A Dirty Paper Coding Approach Without Modulo Operation at the Receiver

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ABSTRACT

We consider a precoding scheme without modulo operation at the receiver for the Gaussian channel with Gaussian interference which is known causally at the transmitter. Modulo operation which is conventionally applied to the received signal for the sake of convenience in analysis causes information loss. In this paper, we investigate the gain in capacity by removing modulo operation at the receiver. We also propose an end-to-end communication scheme based on the proposed precoding scheme by concatenating the precoder with LDPC codes. Bit error rate curves also confirm the superiority of the precoding method without modulo operation at the receiver.

1 INTRODUCTION

In this paper, we consider power-constrained information transmission over AWGN channel with Gaussian interference known at the transmitter. The above channel is characterized by

$$Y = X + S + N, \qquad (1)$$

where S is i.i.d Gaussian interference, which is known at the transmitter, with pdf $f_S(s) = \frac{1}{\sqrt{2\pi P_S}} \exp\left(-\frac{s^2}{P_S}\right)$.

The noise N is also i.i.d. Gaussian and independent of the interference with pdf $f_N(n) = \frac{1}{\sqrt{2\pi P_N}} \exp\left(-\frac{n^2}{P_N}\right)$.

The transmitted signal X with power constrained to P_X is the channel input, and Y is the channel output. Costa showed that the capacity of the above channel is the same as the capacity of the additive white Gaussian noise (AWGN) channel [1]. In other words, he showed that the interference does not incur any loss in the capacity.

In his proof, Costa assumes that the whole sequence of the interference symbols is known at the transmitter. This setting is called the non-causal knowledge of the interference at the transmitter. On the contrast is the causal knowledge setting, where the encoder uses the interference symbols up to the current symbol to generate the current channel input symbol. In the causal case, the capacity is not the same as the capacity of AWGN channel any more. Dirty paper coding comes from Costa's paper title and means coding for the channel defined in (1). In this paper, we consider the case where the interference is known causally at the transmitter.

In the causal case, the encoder maps the message v into X^n using functions

$$x_i = f_i(v, s_1^i), \quad 1 \le i \le n \qquad (2)$$

where $s_1^i = s_1, \ldots, s_i$ are interference symbols up to the time i and X is the channel input alphabet. The corresponding capacity formula is given by [2]

$$C(P_X) = \max_{f_V(v),\, f: \mathsf{V}\times\mathsf{S}\mapsto \mathsf{X},\, E[f(V,S)^2]\le P_X} I(V;Y), \qquad (3)$$

where V and S denote alphabets of the input V and the interference S, respectively. The maximization in (3) is taken over function f, which maps $\mathsf{V}\times\mathsf{S}$ to X, and $f_V(v)$, the pdf of the random variable V. The function f in (3) is a deterministic function of V and S, and is called as *strategy function*.

The capacity formula in (3) is the extension to the continuous case of the capacity formula obtained by Shannon for discrete memoryless channels with side information at the transmitter [3].

2 PRECODING WITHOUT MODULO OPERATION AT THE RECEIVER

As it can be seen, the capacity formula in (3) is not an explicit formula because it requires maximization over input pdfs and strategy functions. In [2], it is proved that by using a dithered quantizer as the strategy function the capacity of dirty paper channel with causally known interference in the limit of high signal-to-noise ratio

(SNR) is achievable. For the general case, however, the maximizing input pdf and the strategy function are not known.

It has been shown that the dithered quantization precoding incurs loss in capacity for low interference powers [4]. In the revised precoding method [4], which avoids the loss, the encoder transmits

$$X = [V - \alpha S] \bmod \Delta , \qquad (4)$$

and the receiver computes

$$Y' = [\alpha Y] \bmod \Delta , \qquad (5)$$

where $Y = X + S + N$ is the channel output.

Due to modulo operation, the channel input X lies within the interval $A_\Delta = [-\Delta/2, \Delta/2)$. Indeed, modulo operation can be considered as a uniform quantizer with infinite quantization levels located at points $n\Delta$, $n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$. The channel input X is then the error of the quantizer.

