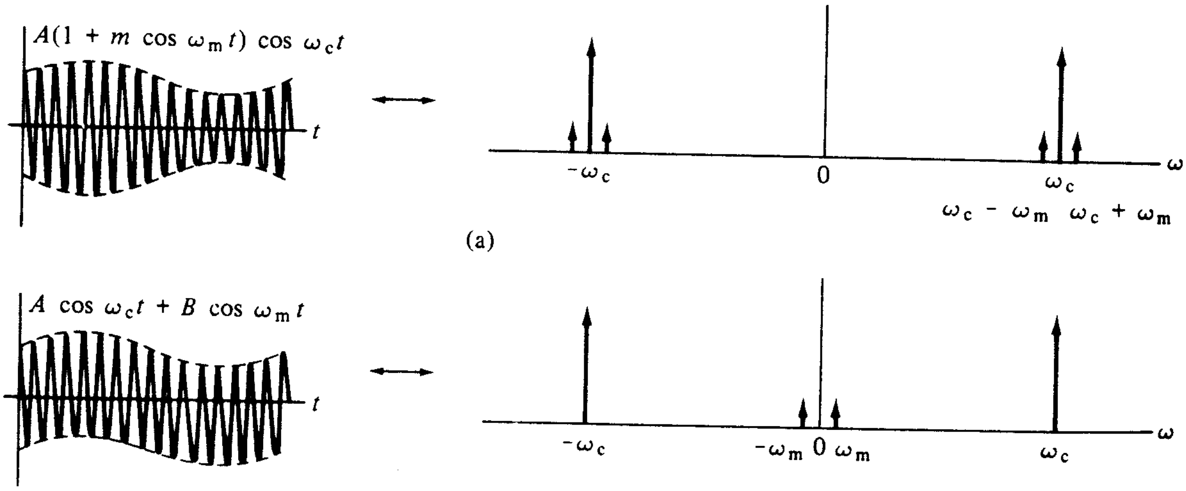

Figure 32: Addition of the carrier to produce DSB-LC waveform.

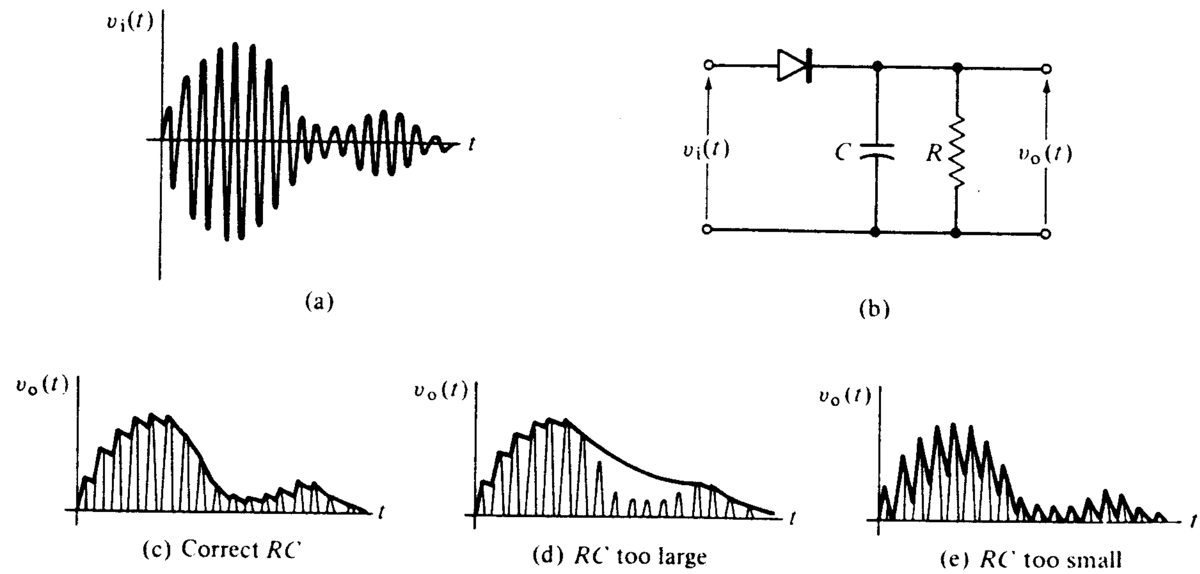

Figure 33: Importance of sufficient carrier in DSB-LC waveform.

