Problem 5.1.3

Figure P5.2 shows a PCM wave in which the amplitude levels of +1 volt and -1 volts are used to represent binary symbols 1 and 0 respectively. The codeword used consists of three bits. Find the sampled version of an analog signal from which this PCM is derived.

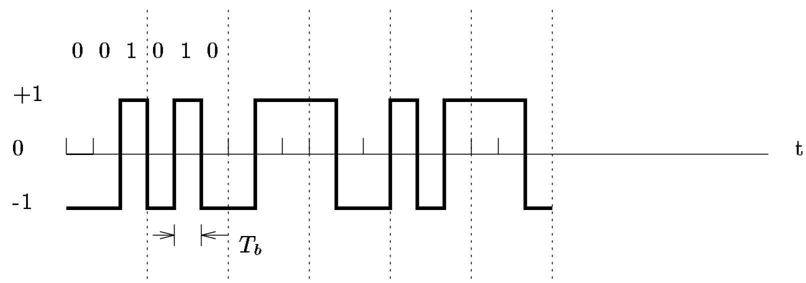

Figure 1: Problem P5.2

Solution:

The sample analog signal has the following waveform.

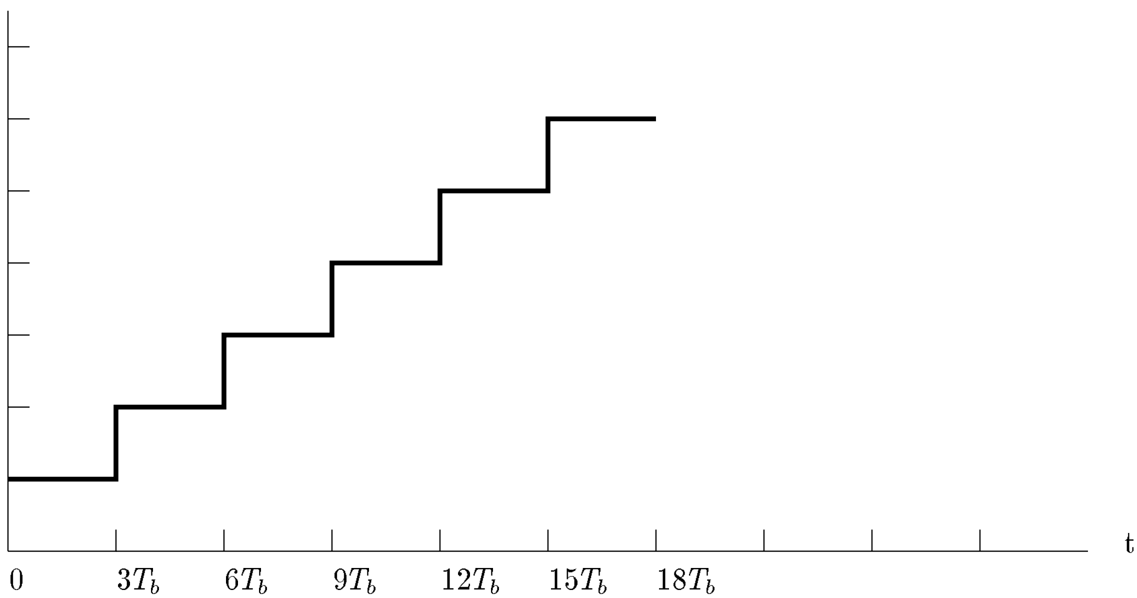

Figure 2: Problem 5.1.3

Problem 5.4.4

Show that the use of A-law companding provides a ratio of maximum step size to minimum step size equal to the parameter A.

Solution:

For the A-law companding we have,

$$\frac{c(|x|)}{x_{xmax}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\ln A} & 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\ \\ \dfrac{1+\ln(A|x|/x_{max})}{1+\ln A} & \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$

For small input x, the A-law is characterized by its derivatives:

$$\frac{dc(x)}{dx}|_{x=0} = \frac{A}{1+\ln A}$$

For large input, on the other hand, it is characterized by

$$\frac{dc(x)}{dx}|_{x=x_{\max}} = \frac{1}{1+\ln A}$$

Accordingly,

$$\begin{aligned} \frac{\text{Maximum step size}}{\text{Minimum step size}} &= \frac{\frac{dc(x)}{dx}|_{x=0}}{\frac{dc(x)}{dx}|_{x=x_{\max}}} \\ &= A \end{aligned} \tag{1}$$

Problem 5.5.2

(a) Consider a DPCM system whose transmitter uses a first order predictor optimized in the minimum mean squared sense. Calculate the prediction gain of the system for the following values of the correlation coefficient for the message signal:

(i)

$$\rho_1 = \frac{R_x(1)}{R_x(0)} = 0.852$$

(ii)

$$\rho_2 = \frac{R_x(1)}{R_x(0)} = 0.950$$

The correlation coefficient ρ_1 equals the autocorrelation function of the message signal for the delay T, normalized with respect to the mean square value of the signal.

