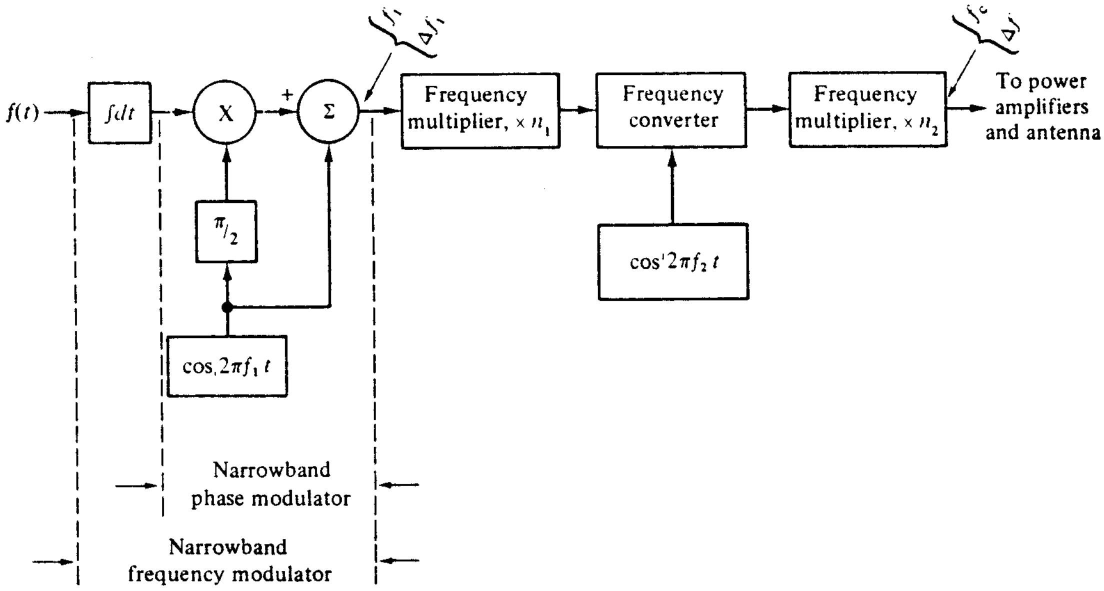

Figure 50: Block diagram of an indirect (Armstrong) FM transmitter.

7.7.2 Direct FM

In the direct method, the modulating signal directly controls the frequency of the carrier. The objective is to have a linear relationship in a wide range of frequency deviation. A common approach is to vary the inductance or the capacitance of a tuned LC circuit. For small variations, the square-root relationship can be approximated by a linear term. One method is to use a reverse biased semiconductor diode as a voltage-controlled capacitance. The relative frequency deviation that can be obtained in this manner is usually quite small. To increase this factor, frequency modulation is performed at a higher frequency and then translated back to the desired frequency.

Another approach is based on generating the PM signal by digital circuits and then shaping the output waveform by a nonlinear shaping circuit or by a linear filter. This is shown in Fig. 51. The clock generator oscillates at the carrier frequency. The ramp generator has the same frequency. Samples of the input signal at this frequency are compared with the amplitude of the ramp. A pulse is generated when these two are equal. In this way, we have one impulse in each period of the clock where the location of this impulse with respect to the start of the period is proportional to the amplitude of the input signal. These impulses signify a zero crossing.

