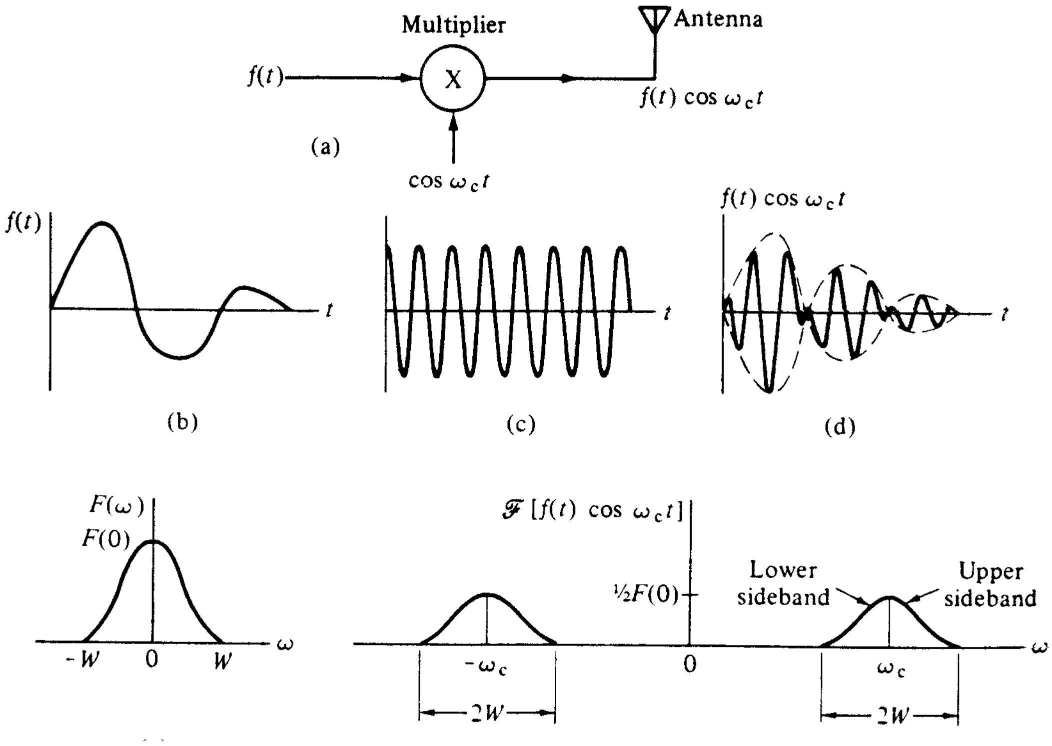

Figure 24: An amplitude modulation suppressed carrier system.

