University of Waterloo
Department of Electrical and Computer Engineering
Thursday, October 25, 2001
4:30pm to 6:00pm

E&CE-318 Midterm Examination

Instructor: W. Zhuang

Time allowed: 1.5 hours.
NO AIDS ALLOWED (Some mathematical formulas are given on page 4).
Attempt all the questions. **JUSTIFY ALL YOUR ANSWERS.**

The marking scheme is shown in the left margin and [30] constitutes full marks.

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[10] 1. Consider the single-tone sinusoidally modulated DSB-LC signal as shown in Figure 1, where the carrier frequency is ω_c and the modulating signal frequency is ω_m.

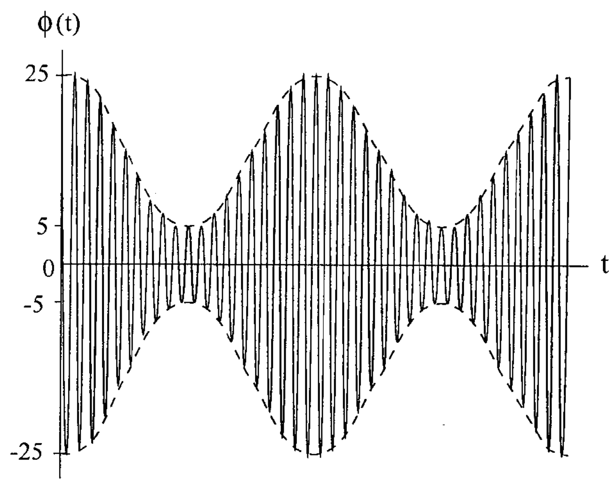

Figure 1

[2.5] (a) Determine the modulation index m.

[2.5] (b) Write an expression for the modulated signal $\phi(t)$.

[2.5] (c) Specify the upper sideband component $\phi_{USB}(t)$ and the lower sideband component $\phi_{LSB}(t)$.

[2.5] (d) Determine the modulation efficiency μ.

[10] 2. This question studies the carrier recovery for coherent demodulation of SSB-SC signals. Two single-tone pilot signals are transmitted for the carrier recovery, as illustrated in Figure 2(a) for transmission of the lower sideband. The two pilot frequencies ω_1 and ω_2 are given by

$$\omega_1 = \omega_c - B - \Delta\omega \qquad \text{and} \qquad \omega_2 = \omega_c + \Delta\omega$$

where ω_c is the carrier frequency and B is the message bandwidth in rad/s. The $\Delta\omega$ is chosen as to satisfy the relation

$$n = \frac{B}{\Delta\omega}$$

where n is an integer. Carrier recovery is accomplished by using the system shown in Figure 2(b). The outputs of the two narrowband filters centered at ω_1 and ω_2 are given by

$$v_1(t) = A_1 \cos(\omega_1 t + \phi_1) \qquad \text{and} \qquad v_2(t) = A_2 \cos(\omega_2 t + \phi_2)$$

The lowpass filter is designed to select the lower frequency component of $v_3(t)$. The *frequency divider* actually divides both frequency and initial phase (at $t = 0$) of the input signal by $n + 2$.

[2] (a) Describe how the system in Figure 2(b) works.

[5] (b) Derive the output signal $v_7(t)$.

[2] (c) Specify the bandwidths of the lowpass filter and the bandpass filter respectively.

[1] (d) Specify the relation between ϕ_1 and ϕ_2 for $v_7(t)$ to be proportional to the carrier $A_c \cos(\omega_c t)$.

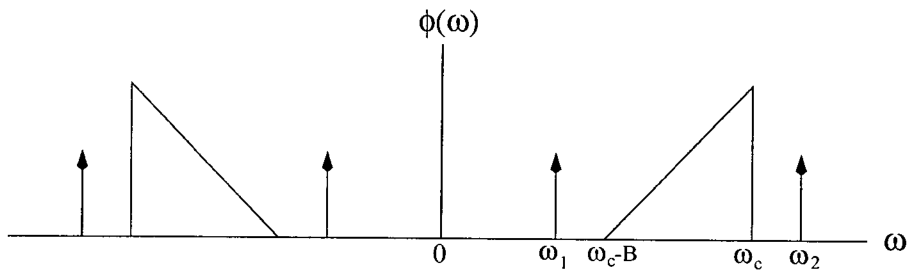

Figure 2(a)

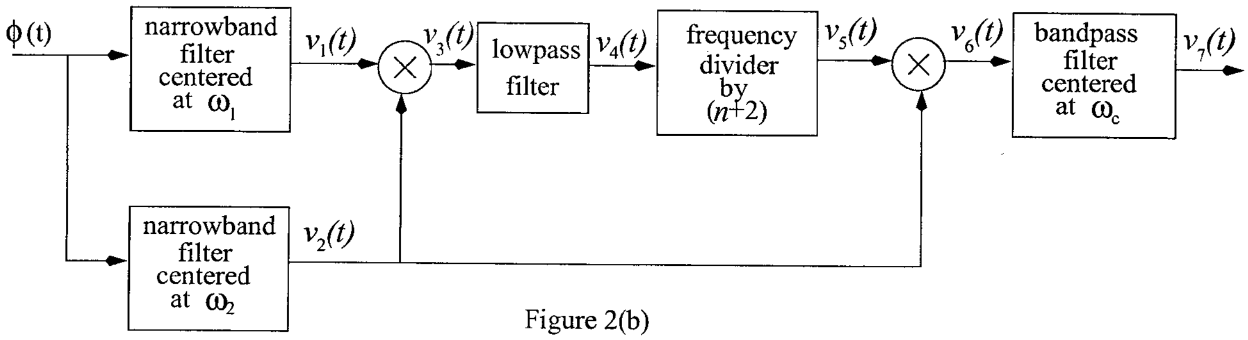

Figure 2(b)

