Adaptive Long-Range Prediction of Mobile Fading

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***Abstract*— Long-range prediction of fading in mobile systems is the key element for many fading-compensation techniques. A linear approach, which is usually used to model the time evolution of the fading process, does not perform well for long-range prediction. In this article, we propose an adaptive channel prediction algorithm by using a novel state-space model for the fading process. Our simulations show that this algorithm significantly outperforms the conventional linear method, for both stationary and non-stationary fading processes, especially for long-range predictions[1].**

I. Introduction

The problem of channel fading modeling and prediction is addressed in this article. Channel fading prediction can be used to improve the performance of the telecommunication systems. Having some estimates of future samples of the fading coefficients enhances the performance of many tasks of the receiver or the transmitter, such as channel equalization, the decoding process of data symbols, antenna beamforming, and adaptive coding and modulation.

Many processes can be represented with a linear model, i.e. an auto-regressive moving-average (ARMA) model. However, linear models fail to show the true time behavior of the fading process. On the other hand, a linear model is easy to use, and has a low complexity. Therefore, an approximate low-order AR model is widely used which can capture most of the fading dynamics [1], [2]. For example, an MMSE linear predictor for mobile fading is proposed in [3].

A linear approach does not perform well for long-range predictions [4]. Also, a linear approach has a poor performance for high mobility channels as it is solely dependent on the correlation properties of the fading process. In this article, a novel channel model is utilized for the prediction of the channel fading. This model is adaptively updated and used in a Kalman filter to introduce a powerful fading prediction algorithm. The simulation results demonstrate the effectiveness of this approach in comparison with the conventional linear approach.

Assuming a two-dimensional isotropic scattering and an omni-directional receiving antenna, it is known that the autocorrelation function of the fading process can be written as [5]

$$R_h(t, t-\tau) = \frac{E[h(t)h^*(t-\tau)]}{\sigma_h^2} = J_0(2\pi f_d \tau), \qquad (1)$$

where f_d is the maximum doppler frequency, $J_0(\cdot)$ is the first-kind Bessel function of the zero order, and τ is the time difference. A Rayleigh fading process with the above correlation property is called Jakes fading [5]. We use a wide-sense stationary (WSS) version of the Jakes fading [6] (which uses 14 low-frequency sinusoids) to examine the performance of the underlying algorithms. Also, we generate a more realistic and non-stationary mobile fading using a ray-tracing approach. The algorithms are examined with this non-stationary fading as well.

The rest of this article is organized as follows. In the next section, first the fading assumptions and notations are introduced. Then, a general fading formulation is explained which is used to propose a state-space model. Section III describes both the linear and our adaptive prediction algorithms. Finally, the algorithms are compared in the simulation results.

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II. Channel Modeling

In this article, we consider a fading channel from a transmit antenna to a receive antenna. A single path flat fading is assumed for the channel. In the case that the path delay variations are not negligible in comparison to the symbol period, the same analysis could apply to each resolvable multipath component [7].

The channel fading coefficient, h_n, is zero mean, and has the variance of $\sigma_h^2 = 1$. The h_n is estimated at the receiver using the available pilots, training sequence, etc. This could be shown as $\overline{h}_n = h_n + v_n$, where $\overline{h}_n$ is the estimate of the channel fading, and v_n is the estimation error modeled as a zero mean Gaussian noise [8] with the variance σ_v^2. As an indicator for the estimation quality, the observation SNR is defined as $\mathrm{SNR}_z = \sigma_h^2/\sigma_v^2 = 1/\sigma_v^2$.

A. A General Fading Model

When the receiver, the transmitter, and/or the scatterers are moving, each scattered component undergoes a Doppler frequency shift given approximately by [9], [10]

$$f(k) = f_d \cos(\theta(k)) \qquad (2)$$

where $\theta(k)$ is the incident angle of the k'th component with respect to the motion direction of the mobile and f_d is the maximum doppler frequency defined as $f_d = \frac{V}{c} f_c$, where f_c is the carrier frequency, V is the mobile speed and c is the speed of light. Assuming N_{sc} scatterers, the complex envelop of the flat fading signal at the receiver is

$$h(t) = \sum_{k=1}^{N_{\mathrm{sc}}} a(k)\, e^{j(\omega(k)\, t + \phi(k))} \qquad (3)$$

where for the k'th scatterer, $a(k)$ is the (real) amplitude, $\phi(k)$ is the initial phase, and $\omega(k) = 2\pi f(k)$. The phase $\phi(k)$ can be absorbed in the amplitude as $\alpha(k) = a(k)\, e^{j\phi(k)}$. Assuming a sampling rate of $f_s = 1/T_s$, the fading samples can be written as

$$h_n = \sum_{k=1}^{N_{\mathrm{sc}}} \alpha(k)\, e^{j\omega(k) n T_s} \qquad (4)$$

where $h_n = h(nT_s)$, and n is the time index. In the realistic mobile environments, there are usually a few main scatterers which construct the fading signal [4]. Note that Jakes model is a special case of the general fading model, and is mathematically valid for a rich-scattering environment.