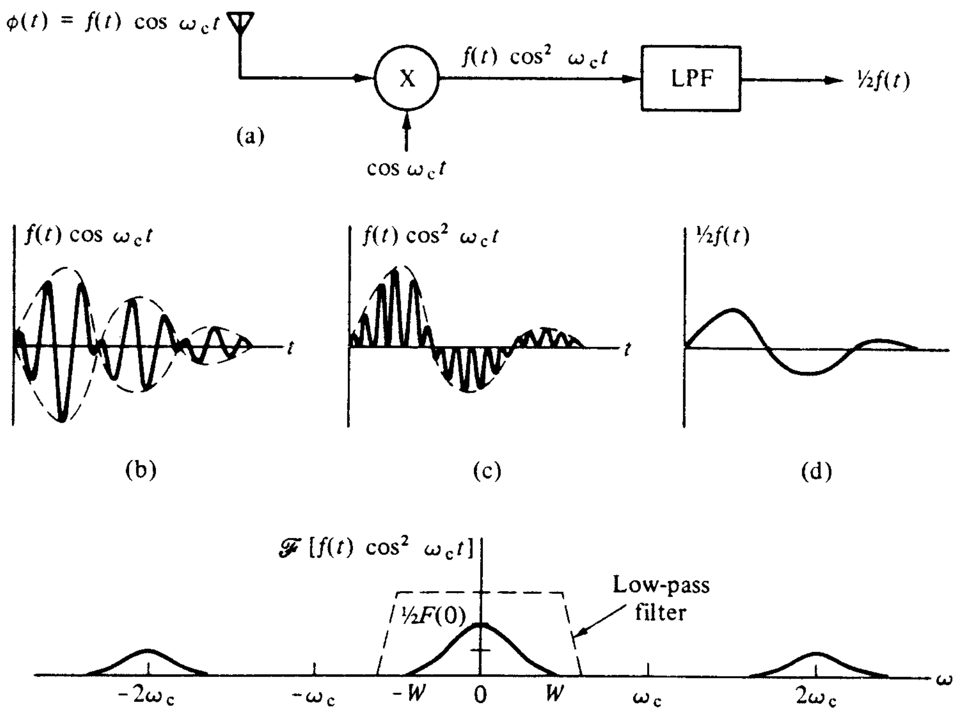

Figure 25: Demodulation of suppressed carrier signals.

6.2 Effect of phase and frequency error in demodulation

Assume that the demodulator signal has a frequency error $\Delta\omega$ and a phase error θ_0. This results in:

$$\phi(t)\cos[(\omega_c+\Delta\omega)t+\theta_0] = f(t)\cos\omega_c t\cos[(\omega_c+\Delta\omega)t+\theta_0] =$$

$$\frac{1}{2}f(t)\cos[(\Delta\omega)t+\theta_0] + \frac{1}{2}f(t)\cos[(2\omega_c+\Delta\omega)t+\theta_0] \tag{318}$$

The output of the low-pass filter is equal to:

$$e_0(t) = \frac{1}{2}f(t)\cos[(\Delta\omega)t+\theta_0] \tag{319}$$

This means that the output signal is not equal to $(1/2)f(t)$ unless both $\Delta\omega$ and θ_0 are equal to zero. In the following, we consider two special cases:

Case I: Assume that $\Delta\omega = 0$, then,

$$e_0(t) = \frac{1}{2}f(t)\cos\theta_0 \tag{320}$$

This means that the phase error results in a variable gain factor in the output signal. A phase error which is *small* and *constant* is quite tolerable. Random phase errors are more deteriorating.

Case II: Assume that $\theta_0 = 0$, then,

$$e_0(t) = \frac{1}{2}f(t)\cos(\Delta\omega)t \tag{321}$$

We obtain the original signal $f(t)$ multiplied by a low frequency sinusoid. This results in undesirable and unacceptable distortion.

In general, the oscillator in the transmitter and receiver should be synchronized. Recovering of the original signal $f(t)$ from the modulated signal $\phi(t)$ using a synchronized oscillator is called *synchronous* or *coherent* detection.

6.3 Quadrature Multiplexing

Quadrature Multiplexing is used to transmit two messages within the same band-width.

$$\phi(t) = f_1(t)\cos\omega_c t + f_2(t)\sin\omega_c t \tag{322}$$

Detection:

$$\phi(t)\cos\omega_c t = f_1(t)\cos^2\omega_c t + f_2(t)\cos\omega_c t\sin\omega_c t = \tag{323}$$
$$\frac{1}{2}f_1(t) + \frac{1}{2}f_1(t)\cos 2\omega_c t + \frac{1}{2}f_2(t)\sin 2\omega_c t$$

and,

$$\phi(t)\sin\omega_c t = f_1(t)\cos\omega_c t\sin\omega_c t + f_2(t)\sin^2\omega_c t = \tag{324}$$
$$\frac{1}{2}f_1(t)\sin 2\omega_c t + \frac{1}{2}f_2(t) - \frac{1}{2}f_2(t)\cos 2\omega_c t$$

After low pass filtering, we obtain,

$$\begin{aligned} e_1(t) &= \tfrac{1}{2}f_1(t) \\ e_2(t) &= \tfrac{1}{2}f_2(t) \end{aligned} \tag{325}$$

6.4 Generation of DSB-SC Signals

In the previous part modulation was achieved by multiplying with a sine wave. In the following we consider the more general case of a periodic signal.