The modulation index is defined as:

$$m = \frac{\text{Peak DSB-SC amplitude}}{\text{Peak carrier amplitude}} = \frac{AB}{A} = B \tag{336}$$

This results in:

$$\phi_{AM}(t) = mA\cos\omega_m t\cos\omega_c t + A\cos\omega_c t = A(1 + m\cos\omega_m t)\cos\omega_c t \tag{337}$$

Parameter m is called the modulation index. Figure 33 shows the corresponding waveforms. The maximum and the minimum value of the corresponding signal is equal to: $(1+m)A$ and $(1-m)A$, respectively. Referring to Fig. 34, it is seen that for $m < 1$, we can use an envelope detector. The cases of $m > 1$ are known as the over-modulated.

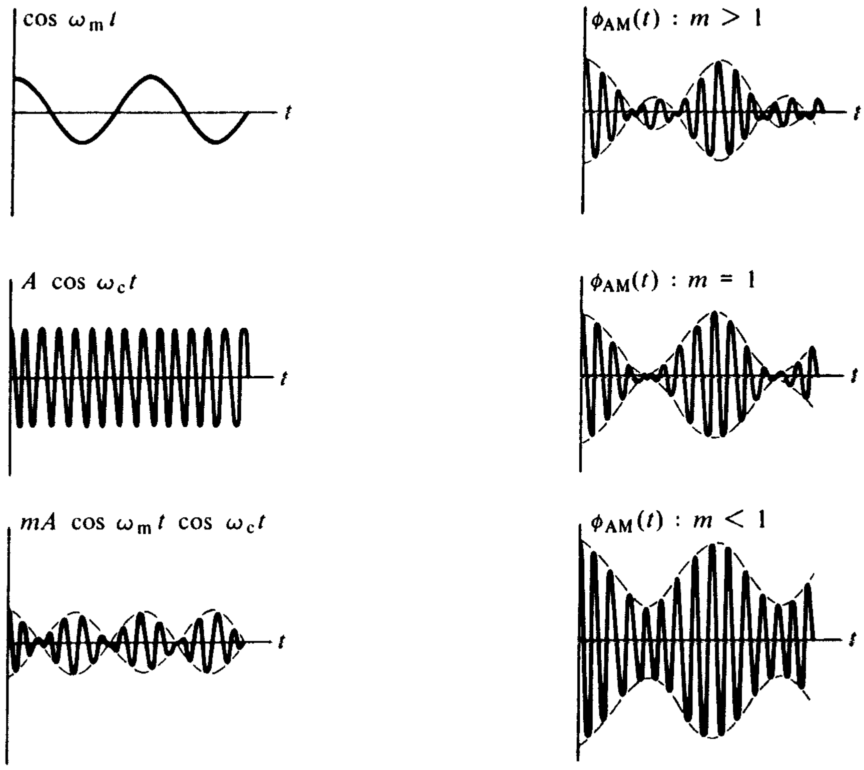

Figure 34: Effect of varying the modulation indexes.

Disadvantage of AM: power in carrier frequency is wasted. This is indeed the price for cheap detector.

6.10 Power in AM

$$\phi_{AM}(t) = A\cos\omega_c t + f(t)\cos\omega_c t \tag{338}$$

$$\overline{\phi^2_{AM}(t)} = A^2\overline{\cos^2\omega_c t} + \overline{f^2(t)\cos^2\omega_c t} + 2A\overline{f(t)\cos^2\omega_c t} \tag{339}$$

where bar indicates time averaging. We assume that:

1. The signal $f(t)$ varies slowly with respect to $\cos\omega_c t$. In this case, we can assume that $f(t)$ is constant within each period of $\cos\omega_c t$. Using this assumption, the average value of functions like $f(t)\cos\omega_c t$, $f^2(t)\cos\omega_c t$, $f(t)\cos^2\omega_c t$ and $f^2(t)\cos^2\omega_c t$ can be written as the product of their average values.

