University of Waterloo
Department of Electrical and Computer Engineering

E&CE-318 – Communication Systems

Final Examination, Winter 1998

Instructor: A. K. Khandani
Time allowed: 3 hours.
NO AIDS ALLOWED except for one sheet (A4, double-sided) of formulas
Attempt all 6 questions.
The marking scheme is shown in the left margin and [120] constitutes full marks.

---

(20) Problem 1: Periodic signal $s(t)$, shown in Fig. 1, is used once to frequency modulate a carrier of frequency f_c and once to phase modulate the same carrier.

(5) **1.1.** Find a relation between k_p and k_f such that the peak phase deviation of the modulated signal in both cases are equal.

(10) **1.2.** In the case of PM, assuming $k_p = 1$, find an expression for the spectral density of the resulting modulated signal assuming that the carrier frequency is equal to 8π and the total power of the modulated signal is equal to $1/2$. What is the maximum instantaneous frequency of the resulting PM signal?

(5) **1.3.** Compute the fraction of the total power of the resulting PM signal in the frequency range $[-\pi, \pi]$.

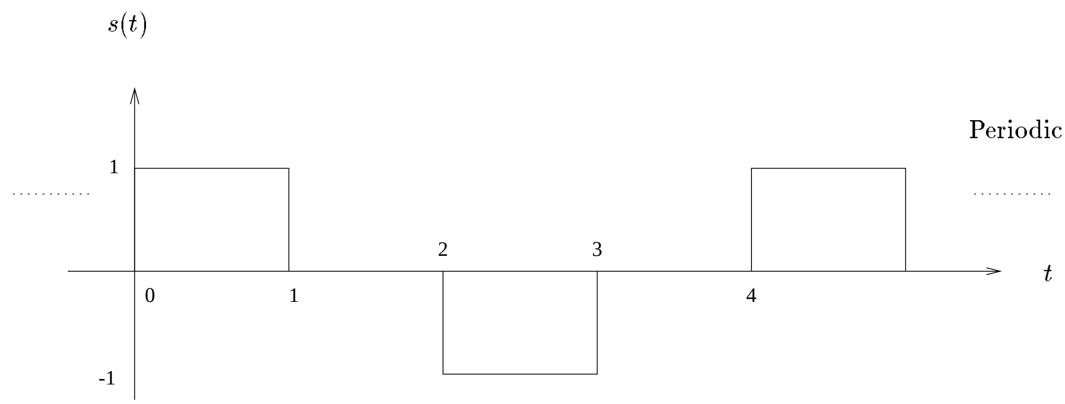


Figure 1: Signal $s(t)$.

(20) Problem 2:

(5) **2.1.** Compute the exponential Fourier series co-efficients of the signal:

$$g(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-n)$$

.

(5) **2.2.** The signal $g(t)$ is passed through the following linear system,

$$H(\omega) = \begin{cases} \dfrac{1}{1+(|\omega|/\pi)}, & |\omega| < \dfrac{3\pi}{2}, \\ 0 & \text{otherwise} \end{cases}$$

compute the signal, $f(t)$, at the output of the filter.

(10) **2.3.** The signal $f(t)$ computed in 2.2 is transmitted in a DSB-SC communication system operating in the presence of noise (this is the equivalent noise at the demodulator input) with the autocorrelation $R_N(\tau) = 10^{-4}\exp(-|\tau|)$ and a path loss (voltage attenuation) of 20dB. Calculate the minimum required bandwidth and the minimum required transmitted power to obtain a 40dB output S/N ratio.

(20) <u>Problem 3:</u> A given set of functions is,

$$\phi_0(t) = \frac{\sqrt{2}}{2}, \qquad \phi_1(t) = \sqrt{\frac{3}{2}}\,t.$$

(4) **3.1.** Show that these functions form an orthonormal set over the interval $[-1, 1]$.

(4) **3.2.** Represent the signal,

$$f(t) = \begin{cases} 1, & t < 0 \\ t+1, & t > 0 \end{cases}$$

over the interval $[-1, 1]$ using this set of functions.

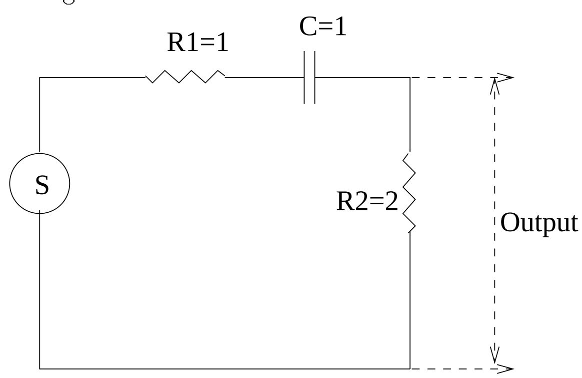
(3) **3.3.** Sketch the function $f(t)$ and its series representation on the same scale.

(3) **3.4.** Compute the energy (in the interval $[-1, 1]$) of each term of the resulting series.