Assume that $P_T(t)$ is a periodic signal of period T. We have,

$$P_T(t) = \sum_{n=-\infty}^{\infty} P_n e^{jn\omega_c t} \tag{326}$$

where $\omega_c = 2\pi/T$. Multiplying by $f(t)$, we obtain,

$$\mathcal{F}\{f(t)P_T(t)\} = \sum_{n=-\infty}^{\infty} P_n F(\omega - n\omega_c) \tag{327}$$

where $F(\omega)$ is the Fourier transform of $f(t)$. Referring to (327), we see that the resulting spectrum is composed of replica of $F(\omega)$ translated around the center frequencies $0, \pm\omega_c, 2 \pm \omega_c, \ldots$ where the amplitude of these successive spectral replica are scaled by constants $P_0, P_1, P_2, \ldots$. This is shown in Fig. 26.

We are usually interested in the spectrum centered around $\pm\omega_c$. This term can be obtained using a proper bandpass filter. To avoid overlap between successive terms, we should have $\omega_c > W$ where W is the bandwidth of $f(t)$. If $\omega_c \gg W$, there will be a large separation between terms in frequency domain and we can use simple filters.

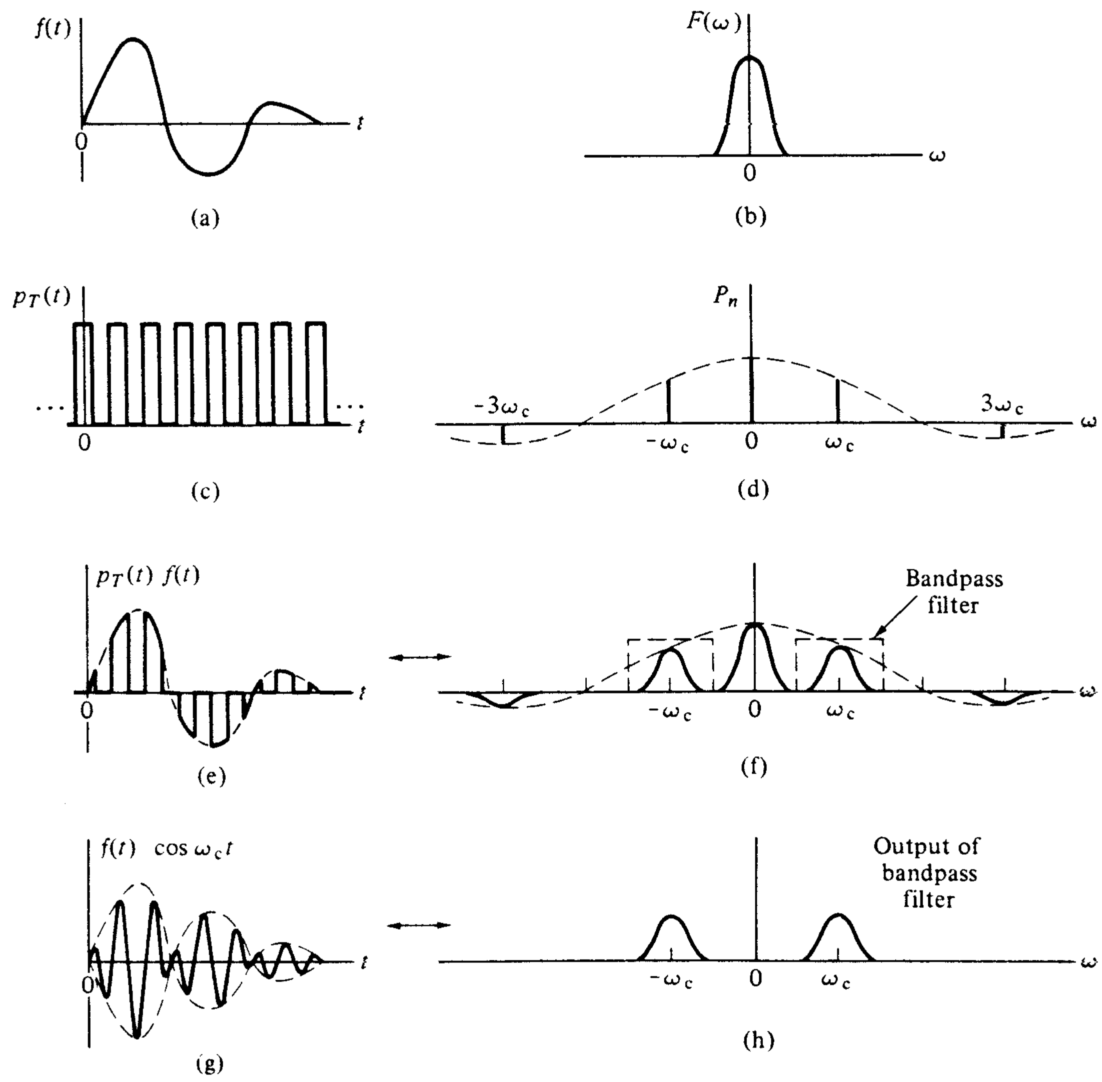

Figure 26: Amplitude modulation using multiplication by periodic signals.

In general, in a modulating system based on frequency translation, we must produce frequencies at the output which are not present at the input. This can not be achieved using a linear time-invariant system and the corresponding system should be either time-varying or nonlinear (or both). In the following, we study some specific examples of such systems.

6.5 Chopper Modulator

This is essentially a linear time-varying system based on a switch. The switch operates (closes/opens) at the rate of the carrier frequency and connects/disconnects the input from the output at that rate (refer to Fig. 27 (a). This chopping operation is essentially equivalent to multiplying of the signal $f(t)$ with a periodic square wave $P_T(t)$ whose two levels are zero and one. A typical practical circuit using four diodes known as a ring modulator is shown in Fig. 27 (b). For best operation, the four diodes should be identical (balanced) so that when $f(t) = 0$ there is no carrier output. For perfect switching we need ideal diodes.

A different configuration, similar to a full wave rectifier, is shown in Fig. 28. This configuration does not need ideal diodes and also the switching operation results in changing the sign of the input at the carrier rate. This is equivalent to multiplying the input signal with a gate function which has zero dc value. This omits the output frequency replica around zero frequency, and consequently, facilitates the subsequent filtering. In order to guarantee that the $e_i(t)$ does not interfere with the switching operation, we should have,

$$[e_i(t)]_{\max} < \frac{1}{2}[e_R(t)]_{\max} \tag{328}$$

6.6 Use of Nonlinear Devices

Consider the nonlinearity (for example a diode):

