Acyclic Tanner Graphs and Two-level Decoding of Linear Block Codes[1]

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Abstract — **For a linear block code C, the Wagner rule together with a subcode having an acyclic Tanner graph are applied to decode C in a two-level soft-decoding technique.**

I. SUMMARY

A Tanner graph [1] (TG) representing a linear block code with check matrix $H = [h_{ij}]$ is a bipartite graph in which one of the two sets of vertices denote the parity nodes, the rows of H, and the other set denote the symbol nodes, the columns of H. A parity node u_i is connected to a symbol node v_j iff $h_{ij} \neq 0$. The single parity check codes can be easily decoded by using the Wagner rule [2] where a bit-by-bit hard decision of the received channel output is considered as the transmitted codeword unless the parity is not satisfied in which case the least reliable bit is flipped. The corresponding TG has a single parity node adjacent with all symbol nodes.

It is natural to think of a generalization of the Wagner rule on codes with an Acyclic Tanner Graph (ATG) by focusing on one of the parity nodes, to be called the *root parity*. This, together with coset decoding techniques, lead us to the application of the Wagner rule on codes having TGs with cycle. There are obviously two main parameters involved in this approach. Given a linear block code C, one first needs to determine a relatively large subcode C_0 of C with ATG to reduce the number of cosets. Our experience has shown that always the largest subcode results in the minimum overall complexity. Another important property of C_0 is the structure of the corresponding minimal Tanner graph (MTG). To reduce the decoding complexity it is essential: (i) to have the number of branches leaving the root parity node as large as possible, and (ii) branches to be as similar as possible. We refer to this property as the uniformity of the Tanner graph. The class of product codes $(n, n-1, 2) \otimes (m, 1, m)$, called *uniform generalized single parity* (UGSP) codes, satisfy this uniformity condition fully. Other than the aforementioned two factors, the method to deal with the cosets of C_0 for finding the most overall reliable codeword is another important issue.

Suppose $M = M_0 + M_c$ where M_c is a generator matrix for the space of coset representatives, and $M_0^{\perp}$ stands for a generator matrix of the dual code $C_0^{\perp}$. A coset $C_0 + c$ is specified by $C_0 + c = \{x + c \mid M_0^{\perp} x = 0\}$. If $M_0^{\perp} c = b$, then $M_0^{\perp}(x + c) = M_0^{\perp} x + M_0^{\perp} c = b$. Therefore, the TG corresponding to the coset $C_0 + c$ is the same as that of C_0 except for the values of the parity nodes, i.e., the sequence of zeros for the parity nodes of C_0 has to be replaced by $b = M_0^{\perp} c$. It follows that any two cosets are distinguished by the values of their corresponding sequences of parity nodes. The set of all such parity node sequences is a vector space called *the parity space* corresponding to C_0 and is denoted by $PS(C_0)$.

The parity space is given by the generator matrix $M_{PS} := M_0^{\perp} \times M_c$. The ATG of C_0, $G_T(C_0)$, together with $T_{PS}(C)$, a minimal trellis diagram of the parity space in which the root parity is ignored, is considered as a graphical representation of C, called the Tanner graph-trellis (TG-T) of C.

If the root parity in $G_T(C_0)$ is of degree m then we may think of T_{PS} as an m-section trellis diagram. The edge label set at each section of T_{PS} is generated by the parity sequences of the corresponding branch of $G_T(C_0)$. If C is over field F_q, then any element of F_q can be the contribution of each edge e of T_{PS} to the root parity. Therefore, an edge e can be thought of as a q-tuple $e := (e_1, e_2, \cdots, e_q)$ where e_i is the version of e which provides the root parity with contribution $i \in F_q$. To each version e_i of e, $1 \leq i \leq q$, a confidency value is associated. In the q-tuple of confidency values, the maximum and the differences of the maximum with the rest are determined. The differences are referred to as the confidency deviations.

All the edges lying on a path of T_{PS} are originally considered with their preferred version, the version with maximum confidency, unless the root parity is not satisfied in which case a group of edges of the path are changed with their other versions such that the changes causes the least total confidency deviation and satisfies the root parity. One way to implement this procedure is to substitute each edge of T_{PS} by a q-tuple $e = (e_1, e_2, \cdots, e_q)$ and then ignore the paths that do not satisfy the root parity. The so obtained trellis, denoted $TT_{PS}(C)$ or simply TT_{PS}, is referred to as the *twisted trellis* of parity space.

We call an n-section trellis diagram T regular if: (i) The number of vertices of T is the same for all time indices, except for the initial and final time indices that have a single vertex. (ii) Each section of the trellis is a complete bipartite graph. (iii) The set of label of edges leaving or entering any vertex of a section of T, except for the first and last sections, is the whole set of edge labels of that section. One can simply apply the Viterbi algorithm on TT_{PS} and find the optimal path. However, we show that if TT_{PS} consists of disjoint regular subtrellises then it can be decoded more efficiently.

II. RESULTS

The Reed-Muller codes, Hamming codes, Hexacode, extended Golay codes, $(32, 16)$ quadratic residue code have been studied and for each of them the best maximal acyclic subcodes, from decoding complexity point of view, has been determined. The given systematic decoding technique has unified the best previously known decoding methods for these codes. Our approach has revealed the importance of the acyclic UGSP codes.

REFERENCES

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