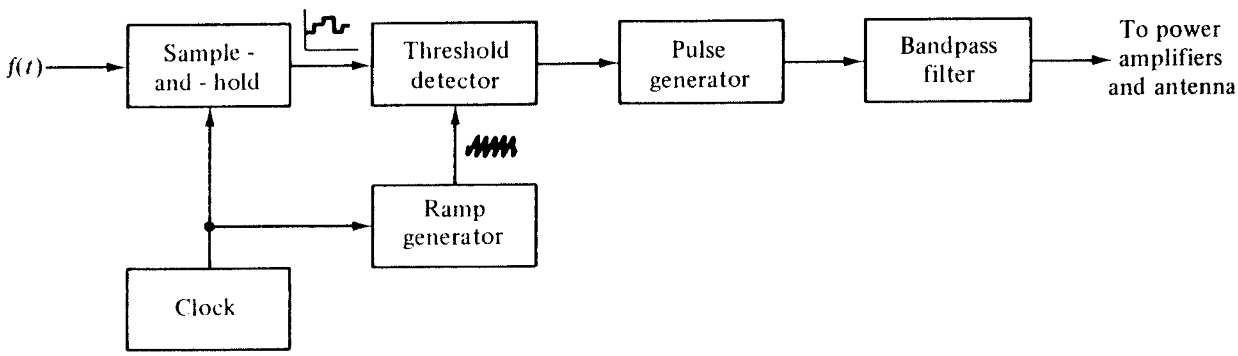

Figure 51: Digital generation of Wideband PM.

7.8 Demodulation of FM Signals

The first method is based on using a system with a linear frequency-to-voltage transfer characteristic. (Like a differentiators, $H(\omega) = j\omega$). Such a devise is called a *discriminator*. Consider the FM signal:

$$\phi_{FM}(t) = A\cos\left[\omega_c t + k_f\int_0^t f(\tau)d\tau\right] \tag{433}$$

Using a differentiators results in,

$$\frac{d\phi}{dt} = -A[\omega_c + k_f f(t)]\sin\left[\omega_c t + k_f\int_0^t f(\tau)d\tau\right] \tag{434}$$

If $k_f f(t) \ll \omega_c$, we see that (434) is in the form of an AM signal with the envelope: $A[\omega_c + k_f f(t)]$. This means that the differentiators has changed the FM signal into AM with the slight difference that the new carrier frequency has some frequency variations. The resulting AM signal can be detected using an envelope detector. The slight variations in the carrier frequency are not detectable by the envelope detector.

The action of the ideal differentiators can be approximated by any devise whose magnitude transfer function is reasonably linear within the range of frequencies on interest. In Fig. 52 (b), an RL circuit is used to approximate a differentiators. In Fig. 52 (c), an LC-tuned circuit is used to approximate a band-pass version of a differentiator. The circuit in Fig. 52 (d) uses a combination of two LC circuits to increase its range of linearity. This combination also produces a zero output response at the carrier frequency which is an advantage.