(b) Suppose that the predictor is made suboptimal by setting the coefficient $h_1 = 1$. Calculate the resulting values of the prediction gain for the two values of ρ_1 specified in part (a). What is the minimum value of ρ_1 for which the suboptimum system produces a prediction gain greater than one?

Solution:

(a) The prediction gain is

$$G_P = \frac{\sigma_X^2}{\sigma_E^2}$$

As shown in the class, the prediction error variance σ_E^2 is related to the variance of the input signal, σ_X^2, as

$$\sigma_E^2 = \sigma_X^2(1 - \rho_1^2)$$

(i) $\rho_1 = 0.825$

$$G_P = \frac{1}{1 - (0.825)^2} = 3.13 = 5 \text{ dB}$$

(ii) $\rho_1 = 0.950$

$$G_P = \frac{1}{1 - (0.950)^2} = 10.26 = 10.1 \text{ dB}$$

(b) The prediction error is

$$e(nT) = x(nT) - x(nT - T) \tag{1}$$

Correspondingly, the prediction error variance is

$$\sigma_E^2 = E[e^2(nT)] \tag{2}$$

Substituting (1) in (2), and simplifying, we get

$$\sigma_E^2 = 2\sigma_X^2(1 - \rho_1)$$

The prediction gain is therefore

$$G_P = \frac{1}{2(1 - \rho_1)} \tag{3}$$

(i) $\rho_1 = 0.825$

$$\begin{aligned} G_P &= \frac{1}{2(1 - 0.825)} \\ &= 2.8571 \end{aligned}$$

(ii) $\rho_1 = 0.950$

$$G_P = \frac{1}{2(1-0.950)} = 10$$

The minimum value for which the prediction gain in (3) exceeds unity is when the correlation coefficient ρ_1 is less than 0.5.

Problem 5.6.1

The ramp signal $x(t) = a(t)$ is applied to a delta modulator that operates with a sampling period T_s, and step size $\Delta = 2\delta$.

(a) Show slope-overload distortion occurs is $\delta < aT_s$.

(b) Sketch the modulator output of the following three values of the step size:

(i) $\delta = 0.75aT_s$ (ii) $\delta = aT_s$ (iii) $\delta = 1.25aT_s$

Solution:

(a) In a period T_s, the input signal rises by aT_s, i.e., $dx(t)/dt = aT_s$, whereas the output of the delta modulator rises by an amount equal to δ. Hence, slope overload distortion occurs if

$$\delta < aT_s$$

(b) For each of the three values of δ, the modulator output is as shown in Fig. (3).

This figure illustrates that slope overload distortion occurs if $\delta > aT_s$.

Problem 5.6.5

Consider an adaptive delta modulator, for which the input signal $x(nT_s)$ is given by

$$x(nT_s) = \begin{cases} -0.5 & n < 5 \\ 19.5 & 5 \leq n \leq 15 \\ -0.5 & 15 < n \end{cases}$$

The adaptation algorithm for the modulator is defined by Eq. 5.90 (Haykin) with the constant K equal to 2. The maximum and minimum permissible values of the step size are as follows:

