

E&CE 318: Introduction to Communication Systems
Instructor: E. Yang
Midterm Exam, Winter 2000, Feb. 14, 2000, 4:30–6:00 p.m.
University of Waterloo
Dept. of E&CE

Special Instructions

- Time allowed: 90 minutes.
- Closed book & notes. No crib sheet is allowed.
- Answer all questions.
- Justify your answers.
- Cheating will not be tolerated. Any instances of cheating will be handled according to university rules.

Problem 1 A modulating signal $f(t) = 12 \cos 20\pi t + 4 \cos 30\pi t$ is applied to a double-sideband suppressed-carrier modulator operating at a carrier frequency of 200 Hz.

- (a) (10 points) Sketch the spectral density of the resulting DSB-SC modulated signal, and identify the upper and lower sidebands.
- (b) (10 points) Compute the power spectral densities of $f(t)$ and the modulated signal.
- (c) (5 points) Compute the average powers of $f(t)$ and the modulated signal.
- (d) (15 points) Repeat Parts (a) and (b) if the modulating signal is $f^2(t)$ rather than $f(t)$.

Problem 2 Consider the chopper demodulator of DSB-SC signals shown in Fig. 1. The balanced diode bridge is driven by a locally generated carrier $\cos \omega_c t$.

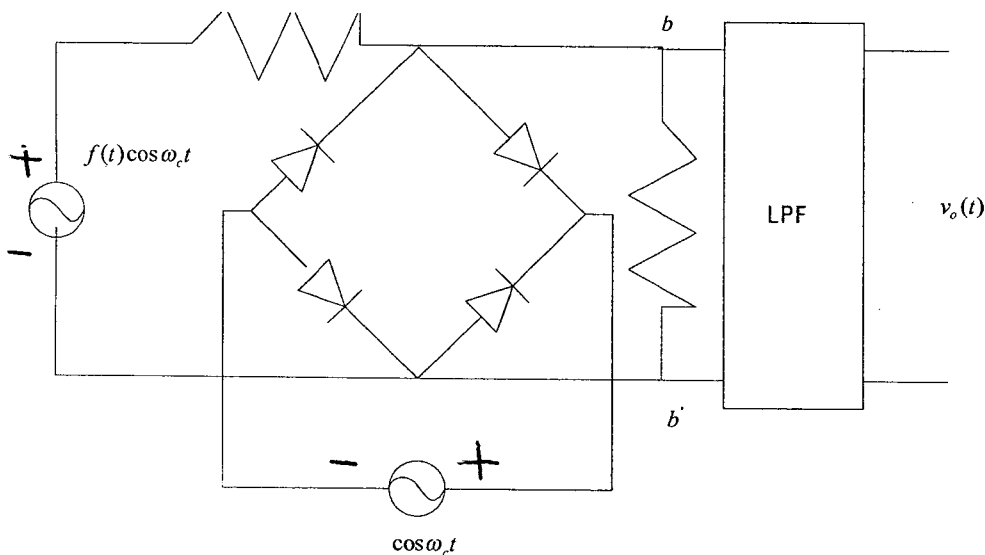


Fig 1.

- (a) (10 points) Determine and sketch the signal $p(t)$ so that the voltage across the terminals b and b' in response to the input $\phi(t) = f(t) \cos \omega_c t$ can be represented as $\phi(t)p(t)$.
- (b) (10 points) Determine the output $v_o(t)$ of the lower-pass filter.
- (c) (10 points) Suppose now that there are frequency and phase errors between the carrier at the transmitter and the locally generated carrier at the receiver. That is, suppose that the balanced diode bridge is now driven by $\cos((\omega_c + \Delta\omega)t + \theta_0)$. Repeat Parts (a) and (b).

Problem 3 Consider DSB-LC modulation. Let $f(t) = 40 \cos 40\pi t$ be a modulating signal.

- (a) (10 points) Sketch the DSB-LC modulated signal corresponding to the modulation index $m = 0.5$, where $\omega_c \gg 40\pi$.
- (b) (5 points) Under what conditions is the information regarding the phase and frequency of the carrier at the transmitter built in automatically to the modulated signal? Explain why.
- (c) (5 points) Sketch a detector which does not estimate the phase and frequency of the incoming signal, but can use the built-in information regarding the phase and frequency implicitly to detect the information signal $f(t)$ from the incoming modulated signal.
- (d) (10 points) Determine the power efficiency μ in terms of the modulation index m . What is the minimum percentage of the total power that must be wasted in order to use a simple, noncoherent detector at the receiver end.

A.1 TRIGONOMETRIC IDENTITIES

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin A \sin B &= \frac{1}{2}[\cos(A - B) - \cos(A + B)] \\ \cos A \cos B &= \frac{1}{2}[\cos(A + B) + \cos(A - B)] \\ \sin A \cos B &= \frac{1}{2}[\sin(A + B) + \sin(A - B)] \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= 2 \cos^2 A - 1 = 1 - 2 \sin^2 A = \cos^2 A - \sin^2 A \\ \sin^2 A &= \frac{1}{2}(1 - \cos 2A) \\ \cos^2 A &= \frac{1}{2}(1 + \cos 2A) \\ \sin A &= \frac{1}{2j}(e^{jA} - e^{-jA}) \\ \cos A &= \frac{1}{2}(e^{jA} + e^{-jA}) \\ e^{\pm jA} &= \cos A \pm j \sin A \end{aligned}$$

Selected Fourier Transform Pairs

$f(t)$	$F(\omega) = \mathcal{F}\{f(t)\}$	$f(t)$	$F(\omega) = \mathcal{F}\{f(t)\}$
1. $e^{-at}u(t)$	$1/(a + j\omega)$	11. $\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
2. $te^{-at}u(t)$	$1/(a + j\omega)^2$	12. $\sin \omega_0 t$	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
3. $e^{-a t }$	$2a/(a^2 + \omega^2)$	13. $\text{rect}(t/\tau)$	$\tau \text{Sa}(\omega\tau/2)$
4. $e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi} e^{-\sigma^2\omega^2/2}$	14. $\frac{W}{2\pi} \text{Sa}(Wt/2)$	$\text{rect}(\omega/W)$
5. $\text{sgn}(t)$	$2/(j\omega)$	15. $\frac{W}{\pi} \text{Sa}(Wt)$	$\text{rect}(\omega/2W)$
6. $j/(\pi t)$	$\text{sgn}(\omega)$	16. $\Lambda(t/\tau)$	$\tau[\text{Sa}(\omega\tau/2)]^2$
7. $u(t)$	$\pi\delta(\omega) + 1/(j\omega)$	17. $\frac{W}{2\pi} [\text{Sa}(Wt/2)]^2$	$\Lambda(\omega/W)$
8. $\delta(t)$	1	18. $\cos(\pi t/\tau) \text{rect}(t/\tau)$	$\frac{2\tau}{\pi} \frac{\cos(\omega\tau/2)}{1 - (\omega\tau/\pi)^2}$
9. 1	$2\pi\delta(\omega)$	19. $\frac{2W}{\pi^2} \frac{\cos(Wt)}{1 - (2Wt/\pi)^2}$	$\cos[\pi\omega/(2W)] \text{rect}[\omega/(2W)]$
10. $e^{\pm j\omega_0 t}$	$2\pi\delta(\omega \mp \omega_0)$	20. $\delta_T(t)$	$\omega_0\delta_{\omega_0}(\omega),$ where $\omega_0 = 2\pi/T$