[10] 3. Figure 3 shows a block digram of the VSB-SC modulator, where $f(t)$ is the baseband message signal with bandwidth B Hz and Fourier transform $F(\omega)$. The VSB-SC signal can be represented by

$$\phi(t) = \frac{1}{2}A_c[f(t)\cos\omega_c t - f_q(t)\sin\omega_c t]$$

where $f_q(t)$ is a quadrature component at baseband, depending on $f(t)$ and $H(\omega)$.

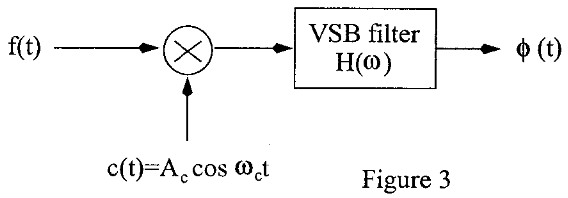

Figure 3

[3] (a) Derive $\Phi(\omega)$ of the VSB-SC signal in terms of $F(\omega)$ and $H(\omega)$.

[3] (b) In order to use an envelope detector for demodulation, the following VSB-LC signal is transmitted

$$\phi_{LC}(t) = \alpha A_c \cos\omega_c t + \frac{1}{2}A_c[f(t)\cos\omega_c t - f_q(t)\sin\omega_c t]$$

where α (> 0) is a constant. Derive an expression for the envelope of $\phi_{LC}(t)$ and specify the condition (for the α value) under which the distortion in demodulation is negligible.

[4] (c) The transfer function of the VSB filter is shown in Figure 4, where

$$H(\omega) = U(\omega - \omega_c) - H_\beta(\omega - \omega_c), \qquad \text{for } \omega > 0.$$

$U(\omega)$ is the step function and

$$H_\beta(-\omega) = -H_\beta(\omega) \qquad \text{and} \qquad H_\beta(\omega) = 0, \quad |\omega| > \beta$$

$\beta \in (0,\ 2\pi B)$. (i) Derive an expression for $f_q(t)$ of the VSB-SC signal $\phi(t)$, in terms of $H_\beta(\omega)$, $F(\omega)$ and/or $f(t)$; (ii) Show that $f_q(t)$ is a real-valued function.

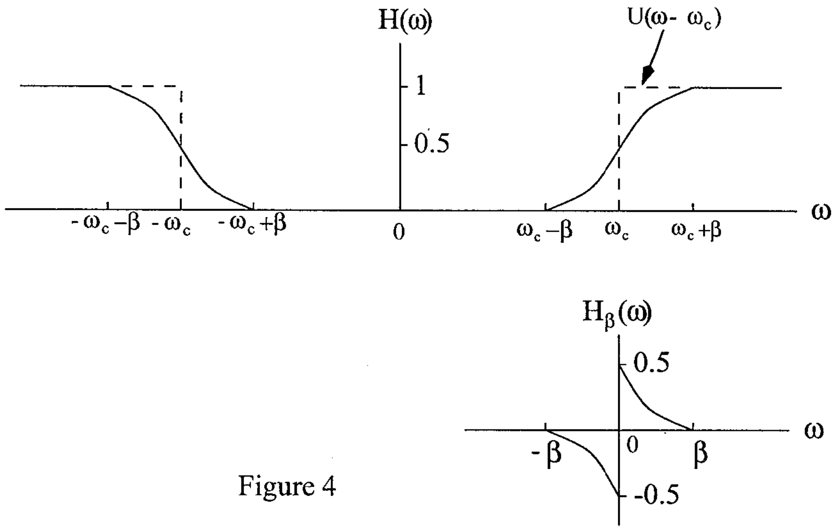

Figure 4

Trigonometric functions

$$\begin{aligned}
\sin\alpha\sin\beta &= -\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)] \\
\cos\alpha\cos\beta &= \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)] \\
\sin\alpha\cos\beta &= \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]
\end{aligned}$$

Fourier transform pairs

(Note: $*$ denotes conjugate and $\star$ denotes convolution.)

time domain	frequency domain
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$\cos(\omega_c t)$	$\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]$
$\sin(\omega_c t)$	$-j\pi[\delta(\omega-\omega_c)-\delta(\omega+\omega_c)]$
$f(t-t_0)$	$\exp(-j\omega t_0)F(\omega)$
$\exp(j\omega_c t)f(t)$	$F(\omega-\omega_c)$
$f^*(t)$	$F^*(-\omega)$
$f_1(t)\star f_2(t)$	$F_1(\omega)F_2(\omega)$
$f_1(t)f_2(t)$	$[F_1(\omega)\star F_2(\omega)]/2\pi$