Spectrally-Efficient Differential Space-Time Coding Using Non-Full-Diverse Constellations

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Abstract— **A method to construct spectral-efficient unitary space-time codes is proposed for high-rate differential communications over multiple-antenna channels. Unlike most of the known methods which are designed to maximize the diversity product (minimum determinant distance), we aim at increasing the spectral efficiency. Simulation results indicate that for high spectral efficiency and for more than one receive antenna, the new method significantly outperforms the other known alternatives. In the special case of two transmit antennas, which is the main focus of this paper, the relation between the proposed code and the Alamouti scheme helps us to provide an efficient maximum likelihood decoding algorithm. We also show that similar ideas can be applied to more than two transmit antennas. As an example, we present a construction for 4 by 4 unitary constellations which has a good performance as compared to the other known codes.**

I. Introduction

Recently, because of the increasing demand for the transmission of video, voice and data in mobile wireless environments, reliable high-rate communications over fading channels has become an important issue. Recent investigations show that using multiple transmit and receive antennas, high rate communications can be achieved [1] [2]. In [2]–[5], some practical schemes are proposed to achieve reliable high-rate communications over multiple-antenna wireless channels. However, these methods require knowledge of the fading coefficients at the receiver. Unfortunately, in many scenarios, such as mobile environments, especially in high-speed vehicles, tracking of channel coefficients is not feasible.

In [6]–[8], differential space-time modulation schemes (based on unitary matrices) are presented. These schemes are suitable for mobile communication applications where fading coefficients change rapidly with time. In the differential techniques proposed in [7] and [8], the code is full-diverse and the codewords form a group structure which helps in simplifying the encoding. In [9], all finite fixed-point-free groups (which correspond to full-diverse unitary constellations) are classified. Although some low rate group codes are introduced in [9] which have excellent performance, no good full-diverse group constellation are obtained for very high rates. Another attractive approach is based on using orthogonal designs [6] [10] which helps to facilitate the decoding. Nonetheless, differential orthogonal space-time codes do not have a good performance for very high rates and large number of receive antennas. To transmit at high rates, in [11], a scheme is proposed for constructing full-diverse codes in the general case (different number of transmit antennas and different rates) which is based on using the Cayley transform to construct unitary matrices.

All the above differential space-time codes have been designed to produce full diversity. However, by sacrificing the diversity, we can increase the rate of the space-time modulation [12]. Indeed, in wireless environments, we can use multiple antenna systems to achieve diversity or spatial multiplexing. For coherent communications, schemes such as BLAST [2] are proposed to exploit the spatial multiplexing to produce communications with high spectral efficiencies over Multiple-Input Multiple-Output (MIMO) channels. Nonetheless, for the noncoherent case, most of the previous works are based on achieving the transmit diversity rather than exploiting the spatial multiplexing.

Although the performance of MIMO fading systems is determined by diversity product for very high SNRs, maximizing the diversity product is not the appropriate criterion for spectral-efficient communications when the number of receive antennas is greater than one. This is similar to the coherent case where for spectral-efficient communications and for a large number of receive antennas, BLAST has a better performance as compared to full-diverse schemes, such as the Alamouti code. In some of the previous works [13] [14], there have been attempts to maximize the diversity sum (minimum Frobenius distance) as well as the diversity product. However, those approaches are useful for very low rates [13], or are based on the exhaustive search [14] which is not feasible in the case of designing high-rate codes.

This paper addresses the problem of constructing unitary constellations which are appropriate for spectral-efficient differential transmission over multiple-antenna channels. The proposed approach is based on sacrificing the transmit diversity and using multiple cosets of a full-diverse (or a partial-diverse) code. Although the main purpose of this paper is to design unitary constellations for two transmit antennas, we show that the same ideas can be applied to construct spectral-efficient codes for more than two antennas. As a special case, we present a construction for four transmit antennas which for a large number of receive antennas considerably outperforms the other known approaches.

