A New Code for High-Rate Differential Space-Time Coding

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In [1], a differential space-time modulation scheme (based on unitary matrices) are presented that can be used without knowledge of the Channel State Information (CSI) at the receiver. This is suitable for mobile communication applications where fading coefficients change rapidly with time. All previous differential space-time codes have been designed to produce full diversity. However, by sacrificing the diversity, one can increase the rate per channel use of the space-time modulation [2]. For very high values of Signal-to-Noise-Ratio (SNR), the performance of multiple-antenna systems is determined by diversity product. However, maximizing the diversity product is not the appropriate criterion for high-rate communications operating in practical ranges of SNR for the case of using multiple receive antennas. Indeed, for reasonable range of SNR values, when the number of receive antennas is greater than or equal to the number of transmit antennas, the diversity sum (minimum squared Euclidean distance) is more important than the diversity product.

In this paper, we present a practical sub-optimum solution to the problem of high-rate non-coherent communication over a Multiple-Input Multiple-Output (MIMO) fading channel with M transmit antennas. We use a differential transmission scheme with unitary code-words in the form of $\mathbf{V}=\mathbf{D}_1\mathbf{A}\mathbf{D}_2$ where $\mathbf{D}_1$ and $\mathbf{D}_2$ are diagonal matrices with unit-norm elements and $\mathbf{A}$ is a fixed unitary matrix. To have a good distance distribution for the code-words, we show that $\mathbf{A}$ must have equal-norm elements [3], e.g., $\mathbf{A}$ can be selected as the Vandermonde matrix with M'th roots of unity, i.e.,

$$\mathbf{A}=\frac{1}{\sqrt{M}}\begin{bmatrix} 1 & 1 & 1 & .... & 1 \\ 1 & w & w^2 & & w^{M-1} \\ 1 & w^2 & w^4 & & w^{2(M-1)} \\ ... & & & & \\ 1 & w^{M-1} & w^{2(M-1)} & & w^{(M-1)(M-1)} \end{bmatrix} \quad \text{where} \quad w=e^{2\pi i/M}$$

If the elements of $\mathbf{D}_1$ and $\mathbf{D}_2$ are chosen from the set of K'th complex roots of unity, then the code-words of the code have elements which are points of a K-PSK constellation. The rate of the code is $2M-1$ symbols per matrix or $(2M-1)/M$ symbols per channel use [3]. We show that the minimum squared Euclidean distance (diversity sum) of the resulting scheme is M times the minimum squared Euclidean distance of the underlying base constellation [3].

Maximum likelihood (ML) decoding of differential codes reported in the literature is, in general, based on exhaustive search. However, for high rates, exhaustive search is not feasible. For the proposed differential scheme, one can have a sub-optimal decoding by decoding the first row separately and then decoding the remaining rows at once [3]. As a simpler method, with a loss in performance, one can decode the first row by nulling and canceling (similar to [4]) and then decode the other rows separately [3]. Also, in the special case of M=2, which is of special practical interest, one can have an efficient exact *ML* decoding by using a different addressing:

$$\mathbf{A}(\theta_1,\theta_2,\theta_3)=\frac{1}{\sqrt{2}}\begin{bmatrix} e^{i(\theta_1+\theta_3)} & e^{i\theta_2} \\ e^{i(-\theta_2+\theta_3)} & -e^{-i\theta_1} \end{bmatrix}$$

Using this addressing, we show that the ML decoding complexity is K times the decoding complexity of Alamouti code.

For M=2, unitary matrices have 4 degrees of freedom and for high rates, we can construct more efficient codes by using the following structure:

$$\mathbf{A} = \begin{bmatrix} ae^{i(\theta_1+\theta_3)} & be^{i\theta_2} \\ be^{i(-\theta_2+\theta_3)} & -ae^{-i\theta_1} \end{bmatrix}$$

where $a^2+b^2=1$. Figure 1 shows that the proposed differential scheme considerably outperforms the best high-rate differential schemes reported in [5], [6]. For the two-layer code, once can send one bit by choosing a=0.895 and b=1.095 (or b=0.895 and a=1.095) and respectively 4, 4, 3 bits are carried by $\theta_1, \theta_2, \theta_3$. We have chosen a and b such that the difference between constellation points is maximized [3].

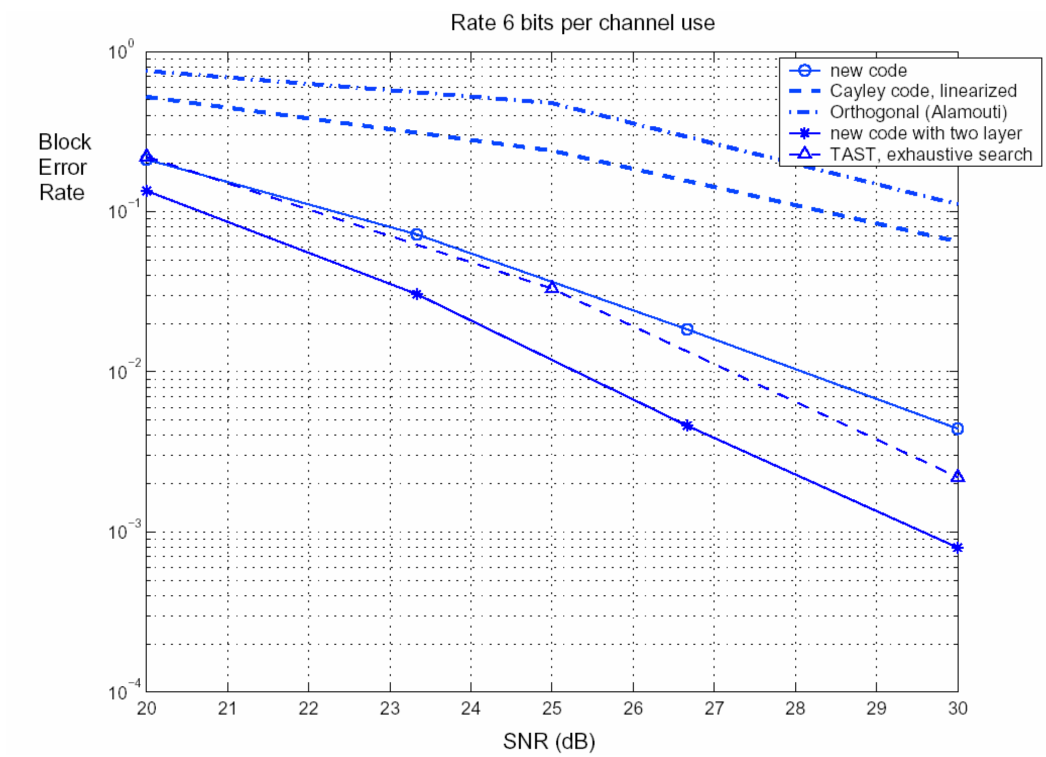

Figure 1. Performance comparison with other differential methods for M=2 transmit and N=2 receive antennas and rate 6 bits per channel use.

References

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