1. Consider the single-tone sinusoidally modulated DSB-SC signal as shown in Figure 1, where the carrier frequency is \( \omega_c \) and the modulating signal frequency is \( \omega_m \).

\[ \phi(t) \]

![Figure 1](image)

(a) Determine the modulation index \( m \).

(b) Write an expression for the modulated signal \( \phi(t) \).

(c) Specify the upper sideband component \( \phi_{USB}(t) \) and the lower sideband component \( \phi_{LSB}(t) \).

(d) Determine the modulation efficiency \( \mu \).
2. This question studies the carrier recovery for coherent demodulation of SSB-SC signals. Two single-tone pilot signals are transmitted for the carrier recovery, as illustrated in Figure 2(a) for transmission of the lower sideband. The two pilot frequencies $\omega_1$ and $\omega_2$ are given by

$$\omega_1 = \omega_c - B - \Delta \omega \quad \text{and} \quad \omega_2 = \omega_c + \Delta \omega$$

where $\omega_c$ is the carrier frequency and $B$ is the message bandwidth in rad/s. The $\Delta \omega$ is chosen as to satisfy the relation

$$n = \frac{B}{\Delta \omega}$$

where $n$ is an integer. Carrier recovery is accomplished by using the system shown in Figure 2(b). The outputs of the two narrowband filters centered at $\omega_1$ and $\omega_2$ are given by

$$v_1(t) = A_1 \cos(\omega_1 t + \phi_1) \quad \text{and} \quad v_2(t) = A_2 \cos(\omega_2 t + \phi_2)$$

The lowpass filter is designed to select the lower frequency component of $v_3(t)$. The frequency divider actually divides both frequency and initial phase (at $t = 0$) of the input signal by $n + 2$.

(a) Describe how the system in Figure 2(b) works.
(b) Derive the output signal $v_\gamma(t)$.
(c) Specify the bandwidths of the lowpass filter and the bandpass filter respectively.
(d) Specify the relation between $\phi_1$ and $\phi_2$ for $v_\gamma(t)$ to be proportional to the carrier $A_c \cos(\omega_c t)$.

![Figure 2(a)]

![Figure 2(b)]
3. Figure 3 shows a block diagram of the VSB-SC modulator, where \( f(t) \) is the baseband message signal with bandwidth \( B \) Hz and Fourier transform \( F(\omega) \). The VSB-SC signal can be represented by

\[
\phi(t) = \frac{1}{2} A_c \left[ f(t) \cos \omega_c t - f_q(t) \sin \omega_c t \right]
\]

where \( f_q(t) \) is a quadrature component at baseband, depending on \( f(t) \) and \( H(\omega) \).

\[ f(t) \xrightarrow{\times} \text{VSB filter} \xrightarrow{H(\omega)} \phi(t) \]

\[ c(t)=A_c \cos \omega_c t \]

Figure 3

(a) Derive \( \Phi(\omega) \) of the VSB-SC signal in terms of \( F(\omega) \) and \( H(\omega) \).

(b) In order to use an envelope detector for demodulation, the following VSB-LC signal is transmitted

\[
\phi_{LC}(t) = \alpha A_c \cos \omega_c t + \frac{1}{2} A_c \left[ f(t) \cos \omega_c t - f_q(t) \sin \omega_c t \right]
\]

where \( \alpha \ (>0) \) is a constant. Derive an expression for the envelope of \( \phi_{LC}(t) \) and specify the condition (for the \( \alpha \) value) under which the distortion in demodulation is negligible.

(c) The transfer function of the VSB filter is shown in Figure 4, where

\[
H(\omega) = U(\omega - \omega_c) - H_\beta(\omega - \omega_c), \quad \text{for } \omega > 0.
\]

\( U(\omega) \) is the step function and

\[
H_\beta(-\omega) = -H_\beta(\omega) \quad \text{and} \quad H_\beta(\omega) = 0, \quad |\omega| > \beta
\]

\( \beta \in (0, 2\pi B) \). (i) Derive an expression for \( f_q(t) \) of the VSB-SC signal \( \phi(t) \), in terms of \( H_\beta(\omega) \), \( F(\omega) \) and/or \( f(t) \); (ii) Show that \( f_q(t) \) is a real-valued function.
Trigonometric functions

\[
\begin{align*}
\sin \alpha \sin \beta &= -\frac{1}{2} [\cos(\alpha + \beta) - \cos(\alpha - \beta)] \\
\cos \alpha \cos \beta &= \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] \\
\sin \alpha \cos \beta &= \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]
\end{align*}
\]

Fourier transform pairs
(Note: * denotes conjugate and \* denotes convolution.)

<table>
<thead>
<tr>
<th>time domain</th>
<th>frequency domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta(t) )</td>
<td>( \frac{1}{2\pi} \delta(\omega) )</td>
</tr>
<tr>
<td>1</td>
<td>( 2\pi \delta(\omega) )</td>
</tr>
<tr>
<td>( \cos(\omega_c t) )</td>
<td>( \pi [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] )</td>
</tr>
<tr>
<td>( \sin(\omega_c t) )</td>
<td>( -j\pi [\delta(\omega - \omega_c) - \delta(\omega + \omega_c)] )</td>
</tr>
<tr>
<td>( f(t - t_0) )</td>
<td>( \exp(-j\omega t_0)F(\omega) )</td>
</tr>
<tr>
<td>( \exp(j\omega_c t)f(t) )</td>
<td>( F(\omega - \omega_c) )</td>
</tr>
<tr>
<td>( f^*(t) )</td>
<td>( F^*(-\omega) )</td>
</tr>
<tr>
<td>( f_1(t) * f_2(t) )</td>
<td>( F_1(\omega)F_2(\omega) )</td>
</tr>
<tr>
<td>( f_1(t)f_2(t) )</td>
<td>( [F_1(\omega) * F_2(\omega)]/2\pi )</td>
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