We call the above precoding method as *method 1* or precoding method with modulo operation at the receiver. Modulo operation is conventionally applied to the received signal for the sake of convenience in decoding. The precoding method without modulo operation at the receiver, called *method 2*, simply does not perform the modulo operation of (5). The maximum achievable rate by the precoding methods 1 and 2 are the maximum of $I(V;Y')$ and $I(V;Y)$ over pdf of V and the precoding parameters α and Δ, respectively. Assuming the same parameters for both methods, data processing inequality [5] implies that the maximum achievable rate by using method 2 is higher than or equal to the maximum achievable rate by using method 1. In this paper, we investigate the gain in capacity by removing modulo operation at the receiver.

Since we are interested in using the above-mentioned precoding methods in a communication scheme with PAM constellation with equiprobable points, we assume that V is uniformly distributed over A_Δ. This makes the pdf of the transmitted signal X uniform in A_Δ, independent of the interference S. Then Δ is determined uniquely by the transmit power constraint as $\sqrt{12P_X}$. Therefore, the maximum achievable rate with precoding methods 1 and 2 will be the maximum of $I(V;Y')$ and $I(V;Y)$ over α, respectively. Following the dirty paper coding of Costa, we choose $\alpha = P_X/(P_X + P_N)$ for both methods.

Fig. 1 depicts the maximum achievable rate using methods 1 and 2 as a function of SNR for different interference powers. The power of the transmitted signal is fixed at 1. As it can be seen, the precoding method 2 offers higher rates than those of method 1. However, the

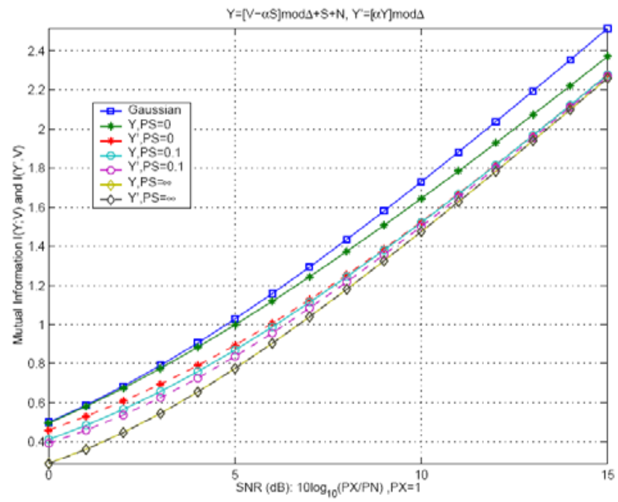

Fig. 1. Maximum achievable rate curves for both methods for different interference powers

difference in maximum achievable rates for the two methods diminishes as the interference power increases. We can also see that at high SNR, the maximum achievable rate is the same for both methods, independent of interference power. These observations are stated more precisely in the following two theorems.

Theorem 1. *At any given SNR, the maximum achievable rate of the precoding method with modulo operation at the receiver tends to the maximum achievable rate of the precoding method without modulo operation at the receiver as the power of interference tends to infinity.*

Theorem 2. *At the limit of high SNR, the power gap to the capacity of AWGN channel for both methods is* $10\log_{10}\left(\frac{\pi e}{6}\right) \approx 1.53\, dB$ *(the shaping gain); independent of the interference power.*

Proof. See the appendix.

3 AN END-TO-END COMMUNICATION SCHEME

The block diagram of an end-to-end communication system employing the precoding method without modulo operation at the channel output is shown in fig. 2. We use a finite-length fast encodable low-density parity-check (LDPC) code with rate $R \approx 1/2$, block length 498, and 248 information bits (250 parity bits) as channel code. The code is designed with simple irregular semi-random parity-check matrix as in [6] with simple modification (without weight one column) and with girth 6. Transmitted symbols v_k are selected from a 2^ν-PAM constellation with Gray labeling.