2. Average value of $f(t)$ is zero.

Then,

$$\overline{\phi^2_{AM}(t)} = A^2\overline{\cos^2\omega_c t} + \overline{f^2(t)\cos^2\omega_c t} = A^2/2 + \overline{f^2(t)}/2 = P_c + P_s \tag{340}$$

where P_c, P_s are equal to the carrier power, signal power, respectively. The fraction of the useful power is equal to:

$$\mu = \frac{P_s}{P_t} = \frac{\overline{f^2(t)}}{A^2 + \overline{f^2(t)}} \tag{341}$$

In the case of a sinusoid modulating signal we have, $f(t) = mA\cos\omega_m t$, resulting in $\overline{f^2(t)} = m^2A^2/2$, and

$$\mu = \frac{m^2}{2 + m^2} \tag{342}$$

From (342), we see that the value of efficiency for $m = 1$ (maximum efficiency) is equal to 33% while the efficiency of DSB-SC is 100% (or slightly less if we include a pilot tone).

6.11 Generation of DSB-LC

The direct approach is to generate a DSB-SC and then add the carrier to it. To generate a DSB-LC with a chopper amplifier, it is enough to add a right amount of dc level, A, to the input signal such that $A + f(t) > 0$. We remind that the chopper action may be viewed as multiplication of the input signal with a periodic square wave $P_T(t)$ whose fundamental frequency is ω_c rad/sec. Another possibility is to add some carrier to $f(t)$ before chopping. In this configuration, if the carrier signal is much larger than $f(t)$, we can use a diode for switching and the diode will turn on and off at the carrier rate (refer to Fig. 35).

