AN EFFICIENT SIGNALING METHOD OVER MIMO BROADCAST SYSTEMS WITH MULTIPLE RECEIVE ANTENNAS

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***Abstract*— A simple signaling method for multi-antenna broadcast channels is proposed. This method converts the interference matrix – but not necessarily the channel matrix – to a lower-triangular form. Dirty paper coding is used to cancel the remaining interference. The proposed scheme offers several desirable features in terms of: (i) accommodating users with different number of receive antennas, (ii) providing fairness and quality-of-service (QoS), (iii) requiring low feedback rate. The simulation results indicate that the achieved sum-rate is close to the sum-capacity of the underlying broadcast channel. An asymptotic analysis shows that the diversity order of the j^{th} data stream, $1 \leq j \leq M$ is equal to $NK(M-j+1)$, where M, N, and K indicate the number of transmit antennas, the number of receive antennas, and the number of users, respectively. Furthermore, it is shown that the throughput of this scheme scales as $M \log\log(K)$ and asymptotically ($K \longrightarrow \infty$) tends to the sum-capacity of the MIMO broadcast channel.**

1 INTRODUCTION

Recently, multiple input multiple output (MIMO) systems have received considerable attention as a promising solution to provide reliable and high data rate communication. More recently, the work on MIMO systems has been extended to MIMO multi-user channels [1], [2]. In [1], a duality between the broadcast channel and the multiple access channel is introduced. This duality is applied to characterize the sum-capacity of the broadcast channel as a convex optimization problem. In [2], a reformulation of the sum-capacity as a min-max optimization problem is introduced and a signaling method which achieves the sum-capacity is presented. It is shown that in an optimal signaling (maximizing the sum-rate), the power is allocated to, at most, M^2 uses (active users), where M is the number of transmit antennas [3]. In practical systems, the number of users is large. In this case, finding the set of active users by solving the optimization problem is a complex operation. In addition, to perform such a computation, all the channel state information is required at the base station which necessitates a high data rate feedback link.

The duality and signaling method introduced in [1], [2] are based on a result, known as *dirty paper coding*, on cancelling known interference at the transmitter [4]. A method for approximate implementation of the dirty paper coding is presented in [5].

A number of research works have focused on practical methods for signaling over MIMO broadcast channels. In [6], a simple method that supports one user at a given time is presented. This method exploits a special kind of diversity, *multiuser diversity*, which is available in the multiuser system with independent channels. Unlike [6], the signaling method presented in other related works support multiple users at a given time. In [7], a variation of channel inversion method is used, where the inverse of the channel matrix is regularized and the data is perturbed to reduce the energy of the transmitted signal. However, in this method, the pre-coding matrix depends on the data, and therefore, the method is computationally extensive. In addition, no method for selecting active users is suggested. In [8], a signaling method based on the QR decomposition and dirty paper coding is introduced. The QR decomposition converts the channel matrix, and consequently the interference matrix, to a lower triangular form, where the entry (p,q) denotes the interference of user p over user q. Dirty paper coding eliminates the remaining interference. By modifying the QR decomposition, a greedy method for selecting active users which exploits multiuser diversity is presented in [9]. References [7]–[9] present methods to support M simultaneous users, each with one receive antenna.

When there is more than one antenna at the receiver, a generalized version of the zero forcing method is utilized in [10]. However, the methods of [10] are highly restrictive in the sense that the number of transmit antennas must be greater than the total number of the receive antennas. In addition, similar to the conventional zero forcing, the method presented in [10] degrades the signal-to-noise-ratio (SNR).

In this paper, an efficient sub-optimum method for selecting the set of active users and signaling over such users is proposed. This method converts the interference matrix – but not necessarily the channel matrix – to a lower-triangular form. This is in contrast to the earlier method proposed in [8], [9] which uses QR decomposition to triangularis the channel matrix.