1) Estimation of the General Fading Parameters: Many channel fading models result from the statistical modeling of the fading shown in (4), including Jakes fading. However, the fading model (4) could be observed as a deterministic equation. Assuming $N_{\rm sc}$ scatterers, there are $2\,N_{\rm sc}$ unknown parameters to be determined in the model given in (4). Using $2\,N_{\rm sc}$ fading samples, an equation set could be solved to find $\omega(k)$ and $\alpha(k)$, for $k = 1, \ldots, N_{\rm sc}$ (refer to [11] for details). This approach uses only a few noisy measurements of the fading process, hence it could result in poor estimation of the parameters. Reference [12] uses an improved method to find the parameters. It finds the doppler frequencies using an ESPIRIT algorithm. Then, it solves a set of linear equations by the Least-Squares method to estimate the complex amplitudes. Alternatively, reference [13] uses Root-MUSIC method to find the doppler frequencies. In this paper, we propose a new method to find the parameters. The details of the method follows.

Assuming a constant scattering model, Fourier transform of the fading signal shown in (4) is

$$H(\omega) = \sum_{k=1}^{N_{\rm sc}} \alpha(k)\,\delta(\omega - \omega(k)) \tag{5}$$

This means that different scattering components are decoupled in the frequency domain and consequently could be estimated. The Fourier analysis provides an accurate estimation of $\omega(k)$'s if the doppler frequencies do not change significantly over the observation window. In practice, the $\omega(k)$'s change slowly with time. Therefore, an adaptive algorithm is used to track the fine changes of the doppler frequencies after the initial estimation. A sudden change in the frequencies may occasionally happen, for example, if the mobile path abruptly changes. In this case, the frequencies should be estimated again.

As it can be seen in (5), the $\alpha(k)$'s also may be estimated from the Fourier analysis. However, $\alpha(k)$'s usually change faster than $\omega(k)$'s as mobile moves around and the scattering environment changes. These changes may even be significant over a short observation window. Therefore, the estimates of the $\alpha(k)$'s should be kept up-to-date by using the most recent fading samples. Knowing $\omega(k)$'s, we use a Kalman filter to efficiently follow the $\alpha(k)$ variations.

B. State-Space Representation

A time evolution model is a useful tool for the prediction of a process. A well-known form of an evolution model is known as the state-space model, which can be written as

$$\begin{cases} \mathbf{x}_n = \mathbf{A}_n\,\mathbf{x}_{n-1} + \mathbf{q}_n \\ z_n = \mathbf{m}_n\,\mathbf{x}_n + v_n \end{cases} \tag{6}$$

where $\mathbf{x}_n$ is an $N_{\rm ray} \times 1$ state vector at time n, $\mathbf{A}_n$ is an $N_{\rm ray} \times N_{\rm ray}$ matrix which controls the transition of the state vector in time, and $\mathbf{q}_n$ is a noise vector with the covariance $\mathbf{Q} = E[\mathbf{q}_n\,\mathbf{q}_n^H]$, which represents the model error. The $\mathbf{m}_n$ is known as the measurement matrix, v_n is the observation noise, and z_n is the system output. In cases of interest, $\mathbf{A}_n$, $\mathbf{Q}$ and $\mathbf{m}_n$ are usually constant or slowly time-varying. A well-known state-space representation of an AR model can be found in [11]. We propose a new state-space model for the mobile fading in the next section.

C. Proposed State-Space Model

Considering the general fading process given in (4), we propose the following state-space model:

$$\mathbf{A}_n = diag\left[e^{j\omega_n(1)\,T_s},\, e^{j\omega_n(2)\,T_s},\, \ldots,\, e^{j\omega_n(N_{\rm ray})\,T_s}\right] \tag{7}$$

and

$$\mathbf{m}_n = [1,\, 1,\, \ldots,\, 1]_{1\times N_{\rm ray}}, \tag{8}$$

where $N_{\rm ray}$ is the model order which is the number of the assumed scatterers here (ideally, $N_{\rm ray} = N_{\rm sc}$). The z_n in (6) is substituted with the available measurement of the fading sample, i.e., $z_n = \overline{h}_n$. Therefore, the state vector $\mathbf{x}_n$ consists of the complex envelops of the scattering components. A Kalman filter can utilize the state-space model to estimate the state $\mathbf{x}_n$ at each time.

