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AN EFFICIENT MMSE SOURCE DECODER FOR NOISY CHANNELS

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ABSTRACT

Exploiting the residual redundancy in a source coder output stream during the decoding process has been proven to be a bandwidth efficient way to combat the noisy channel degradations [2]-[7]. Researchers have recently developed techniques to employ this redundancy to either assist the channel decoder for improved performance or design effective source decoders. However, the dominant method used for modeling the redundancy is a first order Markov model which fails to encapsulate all the remaining redundancies. In this work, we present a general solution for instantaneous MMSE reconstruction of a source over noisy channel when the redundancy is exploited in the form of a γ-order Markov model ($\gamma \geq 1$). Next, we extend it to the case where a delay of δ, $\delta > 0$ is allowed in the decoding process. This solution is optimized to minimize the computational complexity. Numerical results are presented which demonstrate the efficiency of the proposed algorithms.

1. INTRODUCTION

An important result of the Shannon's celebrated paper [1], is that the source and channel coding operations can be separated without any loss of optimality. This has been the basic idea of enormous research endeavors in separate treatment of source and channel coders. However, in practise, there is *residual redundancy* [2] in the output of the source coders which is due to their suboptimality caused by e.g. a constraint on complexity or delay. As Shannon stated, this redundancy can be used at the receiver to enhance the system performance [1].

Recently, researchers have used the residual redundancy for enhanced channel decoding e.g. [5] or for effective source decoding e.g. [6] and [7]. The problem is formulated in the form of a *Maximum A Posteriori Probability* detection e.g. [3] and [5] or a *Minimum Mean Squared Error* estimation problem e.g. [4]. Several speech error concealment solutions based on MMSE source decoding are presented in [6] and [7]. In [7] a solution to reduce the complexity of the MMSE decoding and its application for error concealment in IS-641 CELP is presented.

The recent literature clearly demonstrate the benefit of exploiting the residual redundancies in reconstructing the data received over noisy channels. However, it has limited itself to modeling the redundancy with a first-order Markov model, which does not necessarily encapsulate all the remaining redundancy. In this work, we present a general solution for instantaneous MMSE reconstruction of a source over noisy channels when the redundancy is exploited in the form of a γ-order Markov model ($\gamma \geq 1$). Next, we extend it to the case where a delay of δ, $\delta > 0$, is allowed in the decoding process. This solution is optimized to minimize the computational complexity. The key idea is based on modeling the source encoder output and the a priori information with a trellis structure.

2. SYSTEM OVERVIEW

The block diagram of the system is shown in Figure 1. The source encoder $\mathcal{E}$ is a mapping from an N-dimensional Euclidean space, $\mathcal{R}^n$, into a finite index set $\mathcal{J}$ of M elements. It is composed of two components: the quantizer Q and the index generator $\mathcal{I}$. The quantizer maps the input sample $\mathbf{X} \in \mathcal{R}^N$ to one of the reconstruction points or *codewords* in $\mathcal{R}^{N\ 1}$. The index generator is a mapping from the code-book $\mathcal{C}$ to the index set $\mathcal{J}$. The bitrate of the quantizer r is given by $\lceil(\log_2 M)\rceil$ bits/symbol (or $\lceil(\log_2 M)\rceil/N$ bits/dim). At the receiver, for each transmitted r-bit index (symbol) I_n,

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[1]Capital letters (e.g. I) represent random variables, while small letters (e.g. i) is a realization. For simplicity $P(I = i)$ is represented by $P(I)$. Vectors are shown bold faced (e.g. $\mathbf{X}$). Lower index indicate time instant. Upper index in parenthesis indicate components of a vector or bit positions representing an integer value.

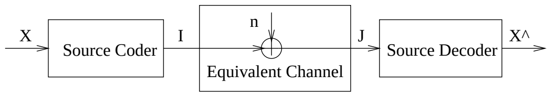

Fig. 1. Overview of the system

a vector J_n with r components is received. The reconstructor (source decoder) maps this information to an output sample $\hat{\mathbf{X}}$. The noisy channel together with the channel encoder and decoder is replaced by a channel model. We assume that the equivalent channel between I_n and J_n is memoryless, i.e.,

$$P(J_n|I_n) = \Pi_{m=1}^{r} P(J_n^{(m)}|I_n^{(m)}), \qquad (1)$$

where $I_n^{(m)}, J_n^{(m)}, m = 1, \ldots, r$ are the bit components of I_n and J_n respectively. For a sequence of transmitted symbols, $\underline{I}_n = [I_1, I_2, \ldots, I_n]$ over a memoryless channel, we have,

$$P(\underline{J}_n|\underline{I}_n) = \Pi_{k=1}^{n} P(J_k|I_k). \qquad (2)$$

The equivalent channel considers AWGN and produces disturbed symbols J_n in the form of soft outputs characterized by the instantaneous values of $p(J_n^{(m)}|I_n^{(m)})$ and Equation (1).

