Problem 1: The following system is used for the amplitude modulation of the signal $x(t)$. Derive a condition on $a, b$ to obtain a DSB signal at the output.

![Amplifier of gain $K$](image)

Figure 1: Modulator for problem 1.

Problem 2: The modulating signal $m(t) = A_m \cos(2\pi f_m t)$ is used to generate the VSB signal

$$s(t) = \frac{A_m A_c}{2} \beta \cos[2\pi(f_c + f_m)t] + \frac{A_m A_c}{2} (1 - \beta) \cos[2\pi(f_c - f_m)t]$$

where $\beta$ is a constant, less than unity, representing the attenuation of the upper side frequency.

2.1. Find the in-phase and the quadrature components of the VSB signal $s(t)$.

2.2. The VSB signal, plus the carrier $A_c \cos(2\pi f_c t)$, is passed through an envelope detector. Derive an expression for the output.
(20) **Problem 3:** Periodic signal $s(t)$, shown in Fig. 2, is used once to frequency modulate a carrier of frequency $f_c$ and once to phase modulate the same carrier.

3.1. Find a relation between $k_p$ and $k_f$ such that the peak phase deviation of the modulated signal in both cases are equal.

3.2. If $k_p = k_f = 1$, what is the maximum instantaneous frequency in each case?

3.3. In the case of PM, assuming $k_p = 1$, find an expression for the spectral density of the resulting modulated signal.

![Figure 2: Signal $s(t)$ in problem 3.](image)

(20) **Problem 4:** Assume that the carrier $c(t) = A\cos(2\pi f_c t)$ is used to frequency modulate each of the following two signals:

4.1. $\cos(4\pi f_m t) + \cos(6\pi f_m t)$

4.2. $|\sin(2\pi f_m t)|^2 - 1/2$

Assuming $k_f = 1$, find an expression for the spectral density of the resulting FM signals for each case (in terms of Bessel functions). What is the minimum spacing between the frequency lines in each case?

(20) **Problem 5:** In a DSB modulation system the carrier is $c(t) = \cos(20\pi t)$ and the modulating signal is $m(t) = \sum_{k=-\infty}^{\infty} S_a[2\pi(t - 2k)]$.

5.1. Find the frequency domain representation and the bandwidth of the modulated signal.

5.2. Compute the power of the modulated signal.

5.3. Assume that we want to add an appropriate amount of carrier to the modulated signal to obtain a DSB-LC. What is the minimum amount of carrier needed?
Problem 6: Consider the following DSB transmission scheme where \( x(t) \) is a low pass signal of bandwidth \([-W, W]\), \( H(\omega) \) is the transfer function of the transmission media and \( f_c = 4W \).

![Diagram of the DSB transmission scheme](image)

Figure 3: Modulator and transmission media for problem 6.

Show that we can use the system in Fig. 4 to recover \( x(t) \) for each of the following two cases:

6.1. \( G(\omega) \) is a band pass filter in the frequency range \([f_c - W, f_c + W]\) and is equal to inverse of \( H(\omega) \) in this frequency range.

6.2. \( G(\omega) \) is an ideal band pass filter of unity gain in the frequency range \([f_c - W, f_c + W]\).

In each case (6.1 & 6.2), compute the power of the noise at the output of the filter \( P(\omega) \).

![Diagram of the demodulator system](image)

Figure 4: Demodulator for problem 6.