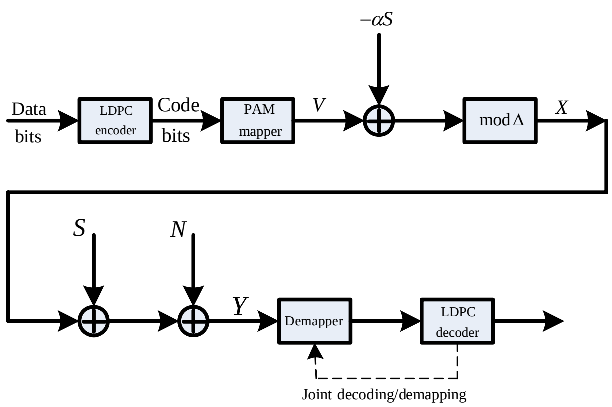

Fig. 2. Block diagram of an end-to-end communication system employing the precoding method without modulo operation at the receiver

Multilevel Coding (MLC) [7]-[8] and Bit-Interleaved Coded Modulation (BICM) [9]-[10] are two well-known coded modulation schemes proposed to achieve both power and bandwidth efficiency. In our work, we select BICM scheme in which we need only one component code as well as one bit-interleaver. We use a special bit-interleaver to reduce the dependency of the constructive bits of the symbols in one encoded block which allows to interleave the transmitted data block by block without any further delay due to the interleaver. Furthermore, the structure of the interleaver allows us to use joint iterative decoding and demapping in BICM system (called BICM-ID [11]-[13]) to improve the error rate performance.

The joint iterative decoding and demapping method is based on finding the updated symbol probabilities as a multiplication of the corresponding bit probabilities which will be exact when the constructive bits of a symbol are independent.

For the case of AWGN channel (zero interference), the transmitted signal x_k is equal to the original symbol v_k. We define $E_b / N_0 = P_X /(2\nu R P_N)$ for both AWGN and DPC channels.

In our simulations, we consider a 2^ν-PAM constellation with unit energy ($E\|V\|^2 = 1$) and with minimum Euclidean distance $d_{\min} = 2\sqrt{3/(4^\nu - 1)}$. We choose $\Delta/2 = V_{\max} + d_{\min}/2$. The transmitted signal power will then range from 1 to $\Delta^2/12 = 1 + 1/(4^\nu - 1)$ depending on the interference power.

The bit error rate curves for 2-PAM ($\nu = 1$) and 8-PAM ($\nu = 3$) transmissions with/without modulo operation at the receiver are illustrated in Figs. 3 and 4 for three different values of P_S. For 8-PAM transmission, the joint iterative decoding and demapping method has been used. We use at most 50 iterations for decoding. As expected, the performance of DPC without modulo operation at receiver is better than the performance of DPC with modulo operation for small P_S. For large interference power, the performances of DPC with and without modulo operation at the receiver are nearly the same. The performance of AWGN channel is also shown for comparison purposes.

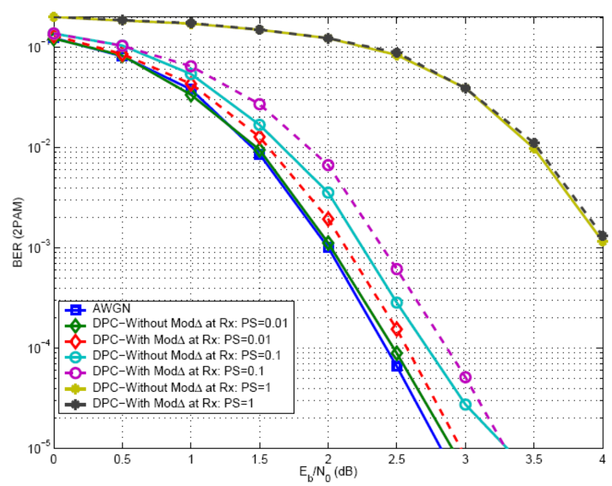

Fig. 3. BER curves for binary transmission with/without modulo operation at the receiver for different interference powers.