3. MMSE DECODING EXPLOITING THE RESIDUAL REDUNDANCIES

Consider the case where due to the sub-optimality of the source coder there is a residual redundancy in its output stream. This redundancy is in the form of a non-uniform distribution or memory in the sequence of the transmitted symbols. For effective reconstruction of the transmitted information at the receiver, our objective is to exploit this redundancy by designing the source decoder, such that it produces the Minimum Mean Squared Error estimate of the source sample $\mathbf{X}_n$ given the received sequence $\underline{J}_n = [J_1, J_2, \ldots, J_n]$. Based on the fundamental theorem of Estimation Theory, this is given by,

$$\hat{\mathbf{x}}_n = E[\mathbf{X}_n|\underline{J}_n] \qquad (3)$$

which minimizes the expected squared error of estimation, $E[\tilde{\mathbf{X}}'_n \tilde{\mathbf{X}}_n|\underline{J}_n]$ where, $\tilde{\mathbf{X}}_n = \mathbf{X}_n - \hat{\mathbf{X}}_n$. The Equation (3), is simplified to

$$\hat{\mathbf{x}}_n = \sum_{I_n \in \mathcal{J}} E[\mathbf{X}_n|I_n] P(I_n|\underline{J}_n) \qquad (4)$$

which describes the MMSE estimate in terms of the weighted average of LBG codewords [9]. The weights are the probability of receiving the corresponding index given the received sequence $\underline{J}_n$.

Let us assume that the encoded sequence contain residual redundancy in the form of a Markov model of order γ, $\gamma \geq 0$. For $\gamma = 0$, i.e. when there is no memory, the Equation (4) collapses to the basic MMSE reconstruction rule,

$$\hat{\mathbf{x}}_n = \sum_{I_n \in \mathcal{J}} E[\mathbf{X}_n|I_n] P(I_n|J_n) \qquad (5)$$

where the probability $P(I_n|J_n) = C.P(I_n).P(J_n|I_n)$ contain information about channel condition and the source a priori probabilities. If we assume that the source coder produces equally probable symbols, the term $P(I_n|J_n)$ in Equation (5) will be replaced with $P(J_n|I_n)$. For $\gamma \geq 1$, the probabilities in Equation (4) can be calculated as follows. First, we model the source I_n by a trellis structure. In this structure, the states at time n corresponds to the ordered set

$$S_n = (I_{n-\gamma+1}, I_{n-\gamma+2}, \ldots, I_{n-1}, I_n), \qquad (6)$$
$$I_{n-k} \in \mathcal{J}, 0 \leq k < \gamma$$

Hence, there are M^γ states in each time step (stage), $S_n \in \mathcal{J}^\gamma$. Each branch leaving the state at time step n corresponds to one particular symbol I_{n+1}. Therefore, there are M branches leaving each state. Each branch is identified by the pair (S_n, S_{n+1}) of the two states that the branch connects together. Having defined the trellis structure as such, there will be one a priori probability $P(I_{n+1}|S_n)$ corresponding to each branch which characterizes the γ-order Markov property of the source. The states now form a first-order Markov sequence, i.e. for the sequence of states $S_1, S_2, \ldots, S_n$ that form an arbitrary path of the trellis, we have,

$$P(S_n|S_{n-1}, S_{n-2}, \ldots, S_1) = P(S_n|S_{n-1}). \qquad (7)$$

Using this property, the memoryless assumption of the channel (see Eq. (1) and (2)), and the BCJR algorithm [8], the probability of a particular state S_n given the observed sequence $\underline{J}_n$ is calculated as follows,

$$P(S_n|\underline{J}_n) = C.P(J_n|I_n) \,. \sum_{S_{n-1} \rightarrow S_n} P(I_n|S_{n-1}) P(S_{n-1}|\underline{J}_{n-1}) \qquad (8)$$

where the summation is over a subset of M states in time step $n-1$ which are connected to the state S_n and C is a factor which normalizes the sum of probabilities to one. The probability of a particular sample used in Equation (4) is then given by,

$$P(I_n|\underline{J}_n) = \sum_{I_{n-1}} \sum_{I_{n-2}} \cdots \sum_{I_{n-\gamma+1}} P(S_n|\underline{J}_n) \qquad (9)$$

The Equations (4), (8) and (9) provide the instaneous MMSE estimate of source samples given the history of the received channel outputs when the residual redundancy is captured by a γ-order Markov model.