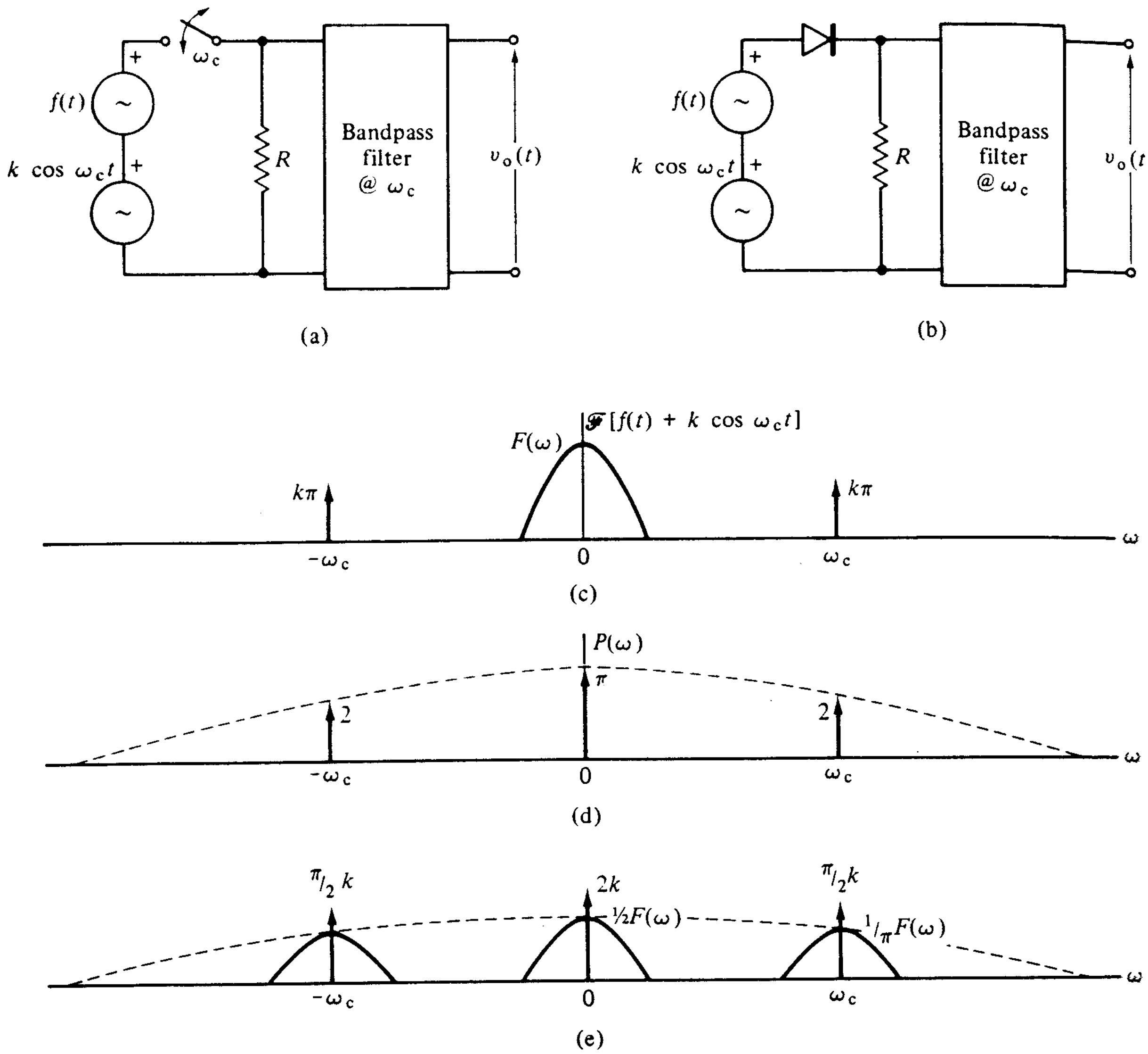

Figure 35: Generation of AM signal using chopper.

We can also use a nonlinearity to generate DSB-LC. Consider the nonlinearity,

$$i(t) = a_1 e(t) + a_2 e^2(t) + \ldots \tag{343}$$

Assume that $e(t) = f(t) + k\cos\omega_c t$. Retaining the first two terms, we obtain,

$$i(t) = a_1[f(t) + k\cos\omega_c t] + a_2[f(t) + k\cos\omega_c t]^2 + \ldots \tag{344}$$

The terms around the carrier frequency are,

$$a_1 k \cos \omega_c t + 2a_2 k f(t) \cos \omega_c t \tag{345}$$

6.12 Demodulation of DSB-LC using envelope detector

In a DSB-LC, the desired signal waveform is present in the envelope of the modulated signal and one can use an envelope detector to recover it. An envelope detector is a nonlinear circuit whose output follows the envelope of the input, like a circuit with a fast charge and a slow discharge time (refer to Fig. 36).

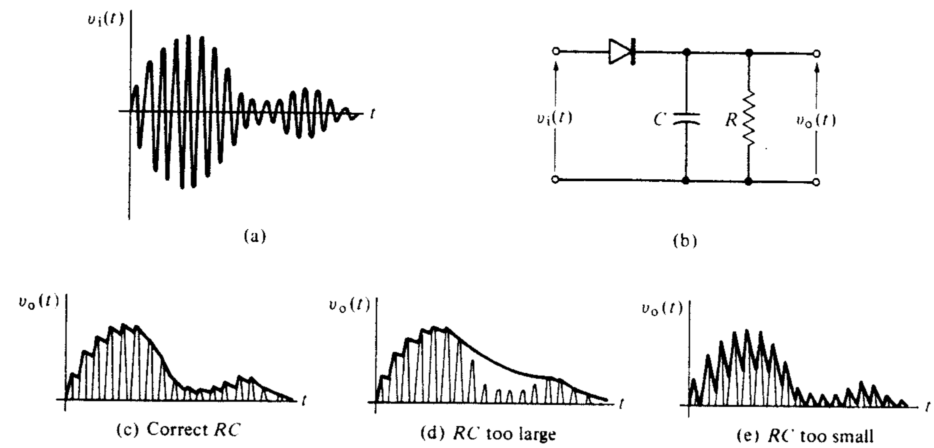

Figure 36: The envelope detector.

6.13 Frequency Division Multiplexing

Frequency Division Multiplexing is based on simultaneous transmission of several signals using different carrier frequencies. If the bandwidth of the signals is ω_m, then two subsequent modulating frequency should be at least $2\omega_m$ apart (refer to Fig. 37).

