E&CE 318: Introduction to Communication Systems
Instructor: E. Yang
Midterm Exam, Fall 2000, Oct. 25, 2000, 5:00–6:30 p.m.

University of Waterloo
Dept. of E&CE

Special Instructions

- Time allowed: 90 minutes.
- Closed book & notes. No crib sheet is allowed.
- Answer all questions.
- Justify your answers.
- Cheating will not be tolerated. Any instances of cheating will be handled according to university rules.

Problem 1 A modulating signal $f(t) = 4\cos 10\pi t + 8\cos 25\pi t$ is applied to a DSB-LC modulator operating at a carrier frequency of $250\sqrt{2}$ Hz. Denote the resulting modulated signal by $\phi_{AM}(t)$.

(a) (5 points) Is $f(t)$ a periodic signal? If no, explain why; if yes, determine the fundamental(i.e., the smallest) period.

(b) (15 points) Determine the time-autocorrelation function, power spectral density, and time-averaged power of $f(t)$.

(c) (10 points) Sketch the power spectral density of $\phi_{AM}(t)$. Is $\phi_{AM}(t)$ periodic? Explain why.

(d) (10 points) Suppose that in $\phi_{AM}(t)$, the carrier power is a doubling of the time-averaged power of $f(t)$. Determine the exact time-domain representation of $\phi_{AM}(t)$ and the power efficiency of the corresponding DSB-LC modulator.

(e) (10 points) Under the condition of Part (d), can $\phi_{AM}(t)$ be correctly demodulated by an envelope detector? If no, explain why; if yes, sketch the corresponding envelope detector and explain how it works.

(f) (10 points) Compute the power spectral density of $f^2(t)$. Is DSB-LC modulation suitable for the transmission of $f^2(t)$? Explain why.

Problem 2 Two signals $f_1(t)$ and $f_2(t)$ are transmitted simultaneously by using QAM(quadrature amplitude modulation). Let $\phi_{QAM}(t)$ be the resulting modulated signal.

(a) (5 points) Is $\phi_{QAM}(t)$ a band-pass signal? If yes, determine its baseband complex envelope; if no, explain why.

(b) (5 points) Sketch a synchronous detector for $\phi_{QAM}(t)$.

(c) (10 points) Analyze the effect of a phase error in the local carrier generated at the receiver on the output of your synchronous detector.

Problem 3 In the frequency domain, DSB-SC modulation simply translates the frequency spectrum of an information signal $f(t)$ by $\pm\omega_c$.

(a) (5 points) Explain why frequency translation can NOT be achieved by linear time-invariant systems.

(b) The spectrum of an information signal $f(t)$ is given in Figure 1. It is desirable to translate the spectrum of $f(t)$ to a high carrier frequency ω_c for transmission.

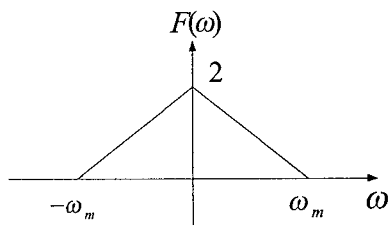

Figure 1.

(b1) (5 points) An engineer, Mr. "Careful", follows the standard approach by multiplying $f(t)$ by $\cos\omega_c t$. Sketch the spectrum of the modulated signal obtained by Mr. "Careful".

(b2) (10 points) Since it is not easy to generate a local carrier of very high frequency ω_c, another engineer, Mr. "Smarter", suggests the following approach:

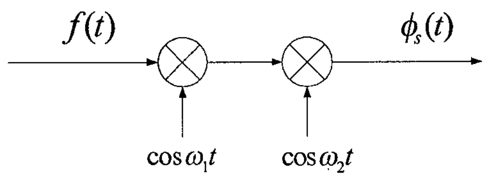

Figure 2.

where $\omega_1 > 0$, $\omega_2 > 0$, and $\omega_1 + \omega_2 = \omega_c$. Determine the spectrum of $\phi_s(t)$. Comment on the approach of Mr. "Smarter"?