The rest of the paper is organized as follows: In Section 2, the system model and the proposed signaling method are presented. In Section 3, an algorithm to select the active users and the corresponding MVs is developed. The asymptotic sum-rate and diversity order, achieved by the proposed method, are derived in Section 4. In Section 5, the simulation results and comparisons with the sum-capacity of the MIMO broadcast are discussed.

2 PRELIMINARIES

Consider a MIMO broadcast channel with M transmit antennas and K users, where the k^{th} user is equipped with N_k receive antennas. In a flat fading environment, the baseband model of this system is given by,

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s} + \mathbf{w}_k, \quad 1 \leq k \leq K, \tag{1}$$

where $\mathbf{H}_k \in \mathcal{C}^{N_k \times M}$ denotes the channel matrix from the base station to user k, $\mathbf{s} \in \mathcal{C}^{M \times 1}$ represents the transmitted vector, and $\mathbf{y}_k \in \mathcal{C}^{N_k \times 1}$ signifies the received vector by user k. The vector $\mathbf{w}_k \in \mathcal{C}^{N_k \times 1}$ is white Gaussian noise with a zero-mean and unit-variance.

In the proposed method, each time, the base station supports M data streams, distributed among at most M users called *active users*, indexed by $\pi(j)$, $j = 1, \ldots, M$. The transmitted vector $\mathbf{s}$ is equal to:

$$\mathbf{s} = \sum_{j=1}^{M} d_j \mathbf{v}_j, \tag{2}$$

where $\mathbf{v}_j \in \mathcal{C}^{M \times 1}$, $j = 1, \ldots, M$, is the MV corresponding to user $\pi(j)$, $\pi(j) \in \{1, 2, \ldots, K\}$, and d_j contains the information for user $\pi(j)$. Vectors $\mathbf{v}_j$, $j = 1, \ldots, M$, form an orthonormal set. Dirty-paper coding is used such that for $i > j$, the interference of data stream i over data stream j is cancelled. To detect the data stream j, user $\pi(j)$ multiplies the received vector by a demodulation vector $\mathbf{u}_j^\dagger$, where $(.)^\dagger$ denotes transpose conjugate operation.

In the next section, we propose a method to select the set of active users $\{\pi(1), \pi(2), \ldots, \pi(M)\} \subset \{1, 2, \ldots, K\}$, modulation vectors $\mathbf{v}_j$, and demodulation vectors $\mathbf{u}_j$ for $j = 1 \ldots M$.

3 SELECTING ACTIVE USERS, MODULATION, AND DEMODULATION VECTOR

Assuming channel state information (CSI) available at the base station, the proposed algorithm works as follows. First, for each user, the maximum *gain* and the corresponding *direction* are determined[1]. Next, the best user, in terms of the largest gain, is chosen as an active user. The MV for the selected user is along the corresponding direction. These steps repeat recursively until the M MVs and the set of active users are determined. In each step, the search for the best user is performed in the null space of the previously selected MVs. It is shown that in this manner, the selected MV has no interference over the previously selected MVs. In the following, the proposed algorithm is presented in details.

1) Set $j = 1$ and $\mathbf{\Xi} = [0]_{M \times M}$.
2) Find σ_j^2, where

$$\begin{aligned} \sigma_j^2 = \quad & \max_r \max_{\mathbf{x}} \mathbf{x}^\dagger \mathbf{H}_r^\dagger \mathbf{H}_r \mathbf{x}. \\ s.t. \quad & \mathbf{x}^\dagger \mathbf{x} = 1 \\ & \mathbf{\Xi}^\dagger \mathbf{x} = 0. \end{aligned} \tag{3}$$

   Set $\pi(j)$ and $\mathbf{v}_j$ equal to the optimizing parameters r and $\mathbf{x}$, respectively.
3) Set

$$\mathbf{u}_j = \frac{1}{\sigma_j} \mathbf{H}_{\pi(j)} \mathbf{v}_j. \tag{4}$$

4) Substitute $\mathbf{v}_j$ in column j of matrix $\mathbf{\Xi}$.
5) Set $j \leftarrow j + 1$. If $j \leq M$, move to step two; otherwise, stop.