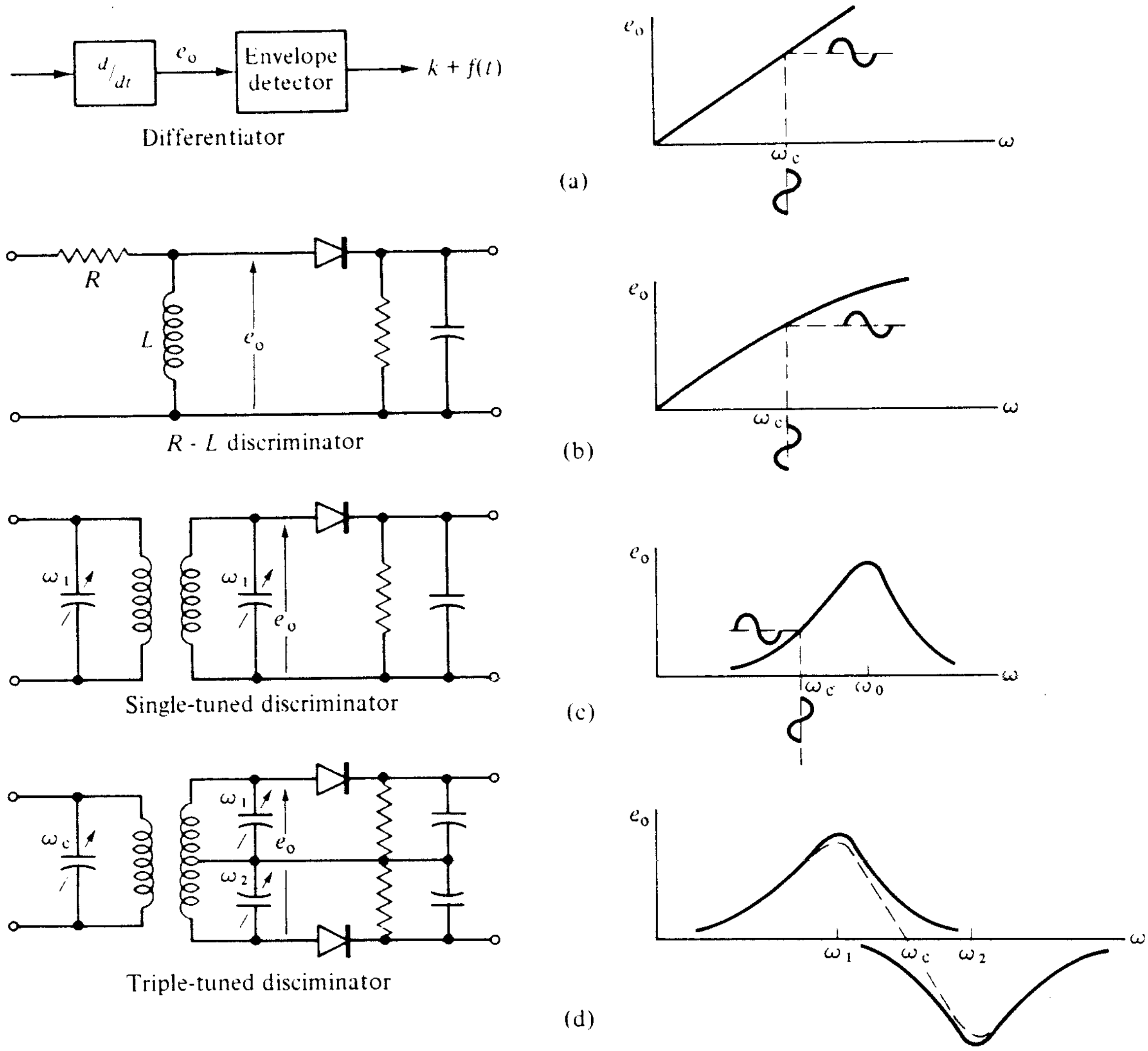

Figure 52: Fm demodulation using discriminators.

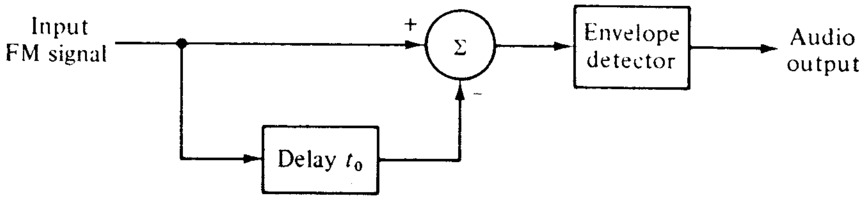

Figure 53: The time-delay demodulator.

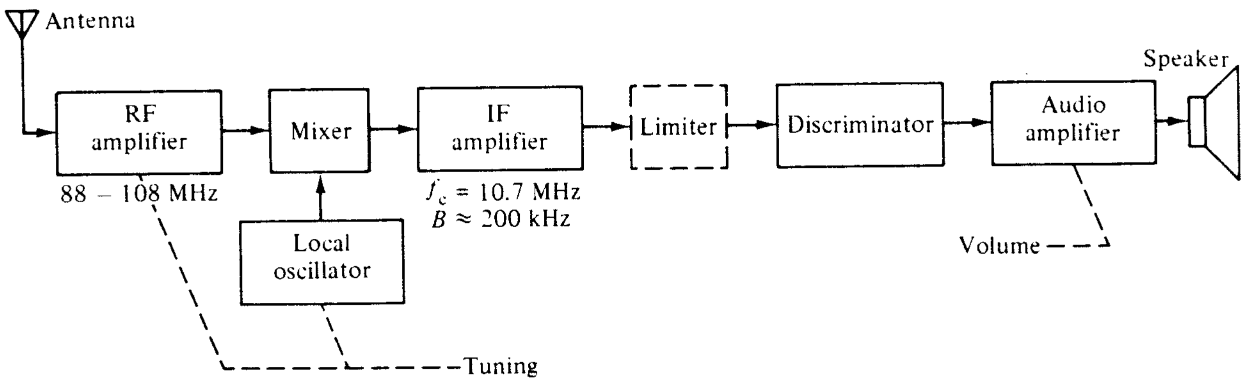

Figure 54: A typical FM receiver.