In practice, the composite signal formed by spacing several signals in frequency may, in turn, be modulated using another carrier frequency. The sinusoids used in modulating within the composite signal are called the sub-carrier.

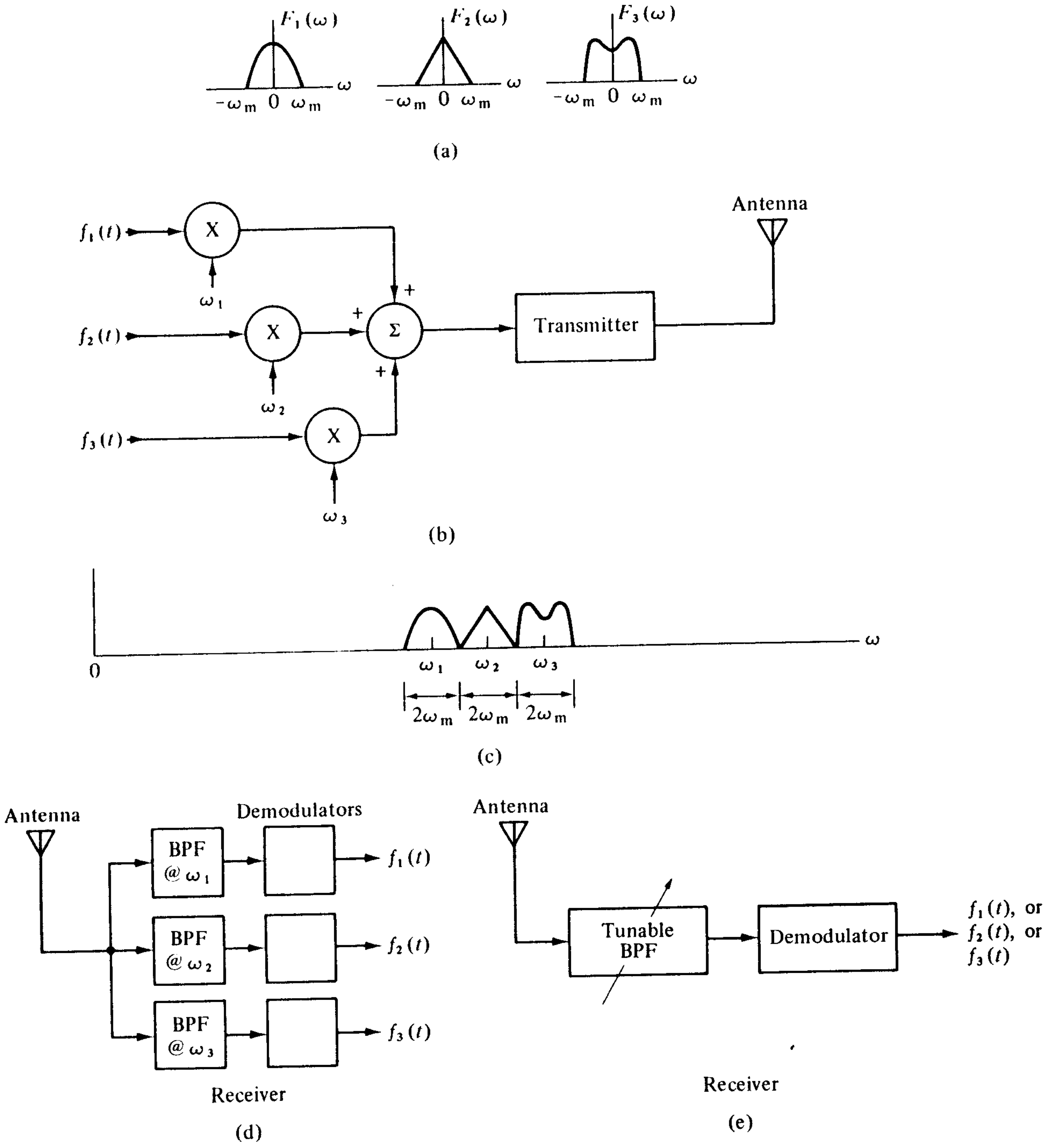

Figure 37: Frequency division multiplexing (FDM).

