Problem 1: When the input to a given audio amplifier is \((4\cos 800\pi t + \cos 1000\pi t)\) mV, the measured frequency components at 500Hz and 1000Hz are equal to 1V and 2V, respectively. Representing the amplifier output-input characteristic by,

\[ e_o(t) = a_1 e_i(t) + a_2 [e_i(t)]^2. \]

(10) Evaluate the numerical values of \(a_1, a_2\) from the test data given.
**Problem 2:** A carrier waveform is frequency-modulated by the sum of two sinusoids:

\[ \phi(t) = A \cos(\omega_c t + \sin \omega_m t + \cos \omega_m t), \]

(10) Find an expression for the resulting spectrum.
**Problem 3:** Consider the signal

\[ v(t) = \sum_{i=0}^{N} (N + 1 - i)[\cos(\omega_c t) \cos(i\omega_0 t) - \sin(\omega_c t) \sin(i\omega_0 t)] \]

(4) 3.1. Show that this is an SSB-\(L\)C signal \((\omega_c > N\omega_0)\). Is it the upper or the lower sideband?

(2) 3.2. Write an expression for the missing side-band.

(2) 3.3. Write an expression for the total DSB signal.

(7) 3.4. For the DSB signal (part 3.3.), what is the minimum value of the carrier needed to make the envelope detection possible?

(5) 3.5. Compute the efficiency of the modulation for the signal obtained in part 3.4.
**Problem 4:** Consider PM modulation ($k_p = 1$) of the signal $x_{T_0}(t) = \sum_{k=-\infty}^{\infty} m(t - kT_0)$,

$$m(t) = \begin{cases} 
1, & 0 \leq t \leq T_0/2 \\
0, & T_0/2 \leq t \leq T_0 
\end{cases}$$

(10) **4.1.** Determine the frequency spectrum of the PM signal assuming that the carrier frequency is equal to $4\omega_0$ (where $\omega_0 = 2\pi/T_0$) and the total power of the modulated signal is unity.

(10) **4.2.** Compute the fraction of the total power of the resulting PM signal in the frequency range $[-3\omega_0/2, 3\omega_0/2]$. 
Problem 5: Consider the periodic signal shown in the following figure:

\[ T \]
\[ \frac{T}{2} \]
\[ \alpha T \]

(8) 5.1. Find the auto-correlation function, the power spectral density and the total power of this signal.

5.2. Assume that a white noise of power spectral density $\eta/2$ is added to this signal and the combination of signal plus noise is passed through a linear filter with the frequency response shown in the following figure (where $\omega_0 = 2\pi/T$).

\[ H(\omega) \]
\[ -\frac{5\omega}{2} \]
\[ \frac{5\omega}{2} \]

(4) 5.2.1. Compute the power of the signal at the filter output.

(4) 5.2.2. Compute the power of the noise at the filter output.

(4) 5.2.3. Compute the value of $\alpha$ in the periodic signal such that the ratio of the signal power to the noise power at the output of the filter is maximized.
**Problem 6:** Consider the following linear system:

\[ H(\omega) \]

The autocorrelation of the noise source, \( n(t) \), is equal to \( R_n(\tau) = \exp(-|\tau|) \). The signal source, \( x(t) \), is a periodic (voltage) signal as shown in the following figure:

\[ \text{Note: } \omega_0 = \frac{2\pi}{T} \]

(7) **6.1.** Compute the Fourier transform of \( x(t) \) (Hint: compute the derivatives of \( x(t) \))

6.2. Compute the power (square voltage) spectral density of the noise source, and the ratio of the signal power to the noise power at:

(4) **6.2.1.** Input of the whole system.
(4) **6.2.2.** Input of \( H(\omega) \).
(5) **6.2.3.** Output of \( H(\omega) \).