Another possibility is to use a time delay approximation to the differentiators (refer to Fig. 53). This approach is specially popular for high carrier frequencies.

Another approach is based on severely clipping (limiting) the amplitude of the received signal and then using a digital circuit to measure the variations in the zero crossings of the resulting square wave. This can be measured by counting the number of zero crossings in a given time interval or by measuring the time interval for a given number of zero crossings.

Figure 54 shows the block diagram of a typical FM receiver.

The carrier frequency is in the range of 88-108 MHz. The receiver is similar to the AM superheterodyne receiver with the exception of the addition of a discriminator and possibly a limiter. The common choice of an intermediate frequency is 10.7 MHz.

7.9 Signal to noise ratios in FM reception

We define the signal-to-noise ratio to be the ratio of the mean signal power without noise to the mean noise power in the presence of an unmodulated carrier.

Assuming that the limiter is ideal and removes all amplitude variations, we can write the FM signal at the discriminator input as,

$$s_i(t) = A\cos\theta(t), \tag{435}$$

$$s_i(t) = A\cos\left[\omega_c t + k_f\int_0^t f(\tau)\,d\tau\right]. \tag{436}$$

The discriminator output is proportional to the difference between the instantaneous frequency of $s_i(t)$ and the carrier frequency. Letting the constant of propotionality be 1 for convenience, we obtain,

$$s_o(t) = \left(\frac{d\theta}{dt} - \omega_c\right) = k_f f(t). \tag{437}$$

The mean square of the output signal is,

$$S_o = \overline{s_o^2(t)} = k_f^2\overline{f^2(t)} \tag{438}$$

Next we return to calculating the mean output noise power in the presence of an unmodulated carrier. Using the bandpass representation of the noise, we obtain,

$$\begin{aligned} A\cos\omega_c t + n_i(t) &= A\cos\omega_c t + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t \\ &= r(t)\cos[\omega_c t - \gamma(t)] \end{aligned} \tag{439}$$

Therefore, the addition of noise introduces both amplitude noise [in $r(t)$] and phase noise [in $\gamma(t)$]. In the AM case we were interested in the effects of noise on $r(t)$ but in the FM case we can assume amplitude limiting and are interested only in $\gamma(t)$.

The phase noise angle is

$$\gamma(t) = \tan^{-1}\left[\frac{n_s(t)}{A + n_c(t)}\right]. \tag{440}$$

Assuming the noise is small, i.e. that $n_c(t), n_s(t) \ll A$, we have

$$\gamma(t) \approx \tan^{-1}\left[\frac{n_s(t)}{A}\right] \approx \left[\frac{n_s(t)}{A}\right]. \tag{441}$$

The discriminator output is proportional to the difference between the instantaneous frequency and the carrier frequency so that

$$n_o(t) = \frac{d\gamma}{dt} = \frac{1}{A}\frac{d}{dt}[n_s(t)]. \tag{442}$$

The corresponding power spectral density is equal to,

$$S_{n_o}(\omega) = \frac{1}{A^2}S_{n_s}(\omega)|H(\omega)|^2 \tag{443}$$

where $H(\omega) = j\omega$ (time differentiator) and $|H(\omega)|^2 = \omega^2$. Then, we obtain

$$S_{n_o}(\omega) = \frac{\omega^2}{A^2}S_{n_s}(\omega) \tag{444}$$

Thus those spectral components at the higher frequencies are emphasized.