In Step 2 of the algorithm, maximization over r selects the best user, and therefore exploits the multiuser diversity. Maximization over $\mathbf{x}$ determines the best MV for each user, and at the same time, converts the interference matrix to a lower triangular form, implying that data stream j has no interference over data stream i, $i = 1, \ldots, j-1$. This property has been proven in the following theorem.

Theorem 1: Consider the following optimization problem:

$$\begin{aligned} \max_x \quad & \mathbf{x}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{x}, \\ s.t. \quad & \mathbf{x}^\dagger \mathbf{x} = 1 \\ & \mathbf{\Xi}^\dagger \mathbf{x} = 0, \end{aligned} \tag{5}$$

where $\mathbf{H}$ and $\mathbf{\Xi} = [\xi_1, \xi_2, \ldots, \xi_\varrho]$ are complex matrices. Let $\mathbf{v}$ be the vector that maximizes (5) and σ^2 be the result of the optimization. Define vector $\mathbf{u}$ as follows:

$$\mathbf{u} = \frac{\mathbf{H}\mathbf{v}}{\sigma}. \tag{6}$$

If there exists a vector $\hat{\mathbf{v}}$ such that $\mathbf{\Xi}^\dagger \hat{\mathbf{v}} = 0$ and $\mathbf{v}^\dagger \hat{\mathbf{v}} = 0$, then

$$\mathbf{u}^\dagger \mathbf{H} \hat{\mathbf{v}} = 0. \tag{7}$$

Proof: Refer to [11]. ■

The interference of data stream i over data stream j is equal to $\mathbf{u}_j^\dagger \mathbf{H}_{\pi(j)} \mathbf{v}_i$. Noting (3) which derives $\mathbf{v}_j$ and according to $\mathbf{v}_j^\dagger \mathbf{v}_i = 0$, Theorem 1 implies that $\mathbf{u}_j^\dagger \mathbf{H}_{\pi(j)} \mathbf{v}_i = 0$, for $i > j$. This means that data stream i has no interference over data stream j, $j = 1, \ldots, i-1$. Note that if $i < j$, the interference of data stream i over data stream j is cancelled by dirty paper coding. Therefore, the MIMO broadcast channel is effectively reduced to a set of parallel sub-channels with gains σ_j, $j = 1, \ldots, M$. As a result, the sum-rate of the system is equal to $R = \sum_{j=1}^{M} \log_2(1 + \sigma_j^2 P_j)$, where P_j is the power allocated to data stream j, and $\sum_{j=1}^{M} P_j \leq P$. To maximize R, the power is allocated based on water-filling. However, regarding the parallel structure of the resulting channel, the power allocation can be easily performed to satisfy the required QoS.

As mentioned, one part of the algorithm is to find the direction in which each user has maximum gain. This part of the processing can be accomplished at the receiver and then if the maximum gain of the user is larger than a threshold, the gain and the corresponding direction are reported to the transmitter. The base station selects the best user in terms of the largest gain. By using this technique, the complete channel state information is not required at the transmitter and the rate of the feedback is significantly reduced.

4 ASYMPTOTIC ANALYSIS OF THE PERFORMANCE

In this section, the asymptotic performance ($K \to \infty$) of the proposed algorithm is investigated. We assume: (i) available power P is divided equally among the active users, (ii) at most, one data stream can be assigned to each user.

To study the performance of the system, we first derive the outage probability of each sub-channel which is defined as $\Pr(\sigma_j^2 < z)$, $j = 1, \ldots, M$, $j = 1, \ldots, K$, for a given z. The following lemma helps us to derive the outage probability functions.