$$i(t) = a_1 e(t) + a_2 e^2(t) + a_3 e^3(t) + \dots \tag{329}$$

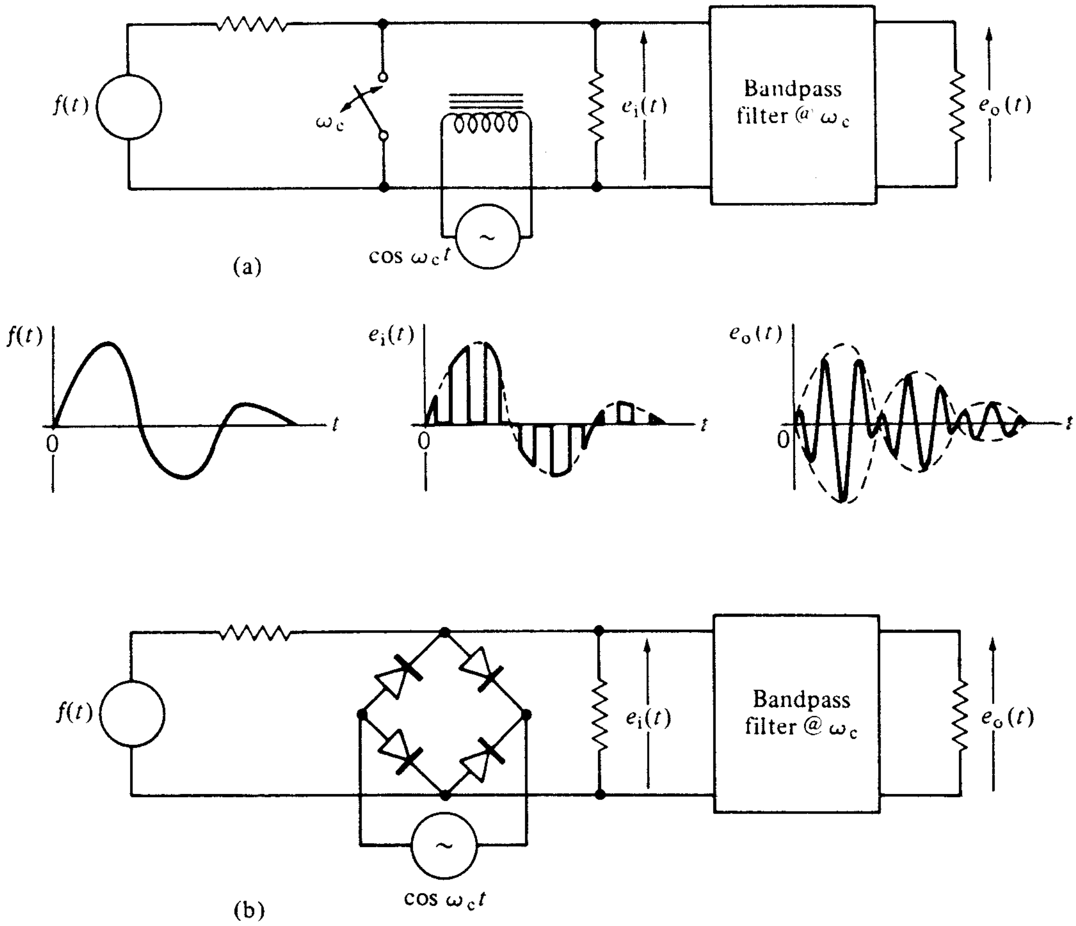

Figure 27: Amplitude modulation using chopper modulator.

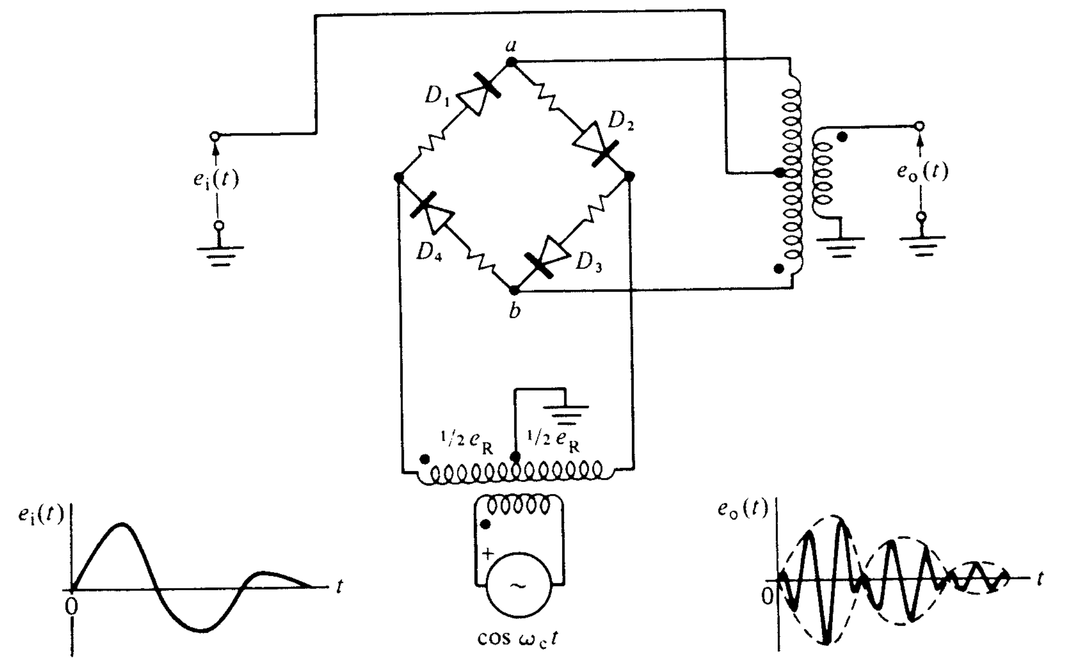

Figure 28: The double-balanced ring modulator.