The bandwidth of the discriminator output is limited by a low-pass filter with a cutoff frequency of ω_m radians per second. We can express the power spectral density within this bandwidth by

$$S_{n_s} = [S_n(\omega - \omega_c) + S_n(\omega + \omega_c)]_{LP}. \tag{445}$$

If the noise at the discriminator is white,

$$S_n(\omega) = \eta/2, \tag{446}$$

Eq. (445) reduces to

$$S_{n_s}(\omega) = \eta, \tag{447}$$

and Eq. (444) becomes

$$S_{n_o} = \eta\omega^2/A^2. \tag{448}$$

The mean-square value of the output noise is

$$N_o = \overline{n_o^2(t)} = \frac{\eta}{\pi A^2}\int_0^{\omega_m}\omega^2 d\omega, \tag{449}$$

$$N_o = \frac{\eta\omega_m^3}{3\pi A^2}. \tag{450}$$

The mean carrier power is

$$S_c = A^2/2. \tag{451}$$

Comparing Eqs (450) and (451), we see that the output noise is inversely proportional to the mean carrier power in FM. This effect of a decrease in output noise power as the carrier power increases is called *noise quieting.* Combining Eqs. (438) and (450), we have,

$$\frac{S_o}{N_o} = \frac{3\pi A^2 k_f^2 \overline{f^2(t)}}{\eta\omega_m^3}. \tag{452}$$

If $f(t)$ is sinusoidal [that is, $f(t) = a\cos\omega_m t$, $\overline{f^2(t)} = a^2/2$ and $\beta = \Delta\omega/\omega_m = ak_f/\omega_m$], Eq. (452) becomes

$$\frac{S_o}{N_o} = \frac{3\pi A^2 k_f^2 a^2}{2\eta\omega_m^3} = \frac{3\pi A^2(\Delta\omega)^2}{2\eta\omega_m^3} = \frac{3\pi A^2\beta^2}{2\eta\omega_m} \tag{453}$$

For a comparison between FM and AM, we know that for AM, the noise at the demodulator output is equal to, [1]

$$(N_o)_{\text{AM}} = \frac{1}{\pi}\int_{\omega_c-\omega_m}^{\omega_c+\omega_m} \frac{\eta}{2}\,d\omega = \frac{\eta\omega_m}{\pi}. \tag{454}$$

---

[1]Recall that for AM, we have,

$$S_i(t) + n_i(t) = [A + f(t)]\cos\omega_c t + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

The envelop of this signal is

$$r(t) = \sqrt{\{[A + f(t)] + n_c(t)\}^2 + \{n_s(t)\}^2}$$

For high input signal-to-noise, we have,

$$r(t) \approx A + f(t) + n_c(t)$$

The detector output gives

$$S_o = \overline{f^2(t)}$$

For $f(t) = mA\cos\omega_m t$, we obtain,

$$S_o = m^2A^2/2$$

For $m = 1$, the signal power at the demodulator output for AM is,

$$(S_o)_{\text{AM}} = A^2/2 \tag{455}$$

Combining (453), (454) and (455), we obtain,

$$\left(\frac{S_o}{N_o}\right)_{\text{FM}} = 3\beta^2 \left(\frac{S_o}{N_o}\right)_{\text{AM}} . \tag{456}$$

The factor $3\beta^2$ reflects the effect of the noise quieting. We conclude that the output signal-to-noise ratio can be made much higher in FM than in AM by increasing the modulation index β. But, an increase in β also increases the bandwidth so that FM systems provide an improvement in signal-to-noise ratio at the expense of an increase in bandwidth. This means that the use of FM allows one to exchange bandwidth for signal-to-noise ratio. For example when $\beta = 5$ the output FM signal-to-noise ratio is 75 times that of an equivalent AM system but the bandwidth required is 8 times larger.

7.10 Threshold effect in FM

The threshold effect in FM in quite pronounced. It occurs when the signal and the noise levels are comparable. In this case, the phase angle $\gamma(t)$ may change by $\pm 2\pi$ in a very short period of time. As the output of the FM discriminator is proportional to $d\gamma/dt$, then impulselike noise spikes will occur in the output. The occurrence of occasional noise spikes are heard as individual clicks in FM discriminator output. For small input signal-to-noise ratio the FM may actually be inferior to AM.