A.1 TRIGONOMETRIC IDENTITIES

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$

$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

$\sin A = \frac{1}{2j}(e^{jA} - e^{-jA})$

$\cos A = \frac{1}{2}(e^{jA} + e^{-jA})$

$e^{\pm jA} = \cos A \pm j \sin A$

Selected Fourier Transform Pairs

	$f(t)$	$F(\omega) = \mathcal{F}\{f(t)\}$		$f(t)$	$F(\omega) = \mathcal{F}\{f(t)\}$
1.	$e^{-at}u(t)$	$1/(a + j\omega)$	11.	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
2.	$te^{-at}u(t)$	$1/(a + j\omega)^2$	12.	$\sin \omega_0 t$	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
3.	$e^{-a\|t\|}$	$2a/(a^2 + \omega^2)$	13.	$\text{rect}(t/\tau)$	$\tau\, \text{Sa}(\omega\tau/2)$
4.	$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}\, e^{-\sigma^2\omega^2/2}$	14.	$\frac{W}{2\pi}\text{Sa}(Wt/2)$	$\text{rect}(\omega/W)$
5.	$\text{sgn}(t)$	$2/(j\omega)$	15.	$\frac{W}{\pi}\text{Sa}(Wt)$	$\text{rect}(\omega/2W)$
6.	$j/(\pi t)$	$\text{sgn}(\omega)$	16.	$\Lambda(t/\tau)$	$\tau[\text{Sa}(\omega\tau/2)]^2$
7.	$u(t)$	$\pi\delta(\omega) + 1/(j\omega)$	17.	$\frac{W}{2\pi}[\text{Sa}(Wt/2)]^2$	$\Lambda(\omega/W)$
8.	$\delta(t)$	1	18.	$\cos(\pi t/\tau)\, \text{rect}(t/\tau)$	$\frac{2\tau}{\pi}\frac{\cos(\omega\tau/2)}{1 - (\omega\tau/\pi)^2}$
9.	1	$2\pi\delta(\omega)$	19.	$\frac{2W}{\pi^2}\frac{\cos(Wt)}{1 - (2Wt/\pi)^2}$	$\cos[\pi\omega/(2W)]\, \text{rect}[\omega/(2W)]$
10.	$e^{\pm j\omega_0 t}$	$2\pi\delta(\omega \mp \omega_0)$	20.	$\delta_T(t)$	$\omega_0\delta_{\omega_0}(\omega)$, where $\omega_0 = 2\pi/T$

Some Fourier Transforms Corresponding to Given Mathematical Operations

Operation	$f(t)$ $\leftrightarrow$	$F(\omega)$
Linearity (superposition)	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Complex conjugate	$f^*(t)$	$F^*(-\omega)$
Scaling	$f(\alpha t)$	$\frac{1}{\lvert\alpha\rvert} F\left(\frac{\omega}{\alpha}\right)$
Delay	$f(t - t_0)$	$e^{-j\omega t_0} F(\omega)$
Frequency translation	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Amplitude modulation	$f(t) \cos \omega_0 t$	$\frac{1}{2}F(\omega + \omega_0) + \frac{1}{2}F(\omega - \omega_0)$
Time convolution	$\int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau)\, d\tau$	$F_1(\omega) F_2(\omega)$
Frequency convolution	$f_1(t) f_2(t)$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) F_2(\omega - u)\, du$
Duality: time-frequency	$F(t)$	$2\pi f(-\omega)$
Symmetry: even-odd	$f_e(t)$	$F_e(\omega)$ [real]
	$f_o(t)$	$F_o(\omega)$ [imaginary]
Time differentiation	$\frac{d}{dt} f(t)$	$j\omega F(\omega)$
Time integration	$\int_{-\infty}^{t} f(\tau)\, d\tau$	$\frac{1}{j\omega} F(\omega) + \pi F(0)\, \delta(\omega)$, where $F(0) = \int_{-\infty}^{\infty} f(t)\, dt$