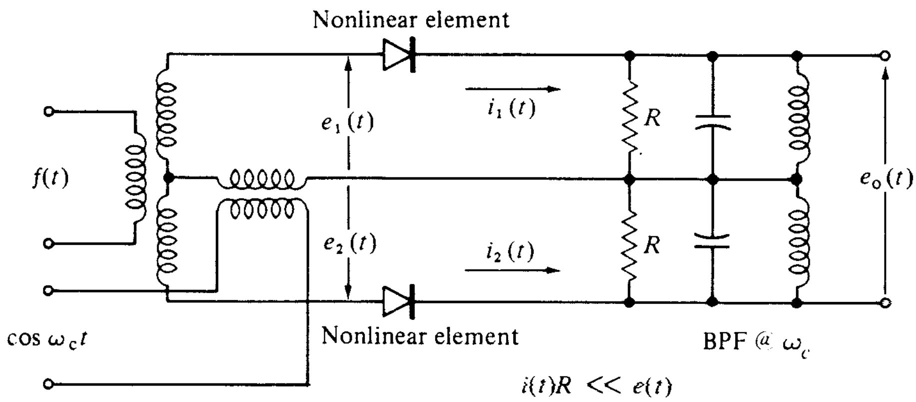

Figure 29: The balanced modulator using nonlinear devices.

For the configuration shown in Fig. 29, we have:

$$e_1(t) = \cos\omega_c t + f(t)$$
$$e_2(t) = \cos\omega_c t - f(t) \tag{330}$$

Keeping the first two terms in (329), we obtain,

$$i_1(t) = a_1[\cos\omega_c t + f(t)] + a_2[\cos\omega_c t + f(t)]^2$$
$$i_2(t) = a_1[\cos\omega_c t - f(t)] + a_2[\cos\omega_c t - f(t)]^2 \tag{331}$$

For a resistive load, the net voltage is equal to: $[i_1(t) - i_2(t)]R$, or,

$$[i_1(t) - i_2(t)]R = 4a_2R\left[f(t)\cos\omega_c t + \frac{a_1}{2a_2}f(t)\right] \tag{332}$$

The second term is filtered out with the band-pass filter, yielding the desired result.

6.7 Demodulation of DSB-SC

To recover the original signal, we must translate the spectrum again. This can be achieved by multiplying $\phi(t)$ by $\cos\omega_c t$ and using a low-pass filter to recover the original signal (synchronous detection). The two sinusoids used in the transmitter and receiver should be synchronized. This is usually achieved by transmitting a small carrier signal, displaced in frequency, to the receiver. This is called the pilot carrier. An example concerning stereo multiplexing in FM is shown in Fig. 30.