In the receiver side, depending of the application, one can demodulate all the signals simultaneously (for example in stereo multiplexing) or use a tunable bandpass filter to separate one of the signals.

Early AM receivers were based on tunable bandpass filters. The problem with those systems was that they did not any amplification for the received signal.

The next step was the tuned-radio-frequency (TRF) receiver (refer to Fig. 38). In these systems all the stages (usually three) were tuned simultaneously to select the receiver. The problem was that the simultaneous adjustment of different stages was a difficult task.

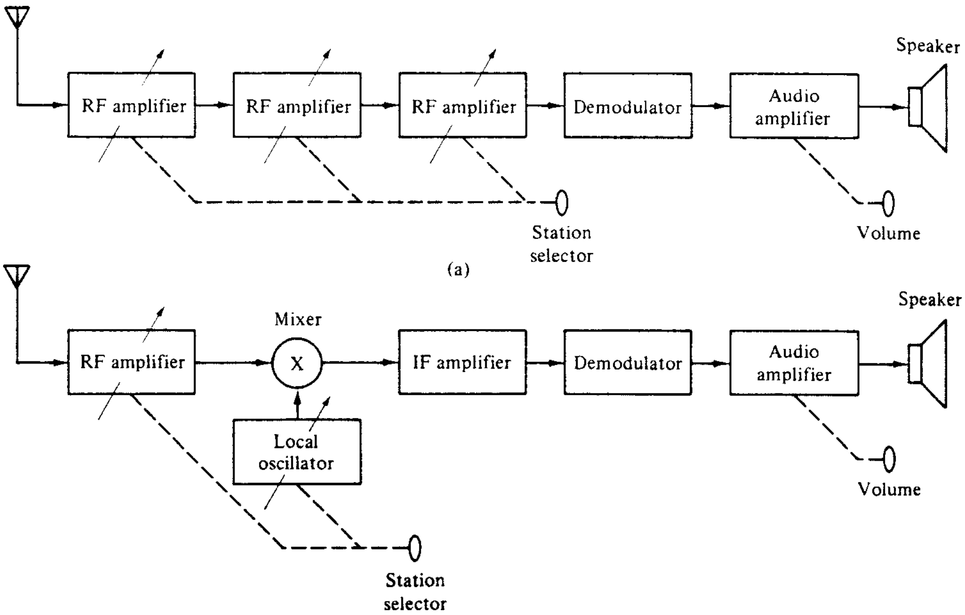

Figure 38: Radio receivers, (a) the TRF, (b) the super-heterodyne.

The next, and the final, improvement stage was the heterodyne receiver which is widely used today. This is based on translating the received signal in frequency such that the desired station is centered around a *fixed* frequency, called the intermediate frequency (IF). Then, the signal is amplified at this intermediate frequency. If the IF frequency is lower than the received carrier frequency but above the final output signal frequency, it is called a super-heterodyne receiver (refer to Fig. 38).

In commercial AM systems, the IF frequency is 455 KHz and different stations are from

540 to 1600 KHz with 10 KHz spacing in frequency. The frequency translation is achieved by mixing the incoming signal with a locally generated signal which differs from the incoming carrier by the IF frequency. This locally generated signal can be either 455 KHz above or 455 KHz below the received carrier. In practice, it is selected to be above the incoming carrier simply because it is easier to build the required tuning oscillators to operate in a higher range of frequencies (1—2 MHz for the frequency band 540 to 1600 KHz).

There are two problems with this system as follows:

2. Suppose that we want to tune a station at 600 KHz, this means the local oscillator is at 455+600 = 1055 KHz. Now if there a station at 1510 KHz, it will be also received (1510 − 1055 = 455 KHz). This second frequency is called an image frequency of the first (refer to Fig. 39). A solution to this problem is to select a high enough IF frequency. The solution used in practice is to attenuate the image frequency before mixing.