III. Prediction Algorithms

A. The Linear Prediction Algorithm (LP)

A linear predictor of order $N_{\rm AR}$, with the prediction depth of D is shown as follows

$$\hat{h}_n = a_D\,\overline{h}_{n-D} + \cdots + a_{D+N_{\rm AR}-1}\,\overline{h}_{n-D-N_{\rm AR}+1} \tag{9}$$

$$= \sum_{i=0}^{N_{\rm AR}-1} a_{D+i}\,\overline{h}_{n-D-i}. \tag{10}$$

Minimizing the mean square error (MSE) provides the prediction coefficients, i.e.,

$$\min E\left[\left|h_n - \hat{h}_n\right|^2\right] = \min_{\mathbf{a}} E\left[\left|h_n - \left(\sum_{i=0}^{N_{\rm AR}-1} a_{D+i}\,\overline{h}_{n-D-i}\right)\right|^2\right]. \tag{11}$$

where $\mathbf{a} = [a_D\, a_{D+1}\, \cdots\, a_{D+N_{\rm AR}-1}]^T$ is the prediction coefficients vector. The solution of (11) can be found by solving the Yule-Walker equations [14], resulting in

$$\mathbf{a} = \mathbf{R}^{-1}\,\mathbf{r_0}, \tag{12}$$

where $\mathbf{R} = [R_{ji}]_{N_{\rm AR}\times N_{\rm AR}}$ is the data correlation matrix and

$$R_{ji} = E[\overline{h}_{n-D-i+1}\,\overline{h}^*_{n-D-j+1}], \tag{13}$$

and $\mathbf{r_0} = [r_{0j}]_{N_{\rm AR}\times 1}$ where

$$r_{0j} = E[h_n\,\overline{h}^*_{n-D-j+1}]. \tag{14}$$

For Jakes fading, the problem is analytically solved, resulting in

$$R_{ji} = \sigma_h^2\,J_0(2\pi f_d T\,|j-i|) + \sigma_v^2\,\delta(j-i), \tag{15}$$

$$r_{0j} = \sigma_h^2\,J_0(2\pi f_d T\,|j+D-1|) + \sigma_v^2\,\delta(D+j-1), \tag{16}$$

where

$$\delta(k) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases} \tag{17}$$

In practical situations, $\mathbf{R}$ and $\mathbf{r_0}$ are estimated using the fading samples. There are other methods to find the prediction coefficients as well. In a non-stationary environment, the coefficients should be frequently updated to follow the model variations.

Here, the linear coefficients are estimated using a Least-Squares approach and solving the equations by the Levinson-Durbin recursion. This method provides the best results for the linear prediction in our simulations. These estimates are frequently updated.

B. Proposed Fading Prediction Algorithm (KF)

We propose an adaptive fading prediction algorithm here. Fig. 1 shows the flowchart of the algorithm, and a description of the main blocks follows.

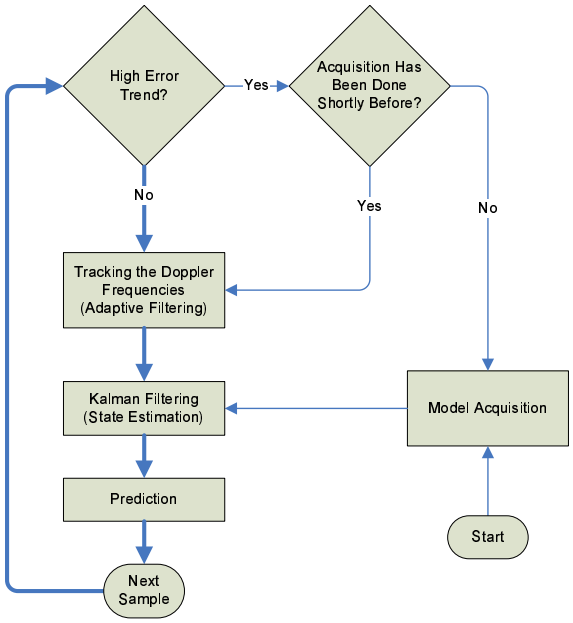

Fig. 1. Block Diagram of our Prediction Algorithm (KF)