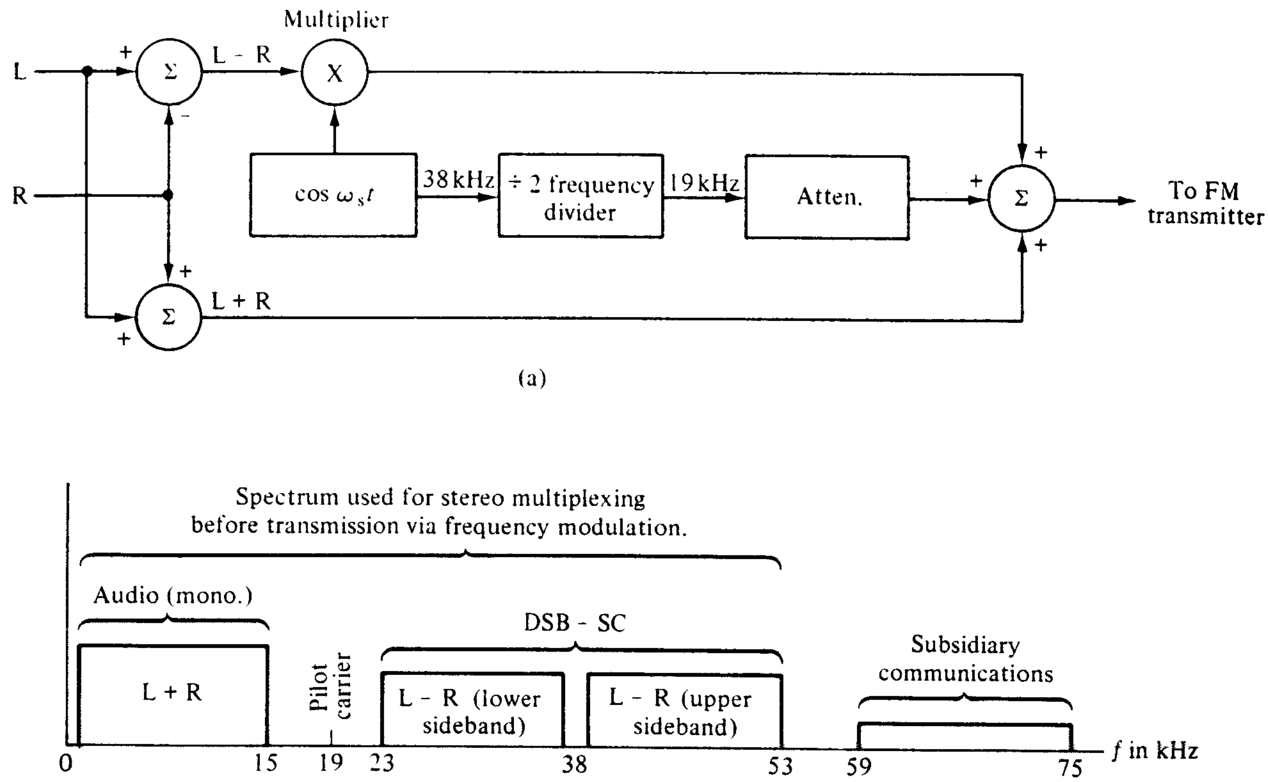

Figure 30: A stereo multiplexing system.

6.8 Some side applications for the DSB-SC modulation

6.8.1 Use in the low-frequency amplifier design

It is difficult to design amplifiers for operating in low range of frequencies. Using modulation, the low frequency signal is shifted in frequency, is amplified and then is shifted back to the original case. In this case, due to the physical proximity of modulator, demodulator we can use the same signal in both operations and synchronous is not a problem (refer to Fig. 31).