1. If there exist a signal at the IF frequency, it will be mixed with the shifted signal.

In practice, both of these problems are solved by placing a frequency selective (tunable) amplifier before the mixer.

Another problem is that the local oscillator also acts as a small transmitter. This effect can be observed by placing two AM receiver near each other and tuning them to two stations 455 KHz apart. This effect can be used to find the location of a receiver. It can be minimized by proper shielding.

6.14 Single-sideband Modulation (SSB)

A DSB-SC or DSB-LC (AM) signal results in doubling the bandwidth. As a real signal has conjugate symmetry, $F(-\omega) = F^*(\omega)$, each pair of sideband (upper or lower) contains the complete information of the original signal. In a Single-sideband Modulation, we use this property and transmit only one of the side-bands (refer to Fig. 40).

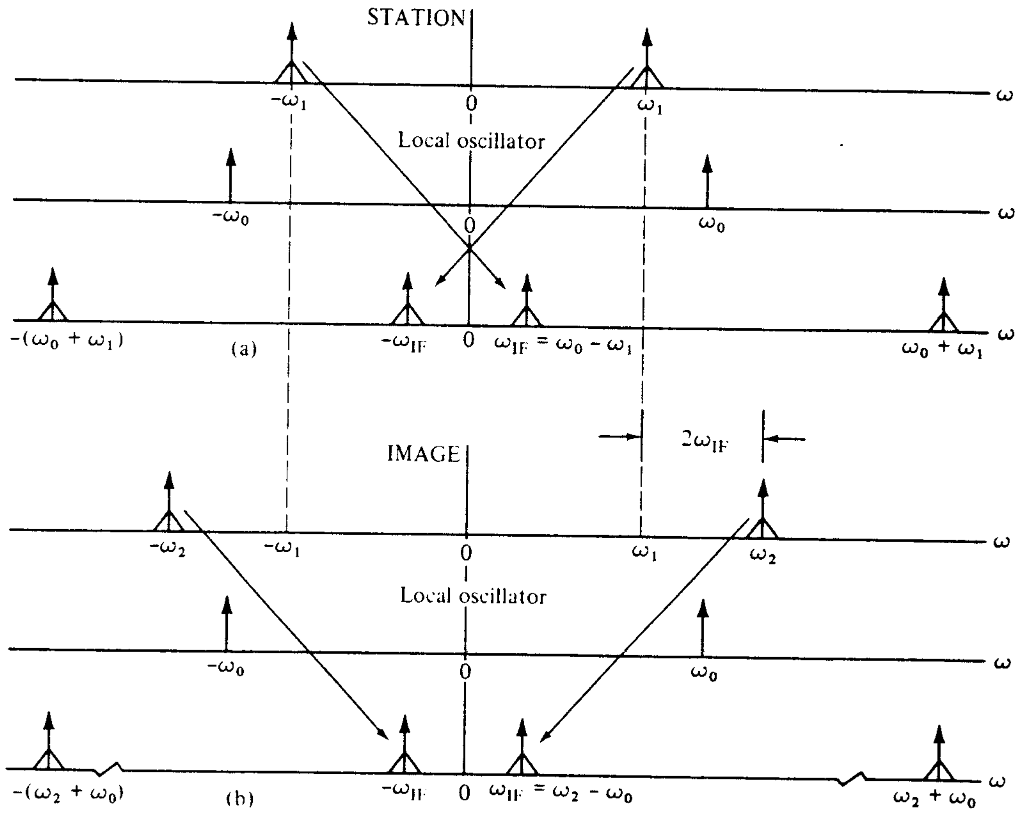

Figure 39: Problem of image frequency.