Lemma 1: Consider a vector space Ω defined by

$$\Omega = \{\mathbf{x} \quad | \quad \mathbf{x} \in \mathcal{C}^{M \times 1}, \quad \mathbf{\Xi}^\dagger \mathbf{x} = 0\}, \tag{8}$$

[1]The gain of the channel $\mathbf{H}$ along the direction (unit vector) $\mathbf{x}$ is defined as the square root of $\mathbf{x}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{x}$.

where $\mathbf{\Xi}$ is a complex matrix. Assume that Ω is spanned by a set of orthogonal vectors $\{\phi_1, \phi_2, \ldots, \phi_\nu\}$, where $\nu \leq M$. Then, for complex matrix $\mathbf{H}$, the result of the following optimization,

$$\begin{aligned} \max_{\mathbf{x}} \quad & \mathbf{x}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{x}, \\ s.t. \quad & \mathbf{x}^\dagger \mathbf{x} = 1 \\ & \mathbf{x} \in \Omega, \end{aligned} \tag{9}$$

is equal to σ^2, where σ is the maximum singular value of matrix $\widehat{\mathbf{H}}$, where

$$\widehat{\mathbf{H}} = \mathbf{H}\mathbf{\Phi} \tag{10}$$

and

$$\mathbf{\Phi} = [\phi_1, \phi_2, \ldots, \phi_\nu]. \tag{11}$$

Proof: Refer to [11] ■

According to Lemma 1, σ_j^2 in (3) is equal to

$$\sigma_j^2 = \max_{r \in \mathcal{T}_j} S_{\max}^2(\widehat{\mathbf{H}}_{r,j}), \tag{12}$$

where set $\mathcal{T}_j = \{1, 2, \cdots, K\} - \{\pi(1), \pi(2), ..\pi(j-1)\}$, $S_{\max}^2(\widehat{\mathbf{H}}_{r,j})$ is the square of maximum singular value of $\widehat{\mathbf{H}}_{r,j} = \mathbf{H}_r \mathbf{\Phi}_j$ (or is the maximum eigenvalue of $\widehat{\mathbf{H}}_{r,j} \widehat{\mathbf{H}}_{r,j}^\dagger$ [2]) and $\mathbf{\Phi}_j$ is a matrix with orthogonal columns which span the complex vector space $\Omega = \{\mathbf{x} | \mathbf{x} \in \mathcal{C}^{M \times 1} \quad \mathbf{\Xi}^\dagger \mathbf{x} = 0\}$.

Note that in (3), $\mathbf{\Xi}^\dagger$ has $j-1$ non-zero orthogonal rows. Therefore, the dimension of the complex vector space Ω is $M-(j-1)$, resulting in $\mathbf{\Phi}_j \in \mathcal{C}^{M \times (M-j+1)}$. In the following, we assume users equipped with N receive antennas. For large K, since the columns of $\mathbf{\Phi}_j$ are orthonormal and the entries of $\mathbf{H}_r$ have independent unit variance Gaussian distributions (Rayleigh channel), the entries of $\widehat{\mathbf{H}}_{r,j} \in \mathcal{C}^{N \times (M-j+1)}$ have independent unit variance Gaussian distributions. Consequently, according to the definition, $\widehat{\mathbf{H}}_{r,j}^\dagger \widehat{\mathbf{H}}_{r,j}$, $r = 1, \ldots, K-j+1$ have a Wishart distribution, which are identical and independent for different r.

The following lemma formulates the distribution of the maximum eigenvalue of a Wishart matrix.