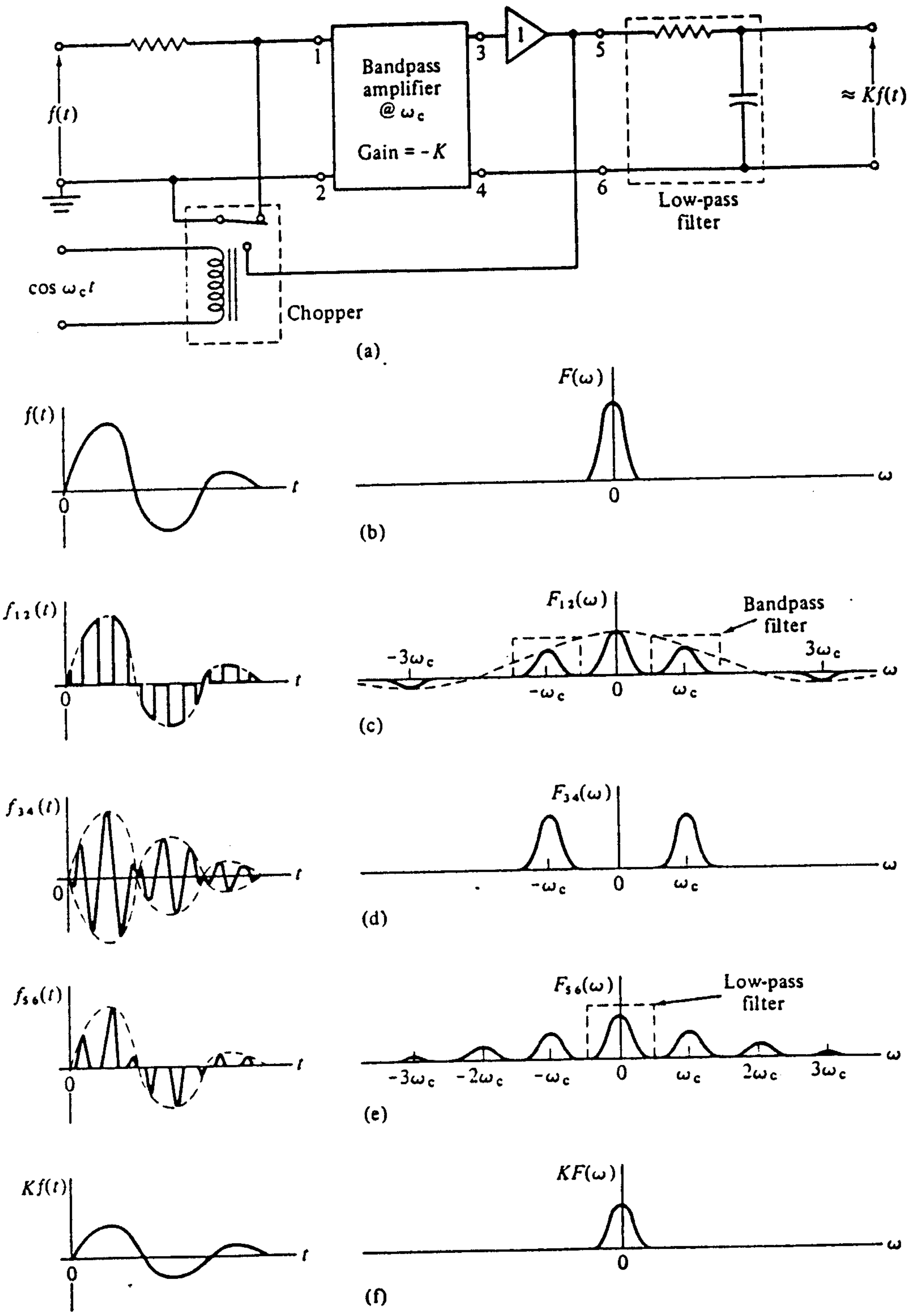

Figure 31: The chopper amplifier.

6.8.2 Use in filter design

Assume that we need a band pass filter with a movable, controllable center frequency. It is usually simpler to use a fixed filter and shift the signal frequency components instead, using modulation.

6.9 Amplitude Modulation: Large Carrier

In DSB-SC modulation, we have the problem of the synchronization. As we will see in this section, one can avoid this problem by transmitting an appropriate amount of the carrier signal. This addition of the carrier allows us to use a simple and cheap demodulator. Obviously, this addition of the carrier signal ruins the low-frequency response of the system. However, fortunately, for many signals, including speech, the low frequency portion is not very important. The resulting signal is called the double-sideband, large-carrier (DSB-LC), or because of its large popularity in commercial systems, it is commonly known as amplitude modulation (AM).
Mathematically, we have:

$$\phi_{AM}(t) = f(t)\cos\omega_c t + A\cos\omega_c t = [A + f(t)]\cos\omega_c t \tag{333}$$

with the Fourier transform:

$$\Phi_{AM}(\omega) = \frac{1}{2}F(\omega+\omega_c) + \frac{1}{2}F(\omega-\omega_c) + \pi A\delta(\omega+\omega_c) + \pi A\delta(\omega-\omega_c) \tag{334}$$

The spectral density is the same as in the case of the DSB-SC modulation except for the presence of two impulses at the carrier frequency (refer to Fig. 32).
Considering (333), if A is large enough $A \geq |\min\{f(t)\}|$, the corresponding envelope is always positive and we can use an envelope detector for the recovery (refer to Fig. 33). This does not depend on any phase or frequency information.

6.9.1 Modulation of a single tone signal

Consider the case of $f(t) = AB\cos\omega_m t$.

$$\phi_{AM}(t) = AB\cos\omega_m t\cos\omega_c t + A\cos\omega_c t \tag{335}$$