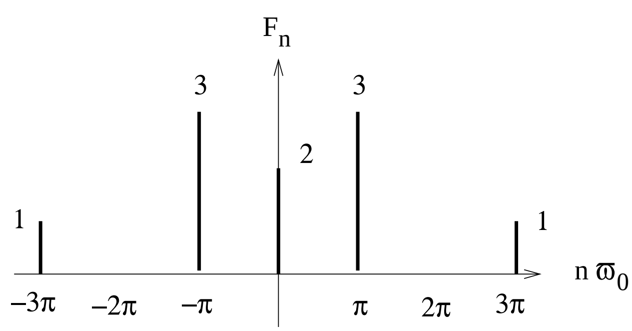
(3) **3.5.** Compute the energy (in the interval $[-1, 1]$) of the error of this representation.

(3) **3.6.** Find the expression(s) for the smallest number of extra basis required to make the representation exact.

(20) <u>Problem 4:</u> Consider the following circuit:



There is a white thermal noise of power (square voltage) spectral density $\eta/2 = 2$ associated with each resistor. The signal source (S) is a periodic (voltage) signal with the Fourier series coefficients shown in the following:



(2) **4.1.** What is the period of the signal?

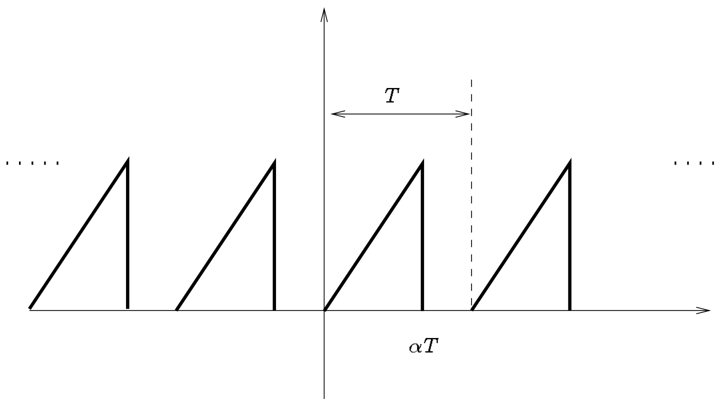
(2) **4.2.** What is the average value (DC value) of the signal?

(4) **4.3.** Compute the autocorrelation function of the signal.

(6) **4.4** Compute the power (square voltage) spectral density and the total mean square voltage of the signal source at the output?

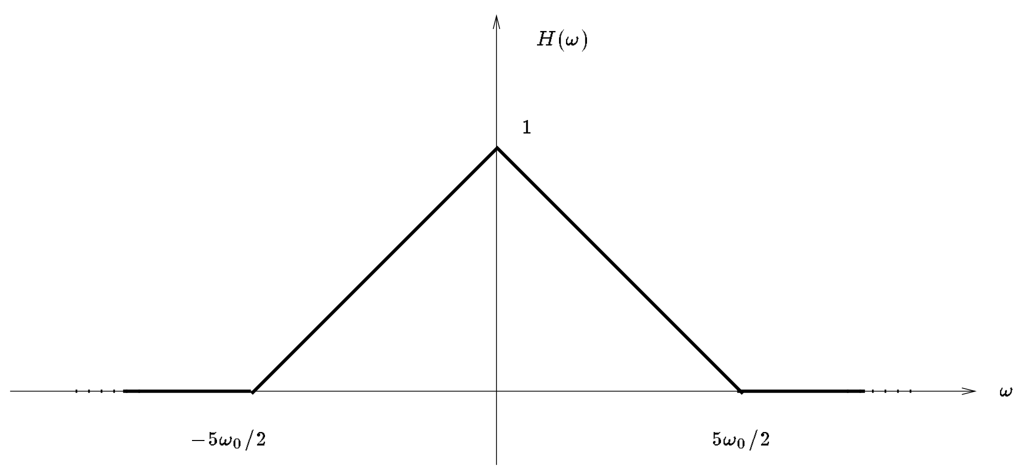
(6) **4.5** Compute the power (square voltage) spectral density and the total mean square voltage of the noise source at the output?

(20) **Problem 5:** Consider the periodic signal shown in the following figure:



(10) **5.1.** Find the autocorrelation function, the power spectral density and the total power of this signal.

(10) **5.2.** Assume that this signal is passed through a linear filter with the frequency response shown in the following figure (where $\omega_0 = 2\pi/T$). Compute the value of α in the periodic signal such that the power of the signal at the output of the filter is maximized.



(20) **Problem 6:** Consider the signal

$$v(t) = \sum_{k=-2}^{2} (|k|^2 - 1)[\cos(\omega_c t)\cos(k\omega_0 t) - \sin(\omega_c t)\sin(k\omega_0 t)]$$

(6) **6.1.** Show that this is an DSB-LC signal ($\omega_c > 2\omega_0$) and determine the modulating signal $m(t)$ and the carrier signal $c(t)$.

(4) **6.2.** Compute the efficiency of the modulation.

(8) **6.3.** Can the signal $m(t)$ be recovered using an envelope detector? If not, then what is he minimum value of the carrier needed to make the envelope detection possible?