate an FM signal at a carrier frequency of $f_c = 10400$ kHz and a maximum frequency deviation of 75 kHz.

(b) If the input signal $f(t)$ is a single-tone sinusoidal signal with frequency 15 kHz, determine the modulation index $\beta$ and the bandwidth (based on the Carson's rule) of the wideband FM signal.

3. Figure 2 shows a block diagram of the VSB-SC modulator, where $f(t)$ is the baseband message signal with bandwidth $B$ Hz and Fourier transform $F(\omega)$. The VSB-SC signal can be represented by

$$\phi(t) = \frac{1}{2} A_c \left[ f(t) \cos \omega_c t - f_q(t) \sin \omega_c t \right]$$

where $f_q(t)$ is a quadrature component at baseband, depending on $f(t)$ and $H(\omega)$.

![VSB signal generator diagram](image)

Figure 2: The VSB signal generator.

(a) From Figure 2, derive $\Phi(\omega)$ of the VSB-SC signal in terms of $F(\omega)$ and $H(\omega)$. 

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(b) In order to use an envelope detector for demodulation, the following VSB-LC signal is transmitted

\[ \phi_{LC}(t) = \alpha A_c \cos \omega_c t + \frac{1}{2} A_c [f(t) \cos \omega_c t - f_{q}(t) \sin \omega_c t] \]

where \( \alpha > 0 \) is a constant. Derive an expression for the envelope of \( \phi_{LC}(t) \) and specify the condition (for the \( \alpha \) value) under which the distortion in demodulation is negligible.

(c) The transfer function of the VSB filter is shown in Figure 3, where

\[ H(\omega) = U(\omega - \omega_c) - H_\beta(\omega - \omega_c), \quad \text{for } \omega > 0. \]

\( U(\omega) \) is the step function and

\[ H_\beta(-\omega) = -H_\beta(\omega) \quad \text{and} \quad H_\beta(\omega) = 0, \quad |\omega| > \beta, \beta \in (0, 2\pi B). \]

![Figure 3: The transfer function of the VSB filter.](image)

Show that

\[ f_{q}(t) = \hat{f}(t) - \frac{1}{\pi} \int_0^\beta \{ \text{Im}[F(\omega) \exp(j\omega t)] \} H_\beta(\omega) d\omega \]

where \( \hat{f}(t) \) is the Hilbert transform of \( f(t) \) and \( \text{Im}(a + jb) = b \) for real-valued \( a \) and \( b \).

4. (a) Consider a WSS random process \( X(t) \) with

\[ E[X(t)X(t + \tau)] = 1. \]

Find the power spectral density (psd) and DC power of \( X(t) \).

(b) Consider the filter consisting of a delay line and a summing device, as shown in Figure 4. Given that the psd of the input WSS random signal \( X(t) \) is \( S_X(\omega) \), derive the psd \( S_Y(\omega) \) of the output signal \( Y(t) \).
5. Figure 5 shows the noisy receiver model in AM reception. The channel introduces white noise with two-sided power spectral density $\eta/2$ W/Hz. The BPF is to let the desired signal go through without distortion and to suppress the input noise as much as possible. $(S_o/N_o)$ and $(S_i/N_i)$ are the signal-to-noise ratio at the demodulator output and input respectively.

[5] (a) Derive the ratio $[(S_o/N_o)/(S_i/N_i)]$ for DSB-SC.
[3] (b) Derive the ratio $[(S_o/N_o)/(S_i/N_i)]$ for SSB-SC.
[2] (c) Compare the noise performance of DSB-SC with that of SSB-SC.

6. The lowpass signal $x(t)$ with a bandwidth of $W$ Hz is sampled with a sampling interval of $T_s$, and the signal

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)p(t - nT_s)$$

is generated, where $p(t)$ is an arbitrary shaped pulse (not necessarily time-limited to the interval $[0, T_s]$).

[6] (a) Find the Fourier transform of $x_p(t)$.
[2] (b) Find the conditions for perfect reconstruction of $x(t)$ from $x_p(t)$.
[2] (c) Determine the required reconstruction filter.