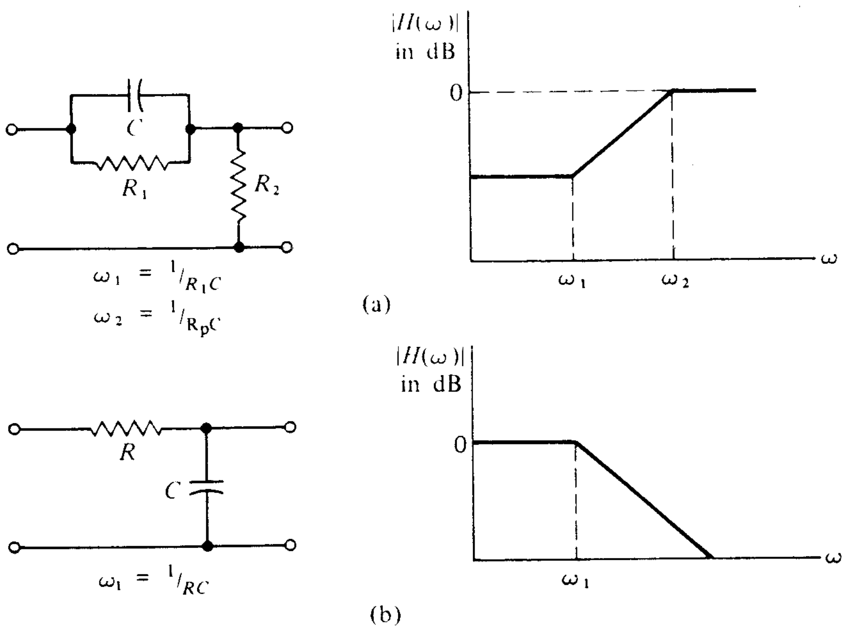

Figure 55: Example of preemphasis/deemphasis network combination

Signal-to-Noise Improvement Using Deemphasis

In the output of an **FM** demodulator, the noise power spectral density rises parabolically with frequency. Therefore (assuming a lowpass modulating signal), the noise power spectral density is largest in the frequency range where the signal spectral density is smaller. To remedy this situation, we emphasize the high-frequency components in the input signal at the transmitter. At the output of the **FM** demodulator in the receiver the inverse operation is performed to deemphasize the high-frequency components.

A simple and straightforward in choosing a preemphasize characteristic is to choose one that will yield a white noise spectral density after demodulation; this requires the $|H(\omega)|^2 = \omega^2$, suggesting the possibility $H(\omega) = j\omega$. This transfer function corresponds to a differentiator and we arrive at the surprising result that we now choose **PM** instead of **FM**. Obviously what we want here is a combination of both modulation methods.

What is needed, is a filter whose transfer function is constant for low frequencies and behaves like a differentiator at the higher frequencies. An example of an RC networks that approximates this type of response is shown in Fig. 55(a).

Assuming that the two networks are chosen properly, there will be no change in the signal. The improvement in the signal-to-noise ratio can be found by calculating the decrease in the noise power. The noise power spectral density at the output of the **FM** demodulator is

$$S_{n_0}(\omega) = \eta\omega^2/A^2$$

The transfer function of the deemphasis filter can be written as

$$H(\omega) = \frac{1}{1 + j\omega/\omega_1} \tag{457}$$

The mean-square value of the noise after the deemphasis filter is

$$\begin{aligned} N_0' &= \frac{1}{\pi}\int_0^{\omega_m} S_{n_0}(\omega)|H(\omega)|^2 d\omega \\ N_0' &= \frac{\eta}{\pi A^2}\int_0^{\omega_m} \frac{\omega^2}{1 + (\omega/\omega_1)^2} d\omega \end{aligned} \tag{458}$$

Without the deemphasis filter, the noise would be

$$N_0 = \frac{\eta}{\pi A^2}\int_0^{\omega_m} \omega^2 d\omega \tag{459}$$

Defining a noise improvement factor

$$\Gamma = N_0/N_0' \tag{460}$$

we find that

$$\Gamma = \frac{1}{3}\frac{(\omega_m/\omega_1)^3}{(\omega_m/\omega_1) - \tan^{-1}(\omega_m/\omega_1)} \tag{461}$$

For example, if $f_m = 15$ kHz, $f_1 = 2.1$ kHz, then $\Gamma = 13$ dB.