Lemma 2: [13] Assume that the entries of $A \in \mathcal{C}^{m \times n}$ have a zero mean, unit variance Gaussian distribution; then, the Cumulative Distribution Function (CDF) of the maximum eigenvalue of the matrix $A^\dagger A$ is equal to

$$F(z) = \Pr(\lambda_{max} \leq z) = \frac{1}{\prod_{k=1}^{a} \Gamma(b-k+1)\Gamma(a-k+1)} \det(\mathbf{\Psi}), \tag{13}$$

where $a = \min\{m, n\}$, $b = \max\{m, n\}$, and $\mathbf{\Psi}$ is an $a \times a$ Hankel matrix which is a function $z \in (0, \infty)$ defined as

$$\mathbf{\Psi}(p, q) = \gamma(b - a + p + q - 1, z) \qquad p, q = 1, \ldots, a, \tag{14}$$

and γ is incomplete gamma function.

Regarding the above statements and using Lemma 1, Lemma 2, and (15), we conclude the following corollary.

Corollary 1: Set $F_j(z)$ be the CDF of $S_{\max}^2(\widehat{\mathbf{H}}_{r,j})$ defined as follows,

$$F_j(z) = \Pr\left(S_{\max}^2(\widehat{\mathbf{H}}_{r,j}) < z\right). \tag{15}$$

[2]The square of the maximum singular value of a matrix A is equal to the maximum eigenvalue of the matrix $A^\dagger A$ [12].

Then, $F_j(z)$ is equal to $F(z)$, defined in Lemma 2, where,

$$\begin{aligned} a &= \min\{M-j+1, N\}, \\ b &= \max\{M-j+1, N\}. \end{aligned} \tag{16}$$

As it has been mentioned, $S_{\max}^2(\widehat{\mathbf{H}}_{r,j})$, $r \in \mathcal{T}_j$ are independent random variables for different r's, therefore according to (12), the outage probability of the sub-channel j is equal to

$$\Pr(\sigma_j^2 < z) = [F_j(z)]^{K-j+1}. \tag{17}$$

In deriving (17), we have used the expression for the CDF of the maximum of $K-j+1$ i.i.d random variables [14].

4.1 Diversity Analysis

The diversity order in a wireless channel is equal to the asymptotic slope ($z \to 0$) of the outage probability curve. This quantity determines the asymptotic slope of the curve of the symbol error rate versus signal-to-noise-ratio. In the following theorem, we use this definition to establish the diversity order of the j^{th} data stream.

Theorem 2: For large K, the diversity order of the sub-channel j is equal to $(K-j+1)N(M-j+1)$.

Proof: The diversity order of sub-channel j is equal to the degree of z in the outage probability $\Pr(\sigma_j^2 < z)$ when $z \to 0$. In [11], it is shown that

$$\lim_{z \to 0} F_j(z) = c_j z^{ab}, \tag{18}$$

where c_j is equal to

$$c_j = \frac{\prod_{i=1}^{a-1}(a-i)!}{\prod_{k=1}^{a}(b-k)! \prod_{i=1}^{a}(b-a+i)^i (b+a-i)^i}. \tag{19}$$

where a and b are defined in (16). Regarding (17) and (18),

$$\lim_{z \to 0} \Pr(\sigma_j^2 < z) = c_j^{K-j+1} z^{(K-j+1)N(M-j+1)}. \tag{20}$$

Therefore, the diversity order of the sub-channel j, $1 \leq j \leq M$, is equal to $(K-j+1)N(M-j+1)$. ■

4.2 Asymptotic Rate Analysis

By using (17), the average sum-rate of the proposed method can be computed. However, an examination of the asymptotic behavior ($K \to \infty$) of the rate provides insight into the performance of the proposed algorithm. When $K \to \infty$, the behavior of σ_j^2 depends on the tail of the distribution function $F_j(z)$, the CDF of $S_{\max}^2(\widehat{\mathbf{H}}_{r,j})$ [6]. The following lemma allows us to derive limiting distribution of σ_j^2.