$$\delta_{max} = 8$$

$$\delta_{min} = 1$$

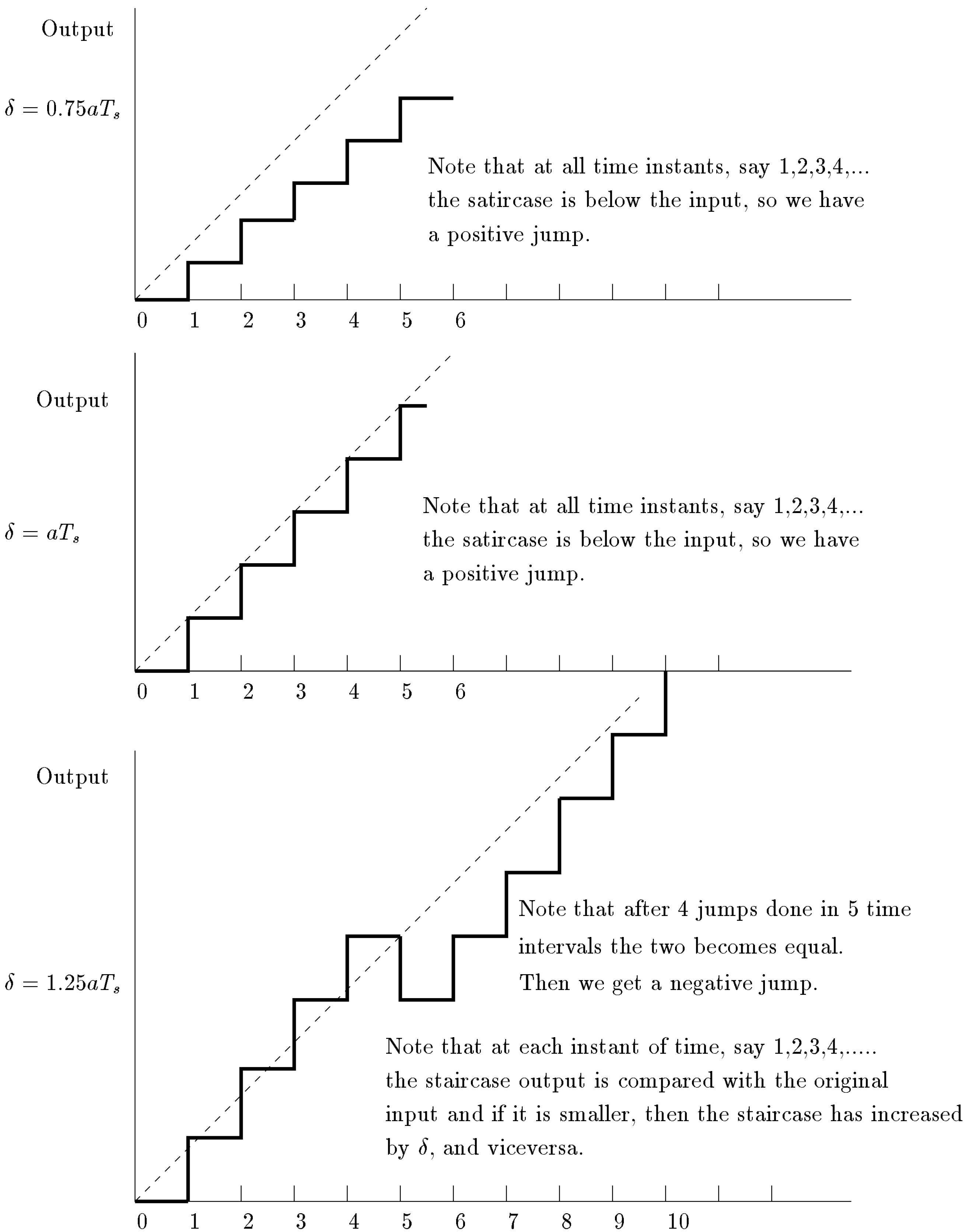

Figure 3: Problem 5.6.1

The initial conditions are given by

$$\delta_{start} = 1$$

$$u(0) = 1$$

$$b(0) = 0 \implies I(0) = -1$$

Plot the staircase approximation $u(nT_s)$ and the binary output for $0 \le n \le 25$.

Solution:

Let the sampling period $T_s = 1$. Then

$$e(n) = x(n) - u(n-1)$$

The binary output, $b(n)$, is equal to zero if $e(n) < 0$ and is equal to one, otherwise.

$$I(n) = \text{sgn}[e(n)]$$

This means that $I(n)$ is one if the binary output is one and is -1 if the binary output is zero.

$$g(n) = \begin{cases} K, & \text{if} \quad b(n) = b(n-1) \\ K^{-1} & \text{if} \quad b(n) \neq b(n-1) \end{cases}$$

$$\delta(n) = \begin{cases} \delta_{\min} & \text{if } \; g(n)\delta(n-1) \le \delta_{\min} \\ \delta_{\max} & \text{if } \; g(n)\delta(n-1) \ge \delta_{\max} \\ g(n)\delta(n-1), & \text{Otherwise} \end{cases}$$

For the reconstruction, we have,

$$u(n) = u(n-1) + \delta(n)I(n)$$

resulting in,

$$u(n) = u(0) + \sum_{i=0}^{n} \delta(i)I(i)$$

Initial conditions:

$$b(0) = 0 \implies I(0) = -1$$

$$u(0) = +1$$

$$\delta(0) = 1$$

Using these initial conditions and inputs in the defining relations for the adaptive delta modulation, we get the approximation $u(n)$, the adaptive variation of $\delta(n)$, and the resulting binary output $b(n)$ as shown in Fig. (4).

