Problem 1: When the input to a given audio amplifier is $(4\cos 800\pi t + \cos 1000\pi t)$ mV, the measured frequency components at 500Hz and 1000Hz are equal to 1V and 2V, respectively. Representing the amplifier output-input characteristic by,

$$e_o(t) = a_1 e_i(t) + a_2[e_i(t)]^2.$$

(10) Evaluate the numerical values of a_1, a_2 from the test data given.

Problem 2: A carrier waveform is frequency-modulated by the sum of two sinusoids:

$$\phi(t) = A\cos(\omega_c t + \sin\omega_m t + \cos\omega_m t),$$

(10) Find an expression for the resulting spectrum.

Problem 3: Consider the signal

$$v(t) = \sum_{i=0}^{N} (N+1-i)[\cos(\omega_c t)\cos(i\omega_0 t) - \sin(\omega_c t)\sin(i\omega_0 t)]$$

(4) **3.1.** Show that this is an SSB-LC signal ($\omega_c > N\omega_0$). Is it the upper or the lower sideband?

(2) **3.2.** Write an expression for the missing side-band.

(2) **3.3.** Write an expression for the total DSB signal.

(7) **3.4.** For the DSB signal (part 3.3.), what is he minimum value of the carrier needed to make the envelope detection possible?

(5) **3.5.** Compute the efficiency of the modulation for the signal obtained in part 3.4.

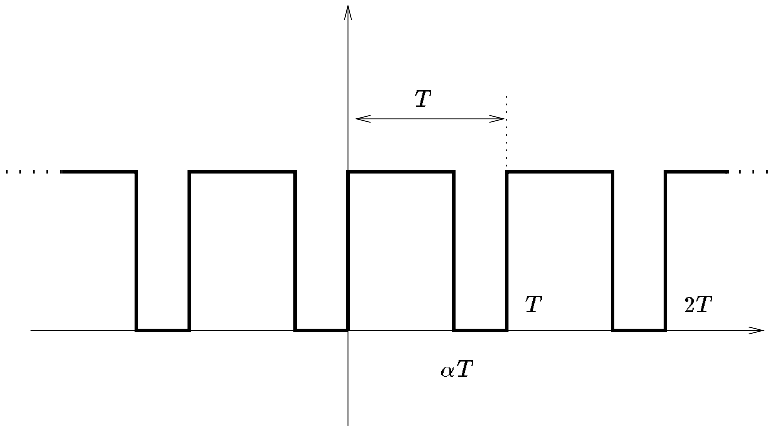
Problem 4: Consider PM modulation ($k_p = 1$) of the signal $x_{T_0}(t) = \sum_{k=-\infty}^{\infty} m(t - kT_0)$,

$$m(t) = \begin{cases} 1, & 0 \le t \le T_0/2 \\ 0, & T_0/2 \le t \le T_0 \end{cases}$$

(10) **4.1.** Determine the frequency spectrum of the PM signal assuming that the carrier frequency is equal to $4\omega_0$ (where $\omega_0 = 2\pi/T_0$) and the total power of the modulated signal is unity.

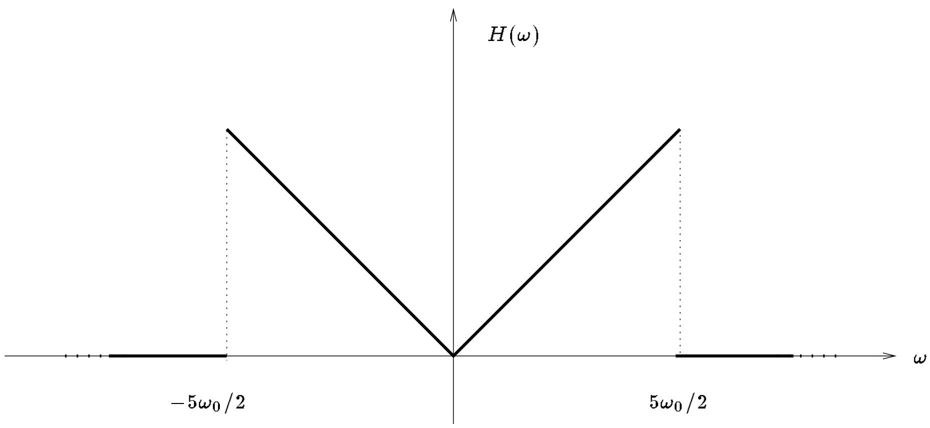
(10) **4.2.** Compute the fraction of the total power of the resulting PM signal in the frequency range $[-3\omega_0/2, 3\omega_0/2]$.