Lemma 3: [14] Let $z_1, z_2, \ldots, z_K$ be i.i.d random variable with a common CDF $F(.)$ and probability density function $f(.)$, satisfying the following conditions: (i) $F(z)$ is strictly less than one for all finite z, (ii) $F(z)$ is twice differentiable, (iii)

$$\lim_{z \longrightarrow \infty} g(z) = c > 0, \tag{21}$$

where $g(z) = \frac{1-F(z)}{f(z)}$ and c is a constant. Then,

$$\max_{1<r<K} z_r - l(K) \tag{22}$$

converges in distribution to a limiting random variable with CDF,

$$\exp[-\exp(-\frac{u}{c})]. \tag{23}$$

where $l(K) = F^{-1}(1 - \frac{1}{K})$ and $F^{-1}(.)$ represents the inverse function of $F(.)$

Lemma 3 states that the maximum of such i.i.d random variables grows like $l(K)$ [6].

It is easy to see that: (i) $F_j(z)$ is less than one for all finite z, (ii) $F_j(z)$ is twice differentiable, and (iii) $\lim_{z\to\infty} g_j(z) = 1$, where $g_j(z) = \frac{1-F_j(z)}{F_j^{(1)}(z)}$, and $F^{(1)}(z)$ denotes the derivative of $F_j(z)$. By substituting the following expansion,

$$\gamma(n+1, z) = n!\left(1 - e^{-z}\sum_{m=1}^{n}\frac{z^m}{m!}\right), \tag{24}$$

into (14) and regarding Corollary 1, we obtain [11]

$$F_j(z) = \Pr(S^2_{\max}(\widehat{\mathbf{H}}_{r,j}) < z) = 1 - \frac{e^{-z}z^{a+b-2}}{(a-1)!(b-1)!}\left(1 + O(z^{-1}e^{-z})\right). \tag{25}$$

According to Lemma 3, σ_j^2 grows like $l_j(K)$ which is given by $l_j(K) = F_j^{-1}(1 - \frac{1}{K})$. Using (25) and (16), $l_j(K)$ is is equal to

$$l_j(K) = \log(K) + (N+M-j-1)\log\log(K) + o(\log\log(K)). \tag{26}$$

Using equation (26), we can prove the following theorem (refer to [11]).

Theorem 3:

$$\lim_{K\to\infty}\frac{R}{M\log(\frac{P}{M}\log(K))} = 1, \tag{27}$$

where R is the sum-rate of the proposed method. In addition,

$$\lim_{K\to\infty} R_{\text{sum-capacity}} - R \longrightarrow 0 \tag{28}$$

where $R_{\text{sum-capacity}}$ indicates the sum-capacity of the MIMO broadcast channel.

Note that these results are derived with two assumptions of equal power distribution among active users (no water-filling) and allocation of at most one data stream to each user. Apparently, Theorem 3 is valid when these two restrictive assumptions are relaxed.

5 SIMULATION RESULTS

Figure 1 depicts the average sum-rate of the proposed method and average sum-capacity versus the number of users for different number of receive antennas. In this simulation, the power is optimally allocated to active users by using the water-filling method. This figure shows that the sum-rate of the proposed method is very close to the sum-capacity of the system, even when the number of the users is small. This result shows that with only M data streams, the major part of the sum-capacity of the MIMO broadcast systems is achieved, no matter what the number of receive antennas is.

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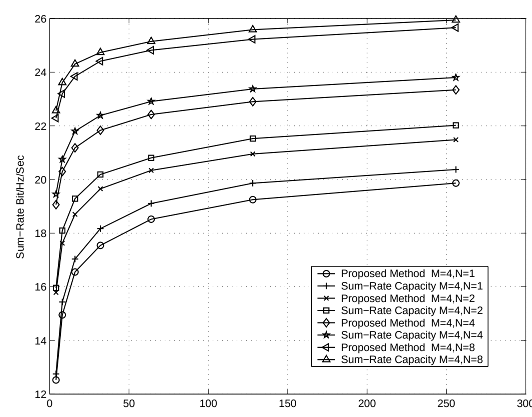

Fig. 1. Average Sum Rate of the Proposed Method and Average Sum Capacity versus Number of Users