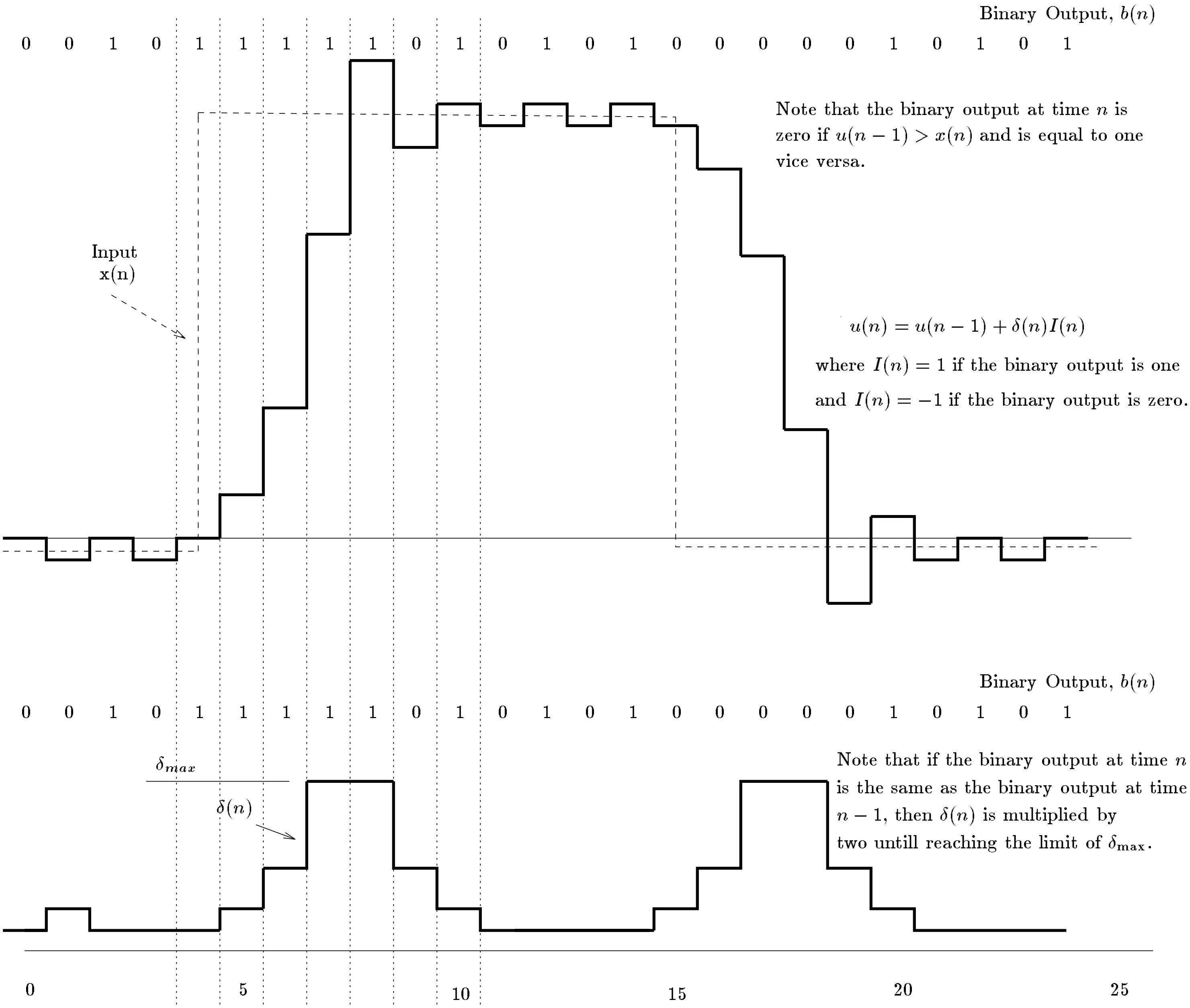

Figure 4: Problem 5.6.5