For the cases, where a delay of δ is allowed in the decoding process, we can also benefit from the information in the future channel outputs while reconstructing a particular sample. In this case, the MMSE estimate is given by,

$$\hat{\mathbf{x}}_n = \sum_{I_n \in \mathcal{J}} E[\mathbf{X}_n | I_n] P(I_n | \underline{J}_{n+\delta}). \qquad (10)$$

To find the symbol probabilities $P(I_n | \underline{J}_{n+\delta})$ in Equation (10), we can use *any of the state probabilities* $P(S_{n+m}|\underline{J}_{n+\delta})$ *as long as* S_{n+m} *includes* I_n *or equivalently,* $0 \le m \le \gamma - 1$. The probabilities of states are calculated as follows,

$$\begin{aligned} & P(S_{n+m}|\underline{J}_{n+\delta}) = \\ & \quad C \,.P(S_{n+m}|\underline{J}_{n+m}) \,.\, P(\underline{J}_{n+\delta}^{n+m+1}|S_{n+m}) \qquad (11) \\ & \quad 0 \le m \le \delta \end{aligned}$$

where $\underline{J}_{n+\delta}^{n+m+1} = [J_{n+m+1}, J_{n+m+2}, \ldots, J_{n+\delta}]$. The above equation can be computed using the well known Forward Backward equation [8]. The first term is referred to as the forward equation and is denoted by $\alpha_{n+m}(S_{n+m})$. This is given in Equation (8) and can be rewritten as,

$$\begin{aligned} \alpha_n(S_n) = & \; P(J_n|I_n) \,. \qquad (12) \\ & \textstyle\sum_{S_{n-1} \rightarrow S_n} P(I_n|S_{n-1}).\alpha_n(S_{n-1}) \end{aligned}$$

The second term in Equation (11) is referred to as the backward equation and is denoted by $\beta_{n+m}(S_{n+m})$. This can be calculated recursively as follows,

$$\begin{aligned} \beta_n(S_n) = & \sum_{I_{n+1} \in \mathcal{J}} P(J_{n+1}|I_{n+1}) \,. \\ & P(I_{n+1}|S_n) \,.\, \beta_{n+1}(S_{n+1}) \qquad (13) \end{aligned}$$

Now, the Equation (11) can be rewritten as,

$$\begin{aligned} & P(S_{n+m}|\underline{J}_{n+\delta}) = \qquad (14) \\ & \quad C \,.\, \alpha_{n+m}(S_{n+m}) \,.\, \beta_{n+m}(S_{n+m}) \\ & \quad 0 \le m \le \delta \end{aligned}$$

where,

$$\beta_{n+\delta}(S_{n+\delta}) = 1, \quad \forall S_{n+\delta} \in \mathcal{J}^{\gamma} \qquad (15)$$

In Equation (14), if the number of computations required for the forward and backward recursions per time step is denoted by NC_{fwd} and NC_{bwd} respectively, we have,

$$NC_{fwd} = (2M+1)\, M^{\gamma} \qquad (16)$$
$$NC_{bwd} = 3(\delta - m)\, M^{\gamma+1} \qquad (17)$$

where $\delta - m$ is the number of backward recursions required per time step. To minimize the overall computational burden, we solve the following for the optimum value of m,

$$\begin{aligned} & Minimize \quad NC_{bwd} = 3(\delta - m)\,.\,M^{\gamma+1} \qquad (18) \\ & subject\ to \quad 0 \le m \le \gamma - 1; \;\; 0 \le m \le \delta \end{aligned}$$

case 1. $\delta < \gamma$ In the cases where the delay is smaller than the assumed residual redundancy order, we are able to choose $m = \delta$ and eliminate the backward term. The probabilities in Equation (10) are then given as follows,

$$\begin{aligned} & P(I_n|\underline{J}_{n+\delta}) = \ldots \textstyle\sum_{I_{n+k}} \ldots \; P(S_{n+\delta}|\underline{J}_{n+\delta}), \qquad (19) \\ & \quad k = \delta - \gamma + 1, \;\ldots, \;\delta, \qquad k \ne 0. \end{aligned}$$

case 2. $\delta \ge \gamma$ Alternatively, when the delay is larger than the assumed redundancy order, the NC_{bwd} is minimized when $m = \gamma - 1$, i.e. $\delta - \gamma + 1$ backward recursions is required. The probabilities in Equation (10) are now given by the Equations (12) to (15) and the following,

$$P(I_n|\underline{J}_{n+\delta}) = \sum_{I_{n+1}} \sum_{I_{n+2}} \cdots \sum_{I_{n+\gamma-1}} P(S_{n+\gamma-1}|\underline{J}_{n+\delta}). \qquad (20)$$