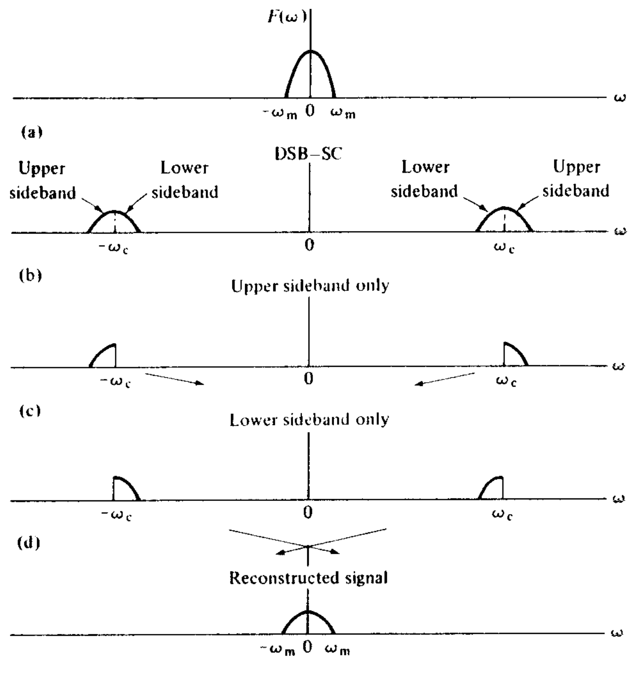

Figure 40: Spectra of DSB and SSB signals.

6.14.1 Generation of SSB signals

The direct approach is to first generate a DSB signal and then suppress one of the sidebands by filtering (refer to Fig. 41). This approach requires very sharp filters. In the case that the signal does not have substantial components near zero frequency, the filters do not need to be very sharp. It is also usually simpler to design the filter to operate at a fixed frequency and then shift the frequency of the filtered signal.

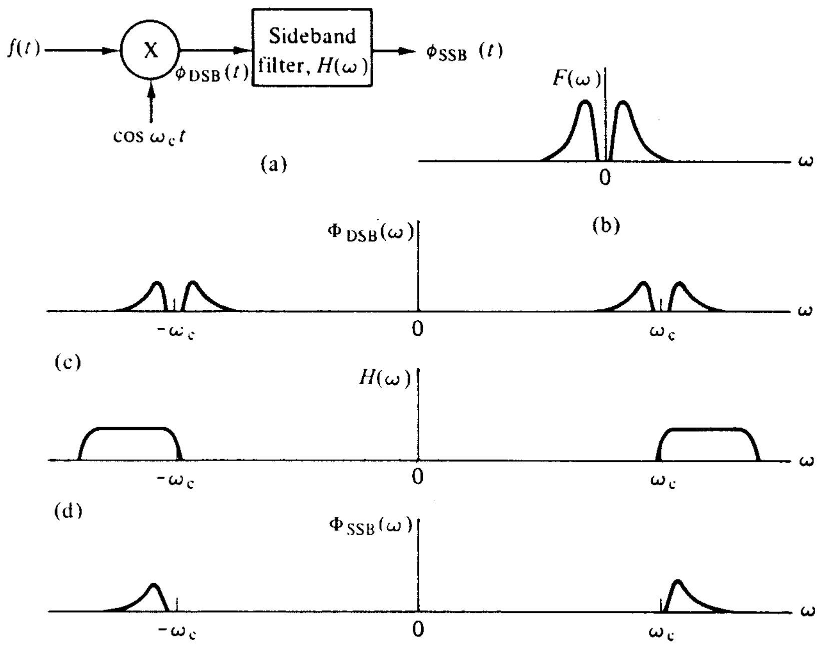

Figure 41: Generation of SSB signal using filtering.

Another approach is to generate the SSB signal using a proper phasing of signal. To explain this method, consider the modulating signal $e^{j\omega_m t}$ and the carrier signal $e^{j\omega_c t}$. We have,

$$\mathcal{R}\{e^{j\omega_m t}e^{j\omega_c t}\} = \mathcal{R}\{e^{j\omega_m t}\}\mathcal{R}\{e^{j\omega_c t}\} - \mathcal{I}\{e^{j\omega_m t}\}\mathcal{I}\{e^{j\omega_c t}\} =$$

$$\cos\omega_m t \cos\omega_c t - \sin\omega_m t \sin\omega_c t \tag{346}$$

This is in fact a SSB signal composed of the upper sidebands, i.e.,

$$\phi_{SSB+}(t) = \cos\omega_m t \cos\omega_c t - \sin\omega_m t \sin\omega_c t \tag{347}$$