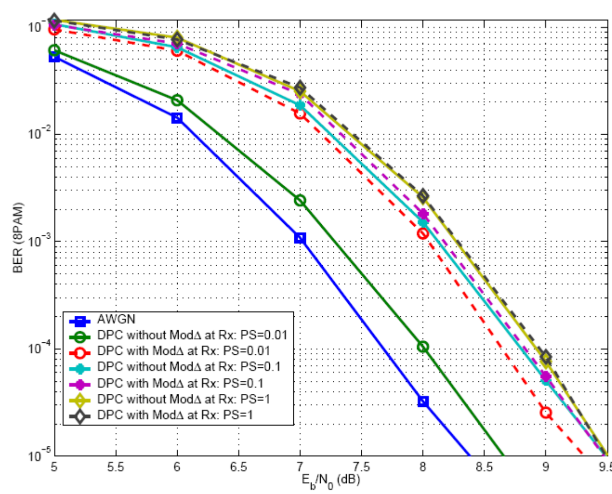

Fig. 4. BER curves for 8-PAM transmission with/without modulo operation at the receiver for different interference powers ($P_X \approx 1$).

4 CONCLUSION

In this paper, we investigated the gain achieved by removing modulo operation at the receiver for channels with causally known interference at the transmitter. Although for high interference powers there is no gain in removing modulo operation at the transmitter, for low interference powers the precoding method without modulo operation at the receiver outperforms in terms of achievable rate. We also proposed an end-to-end communication scheme using our proposed precoding method. In terms of bit error rate, simulation results confirm the superiority of precoding method without modulo operation at the receiver.

APPENDIX

Proof of theorem 2. We use the fact that $I(V;Y)$ and $I(V;Y')$ are decreasing functions of P_S. Hence,

$$\underbrace{I(V;Y)}_{\text{for } P_S=\infty} \le \underbrace{I(V;Y)}_{\text{for any } P_S} \le \underbrace{I(V;Y)}_{\text{for } P_S=0}; \tag{6}$$

and

$$\underbrace{I(V;Y')}_{\text{for } P_S=\infty} \le \underbrace{I(V;Y')}_{\text{for any } P_S} \le \underbrace{I(V;Y')}_{\text{for } P_S=0}. \tag{7}$$

From [4] and [2], for $P_S \to \infty$ we have

$$\lim_{\frac{P_X}{P_N}\to\infty}\left[\frac{1}{2}\log\left(1+\frac{P_X}{P_N}\right)-\underbrace{I(V;Y')}_{\text{for } P_S=\infty}\right]=\frac{1}{2}\log\left(\frac{\pi e}{6}\right). \tag{8}$$

Using theorem 1, we have the same result for method 2 as $P_S \to \infty$:

$$\lim_{\frac{P_X}{P_N}\to\infty}\left[\frac{1}{2}\log\left(1+\frac{P_X}{P_N}\right)-\underbrace{I(V;Y)}_{\text{for } P_S=\infty}\right]=\frac{1}{2}\log\left(\frac{\pi e}{6}\right). \tag{9}$$

For zero-power interference case, we have $X = V$ and $Y = V + N$. Then

$$I(V;Y) = h(Y) - h(Y \mid V)$$
$$= h(Y) - \frac{1}{2}\log(2\pi e P_N) \tag{10}$$

At the limit of high SNR, Y looks like uniformly distributed and its entropy tends to $\log\sqrt{12(P_X + P_N)}$. Or equivalently, for $P_S = 0$

$$\lim_{\frac{P_X}{P_N}\to\infty}\left[\frac{1}{2}\log\left(1+\frac{P_X}{P_N}\right)-\underbrace{I(V;Y)}_{\text{for } P_S=0}\right]=\frac{1}{2}\log\left(\frac{\pi e}{6}\right). \tag{11}$$

Comparing (9), (11), and (6) the theorem is proved for method 2.

Due to data processing inequality, we have $\underbrace{I(V;Y')}_{\text{for } P_S=0} \le \underbrace{I(V;Y)}_{\text{for } P_S=0}$. Therefore,

$$\lim_{\frac{P_X}{P_N}\to\infty}\left[\frac{1}{2}\log\left(1+\frac{P_X}{P_N}\right)-\underbrace{I(V;Y')}_{\text{for } P_S=0}\right]\ge\frac{1}{2}\log\left(\frac{\pi e}{6}\right) \tag{12}$$

Comparing (8), (12), and (7) the theorem is proved for method 1.

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