4. NUMERICAL RESULTS

To analyze the performance of the proposed MMSE decoding schemes, we use a synthesized source similar to [2]. The source is a fifth-order Gauss Markov source with coefficients

$$[1.381, -0.599, 0.367, -0.700, 0.359] \qquad (21)$$

At different values of γ, the mutual information,

$$MI(I_n; S_{n-1}) = H(I_n) - H(I_n|S_{n-1}) \qquad (22)$$

provides an indication of the redundancy to be exploited and hence, the gains to be achieved. Table 1 presents the conditional entropy and the mutual information as a function of γ when the source is quantized by a 3-bit Lloyd/Max scalar quantizer. As

Redundancy Order γ	0	1	2	3
Conditional Entropy	2.813	1.930	1.766	1.715
MI	0	0.884	1.047	1.098

Table 1. Conditional Entropy and the Mutual Information of the source at different values of γ

given in Table 1, the redundancy due to the nonuniform distribution ($\gamma = 0$) is 0.19 bits. The redundancy exploited by means of a first, second and third order Markov model is 0.88, 1.05 and 1.10 bits respectively. Figure 2 demonstrates the performance of the instantaneous MMSE reconstruction scheme presented in the previous section. The performance of the Maximum Likelihood Detection (Hard Decision) is provided as a baseline for comparison. Also the performance of the instantaneous MMSE reconstructor when the residual redundancy is neglected and when it is exploited using a γ-order ($\gamma = 0, 1, 2, 3$) Markov model is provided. To study

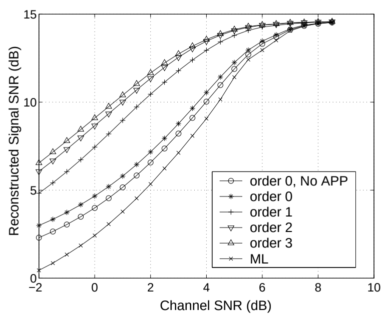

Fig. 2. Performance of the instantaneous MMSE decoder when different levels of residual redundancy is exploited at the decoder

the effect of delay in MMSE reconstruction, Tables 2 and 3 present the performance of MMSE decoding over noisy channel (SNR -2 to 6) using a γ-order residual redundancy model ($\gamma = 1, 3$) at different delays ($\delta = 0$ to 3). It is clear that the performance is substantially improved as the redundancy model order γ and/or the delay δ is increased.

SNR	BER	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$
-2.00	0.1300	4.82	6.70	7.53	7.71
-1.00	0.1031	6.05	8.23	9.15	9.35
0.00	0.0786	7.44	9.86	10.77	10.96
1.00	0.0559	8.96	11.44	12.14	12.26
2.00	0.0371	10.45	12.69	13.13	13.18
3.00	0.0223	11.78	13.54	13.80	13.82
4.00	0.0122	12.95	14.08	14.17	14.17
5.00	0.0055	13.79	14.37	14.38	14.38
6.00	0.0022	14.26	14.53	14.54	14.54

Table 2. Reconstructed signal SNR (dB) for MMSE decoding over noisy channel (SNR -2 to 7) using first-order residual redundancy model at different delays ($\delta = 0$ to 3)

SNR	BER	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$
-2.00	0.1300	6.54	8.36	9.21	9.53
-1.00	0.1031	7.80	9.84	10.69	10.97
0.00	0.0786	9.09	11.25	11.97	12.16
1.00	0.0559	10.40	12.45	12.99	13.07
2.00	0.0371	11.66	13.40	13.70	13.71
3.00	0.0223	12.74	13.99	14.12	14.13
4.00	0.0122	13.56	14.29	14.33	14.33
5.00	0.0055	14.12	14.45	14.46	14.46
6.00	0.0022	14.39	14.57	14.58	14.58

Table 3. Reconstructed signal SNR (dB) for MMSE decoding over noisy channel using third-order residual redundancy model at different delays

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