$\mathbf{Q}$	The covariance matrix of the model noise
z_n	The observation sample
$\mathbf{x}_{n\|n-1}$	The *a priori* estimate of the state $\mathbf{x}_n$ (i.e., the estimation of the state at the time n given the observations upto the time $n-1$)
$\mathbf{x}_{n\|n}$	The *a posteriori* estimate of the state $\mathbf{x}_n$ (i.e., the estimation of the state at the time n given the observations upto the time n)
$\mathbf{P}_{n\|n-1}$	The covariance matrix of the *a priori* error
$\mathbf{P}_{n\|n}$	The covariance matrix of the *a posteriori* error

TABLE I
VARIABLES USED IN KALMAN FILTER

1) Kalman Filtering: Kalman Filtering is now commonly used in communication systems (for example, see [1], [15]). Assuming a state-space model, Kalman filter efficiently estimates the state vector $\mathbf{x}_n$ using the observation samples. The estimation of the state vector given the observations at the time n, shown as $\mathbf{x}_{n|n}$, is optimal in the MMSE sense. This state vector is used to predict the future samples of the fading signal later.

Table I defines the variables used in the Kalman equations, which follow:

Prediction part:

$$\mathbf{x}_{n|n-1} = \mathbf{A}_n\,\mathbf{x}_{n-1|n-1} \quad (18)$$

$$\mathbf{P}_{n|n-1} = \mathbf{A}_n\,\mathbf{P}_{n-1|n-1}\mathbf{A}_n^T + \mathbf{Q} \quad (19)$$

Update part:

$$\mathbf{x}_{n|n} = \mathbf{x}_{n|n-1} + \mathbf{k}_n\left(z_n - \mathbf{m}_n\,\mathbf{x}_{n|n-1}\right) \quad (20)$$

$$\mathbf{P}_{n|n} = \mathbf{P}_{n|n-1} - \mathbf{k}_n\,\mathbf{m}_n\,\mathbf{P}_{n|n-1} \quad (21)$$

where

$$\mathbf{k}_n = \mathbf{P}_{n|n-1}\,\mathbf{m}_n^H\left(\mathbf{m}_n\,\mathbf{P}_{n|n-1}\,\mathbf{m}_n^H + \sigma_v^2\right)^{-1}. \quad (22)$$

f_c	2.15 GHz
f_s	1500 Hz
SNR_z	10 dB

TABLE II
SIMULATION PARAMETERS

2) Model Acquisition: The current parameters for the fading model are estimated according to Section II-A.1. We apply the Fourier method to estimate the $\omega(k), k = 1, \ldots, N_{\mathrm{ray}}$ by performing FFT over an observation window of N_{win} recent samples, $H = \mathrm{FFT}[h]$. We have used an FFT length of $N_{\mathrm{FFT}} = 2\,N_{win}$ to increase the frequency resolution. Therefore, each sinusoid can be projected upto 3 samples in H. First the peak of H is found, and then $\omega(1)$ is calculated by averaging over the amplitudes of the three adjacent frequency samples. An initial estimate of $\alpha(1)$ is also obtained in this way. Other $\omega(k)$ and $\alpha(k)$'s are found by continuing this procedure.

Acquisition could be done frequently to keep the doppler frequency estimates updated. However, to decrease the required computations, it may be done only if the error trend exceeds a threshold (see Section III-B.5 for details). Furthermore, the algorithm does not allow two consecutive acquisitions to happen too close to each other because after each acquisition, other blocks of the algorithm should have enough time to catch up with the new model parameters.

3) Tracking: An adaptive algorithm is used to track the fine changes of the doppler frequencies. Using a gradient-based approximate, the following LMS algorithm can be applied

$$w_{n+1}(k) = w_n(k) + \mu\,\mathbf{x}_{n|n}^H(k)\,\mathbf{m}_n^H(k)\,e_n, \quad (23)$$

where

$$e_n = z_n - h_{n|n}, \quad (24)$$

$$h_{n|n} = \mathbf{m}_n\,\mathbf{x}_{n|n}. \quad (25)$$

4) Prediction: Given the current state $\mathbf{x}_n$, which carries all the information about the past, the future channel state should be predicted. It has been shown in [16] that given the state transition matrix $\mathbf{A}_n$, the MMSE estimate of the D-step prediction is

$$\mathbf{x}_{n+D|n} = \mathbf{A}_n^D\,\mathbf{x}_{n|n}. \quad (26)$$

where $\mathbf{x}_{n+D|n}$ is the estimate of the state vector at the time $n+D$ given the observations until time n. Hence, the predicted fading sample can be obtained as $h_{n+D|n} = \mathbf{m}_n\,\mathbf{x}_{n+D|n}$.