Problem 5: Consider the periodic signal shown in the following figure:



(8) **5.1.** Find the auto-correlation function, the power spectral density and the total power of this signal.

5.2. Assume that a white noise of power spectral density $\eta/2$ is added to this signal and the combination of signal plus noise is passed through a linear filter with the frequency response shown in the following figure (where $\omega_0 = 2\pi/T$).

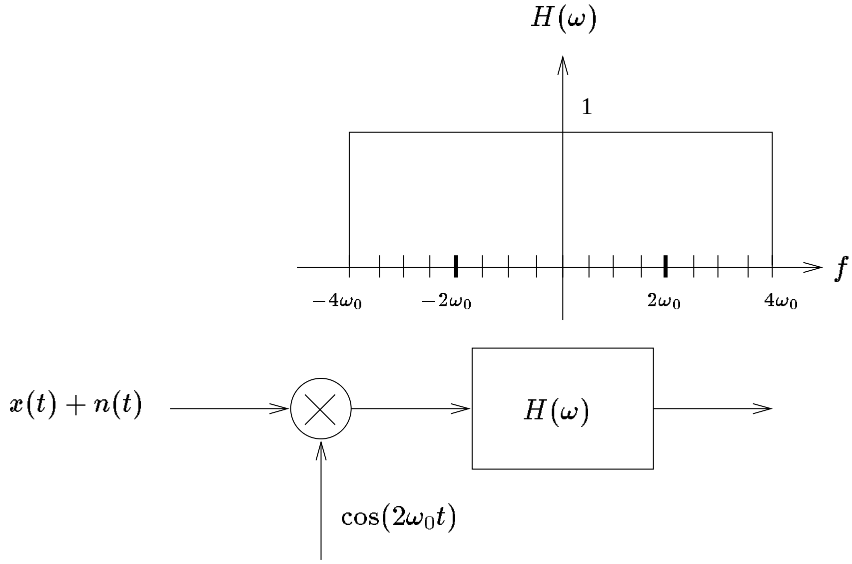


(4) **5.2.1.** Compute the power of the signal at the filter output.

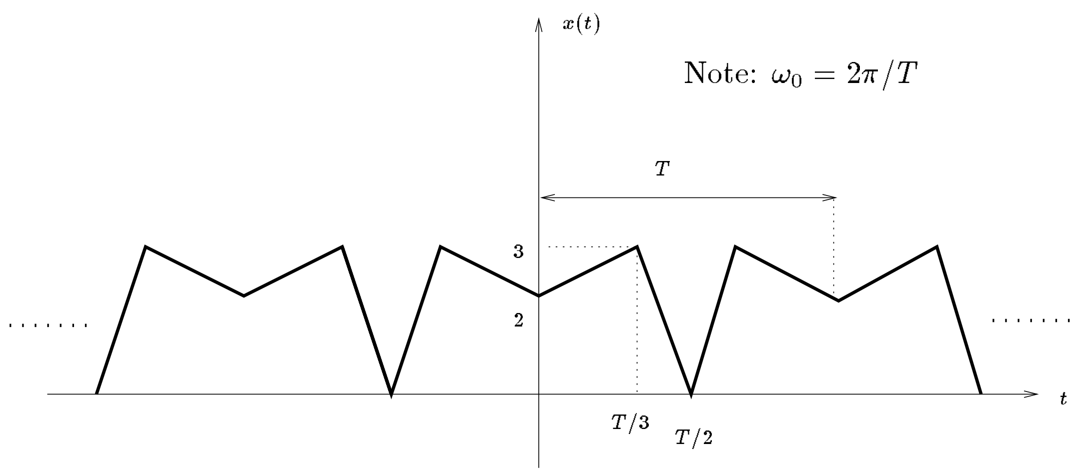
(4) **5.2.2.** Compute the power of the noise at the filter output.

(4) **5.2.3.** Compute the value of α in the periodic signal such that the ratio of the signal power to the noise power at the output of the filter is maximized.

Problem 6: Consider the following linear system:



The autocorrelation of the noise source, $n(t)$, is equal to $R_n(\tau) = \exp(-|\tau|)$. The signal source, $x(t)$, is a periodic (voltage) signal as shown in the following figure:



(7) **6.1.** Compute the Fourier transform of $x(t)$ (*Hint: compute the derivatives of* $x(t)$)

6.2. Compute the power (square voltage) spectral density of the noise source, and the ratio of the signal power to the noise power at:

(4) **6.2.1.** Input of the whole system.

(4) **6.2.2.** Input of $H(\omega)$.

(5) **6.2.3.** Output of $H(\omega)$.