A New Block-Coded Modulation Scheme for Rayleigh Fading Channel, Soft Output Decoding Issues

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Abstract — **We investigate the application of the sum-product algorithm to the decoding of a q-ary Block-Coded Modulation (BCM) scheme which is based on extending the parity check equations of a binary block code to q-ary symbols. This is achieved by decomposing the code into a sub-code with an acyclic Tanner graph and its cosets which are represented by a trellis diagram. The combination of these two cycle-free graphs are used to develop an efficient soft output decoding algorithm for the given code.**

I. Introduction

Our objective has been to develop a soft output decoding method for the BCM codes proposed in [1]. The corresponding construction is based on extending good binary block codes from $GF(2)$ to Z_q. They assume a q-PSK signal constellation where the components of the q-ary code are directly mapped to the q-PSK points using an appropriate labeling. The extension of the binary linear code to a q-ary linear code is based on extending the parity check equations to $\{0, 1, ..., q-1\}$, mod q constraints. In this case, the encoder inputs $\log(q^k) = k \cdot \log(q)$ bits and outputs a length n codeword of elements of $Z_q = \{0, 1, ..., q-1\}$ which are each mapped to the points of a q-PSK constellation. The resulting scheme is $2n$-dimensional with a minimum time diversity of MTD= d, and Band-Width-Efficiency (BWE) of $\eta = k \cdot \log(q)/n = R \cdot \log(q)$ bits/2-D symbol ($R = k/n$ is the binary code rate). Therefore the optimality of these codes for a Rayleigh fading channel in terms of MTD and BWE is tantamount to that of the underlying binary block code. These schemes fall into the category of codes over rings and groups which recently have received a lot of attention among coding theorists [2].

II. Decoding

Consider the communication system in Figure 1 where k information bits $\vec{u} = (u_1, u_2, ..., u_k)$ are first encoded to n channel symbols $\vec{x} = (x_1, x_2, ..., x_n)$ and then transmitted through the channel which outputs $\vec{y} = (y_1, y_2, ..., y_n)$. Channel is memoryless such that each channel output y_i is only related to the channel input at the same time, namely, x_i, by, $y_i = \alpha_i \cdot x_i + n_i$, where α_i is 1 for an AWGN channel and Rayleigh-distributed for a Rayleigh fading channel. For a probability propagation decoding, one can construct a probabilistic model for the system by examining the encoding process and the channel. Then, a soft output decoding method

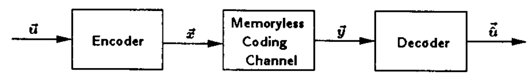

Figure 1: General Memoryless Coding Channel.

that maximizes $\Pr(x_i \mid \vec{y})$, $i = 1, 2, \ldots, n$, will minimize the symbol (q-ary) error probability. The sum-product algorithm provides an efficient way for calculating such marginals using a graphical representation of the code [3, 4].

In [1], a 2-level decoding method based on a generalization of the method discussed in [5] is proposed. To decode the constructed BCM scheme, the codebook which has a cyclic Tanner graph (TG) is decomposed to a sub-code with an Acyclic Tanner Graph (ATG), and its cosets. The significance of representing the code by an ATG is that, one can use a generalization of the well-known *Wagner* rule for their decoding. A composite Tanner graph-Trellis (TG-T) is used to represent the code structure.

On the other hand, it is well known that the probability propagation algorithms for soft output decoding, e.g., BCJR algorithm, can be used on a cycle-free graph to produce an exact probability calculation of code symbols. Examples of such cycle free graphs include a trellis representation and a cycle-free Tanner graph. The focus of the current article is to use the TG-T representation of the code (which is based on the combination of two cycle free graphical representations) to produce an efficient soft output decoding method.

The bit error performance of the resulting code construction will depend on the method used for the bit labeling of the underlying q-PSK constellation. We will present a discrete optimization method to optimize such labeling to minimize the resulting bit error probability.

References

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