5) Calculation of the Error Trend: We use an exponential window for calculation of the error trend from the sample errors, as follows

$$E_{n+1} = \lambda\,E_n + (1-\lambda)\,|e_n|^2, \quad (27)$$

where λ is the forgetting factor ($0 \ll \lambda < 1$).

IV. SIMULATION RESULTS

Table II shows the simulation parameters. The two prediction algorithms (LP and KF) are compared here, with respect to the average MSE versus the prediction depth. The results are reported for various linear orders N_{AR}, and various scattering orders N_{ray}, respectively (N_{ray} is an estimate of N_{sc} in (4)). Fig. 2 shows the results for the Jakes fading for the mobile speeds of $V = 25$ and $V = 100$ kmph. It is observed that KF significantly outperforms LP if N_{ray} is large enough (here, for $N_{\mathrm{ray}} \geq 8$), while LP fails at high prediction depths regardless of the linear order.

Jakes fading is a valid model for a rich scattering area. Furthermore, because the Jakes fading is stationary, it can not model the

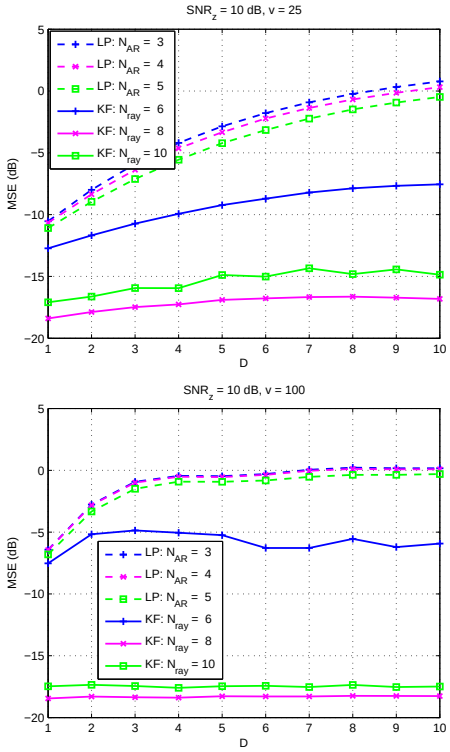

Fig. 2. Comparison of MSE versus prediction depth for Jakes fading at $V = 25$ and $V = 100$

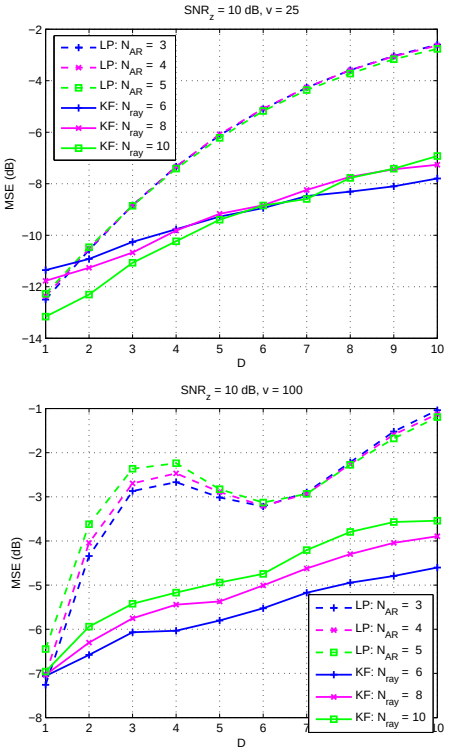

Fig. 3. Comparison of MSE versus prediction depth for (non-stationary) RT fading at v=25 and v=100

changes in the scattering environment. We want to test the algorithms with a more realistic fading signal. So we use the Ray-Tracing simulation environment explained in [11] to generate the "RT fading". The mobile is randomly moving vertically and horizontally in the scattering area and experiences different combination of signal rays. At each point of the mobile path, it undergoes a different doppler frequency and a different signal power for each ray. Therefore, the generated fading can closely resemble the fading in a real mobile environment.

Fig. 3 shows the results for RT fading for $V = 25$ and $V = 100$ kmph. It is observed that KF can always outperform LP. As RT fading represents not a very rich scattering environment, it is observed that increasing N_{ray} does not necessarily improves the performance. Note that LP is sensitive to the linear order at high mobile speeds. In fact, it is observed in our simulations that a linear model is not dependable for higher mobile speeds because the pattern of the performance fluctuation follows the correlation properties of the fading, i.e., a lower correlation results in a higher MSE. In conclusion, the simulations show that the proposed prediction algorithm can perform very well in real mobile environments, and it is significantly more efficient than the linear algorithm.

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