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II. SYSTEM MODEL

We consider a multiple antenna system with *M* transmit and *N* receive antennas. The transmitted and the received signals (in *M* consecutive channel uses) can be considered as $\mathbf{S}$ (an $M \times M$ matrix) and $\mathbf{X}$ (an $M \times N$ matrix), respectively. We have,

$$\mathbf{X} = \sqrt{\rho}\mathbf{S}\mathbf{H} + \mathbf{W}$$

where $\mathbf{H}$ is the $M \times N$ channel matrix, $\mathbf{W}$ is the $M \times N$ additive noise matrix and ρ is the received Signal to Noise Ratio (SNR). Entries of $\mathbf{H}$ and $\mathbf{W}$ are independent identically distributed (i.i.d.) complex-Gaussian $C(0, 1)$. We assume that the transmitted matrix $\mathbf{S}$ has unit energy per time (i.e. $\mathrm{tr}(\mathbf{S}\mathbf{S}^*) = M$).

In differential space-time coding with *M* transmit antennas, an identity matrix $\mathbf{S}_0 = \mathbf{I}_M$ is transmitted in the first *M* channel uses. After that, for the *t*'th block (which consists of *M* consecutive channel uses), the information is encoded into an $M \times M$ unitary matrix $\mathbf{V}_t$, transmitting

$$\mathbf{S}_t = \mathbf{V}_t\mathbf{S}_{t-1}. \tag{1}$$

For differential space-time codes, it is known that the pairwise error probability satisfies the following upper bound [7]:

$$P(\mathbf{V} \longrightarrow \mathbf{V}') \leq \frac{1}{2}\prod_{m=1}^{M}\left[1 + \frac{\rho^2}{4(1+2\rho)}\sigma_m^2\right]^{-N} \tag{2}$$

where σ_m denotes the *m*'th singular value of $V - V'$. At high SNR, we have [7]

$$P(\mathbf{V} \longrightarrow \mathbf{V}') \leq \frac{1}{2}\left(\frac{8}{\rho}\right)^{MN}\frac{1}{|\det(V - V')|^{2N}}. \tag{3}$$

Thus, similar to the case of coherent space-time communication, at very high SNR, the performance of the code is related to the *minimum determinant distance* or the *diversity product* [7]:

$$\zeta = \frac{1}{2}\min_{l \neq l'}|\det(V_l - V_{l'})|^{\frac{1}{M}}. \tag{4}$$

Based on this fact, almost all the design schemes have focused on finding constellations of unitary matrices with large diversity products [7] [8] [6] [9] [15], or an averaged version of it [11]. However, for a large number of receive antennas, practical SNR values and reasonable bit error rates, the low-SNR approximation of (2) is more appropriate (using the first order approximation):

$$Pr(\mathbf{V} \longrightarrow \mathbf{V}') \leq \frac{1}{2}\left[1 + \frac{\rho^2}{4(1+2\rho)}\sum_{m=1}^{M}\sigma_m^2\right]^{-N}$$

Therefore, for a large number of receive antennas, maximizing the minimum Frobenius distance (diversity sum [13]) will be more useful:

$$d_{min} = \min\|\mathbf{V} - \mathbf{V}'\|_F \tag{5}$$

where $\|\mathbf{V} - \mathbf{V}'\|_F = \mathrm{tr}\left[(\mathbf{V} - \mathbf{V}')(\mathbf{V} - \mathbf{V}')^*\right]$.

III. CODE DESIGN FOR TWO TRANSMIT ANTENNAS

The unitary matrices can be parameterized by M^2 real parameters. In order to construct good unitary constellations with high spectral-efficiency, we must exploit all of these degrees of freedom. For this purpose, we can use parametrization methods for unitary matrices. We need simple parametrization methods which easily construct a family of unitary matrices with certain distance properties.

Here, we consider the especial case of two transmit antennas. Every 2×2 complex unitary matrix $\mathbf{A}$ can be represented by

$$\mathbf{A} = \begin{bmatrix} ae^{j(\theta_1+\theta_3)} & be^{j\theta_2} \\ -b^*e^{j(-\theta_2+\theta_3)} & a^*e^{-j\theta_1} \end{bmatrix} \tag{6}$$

where $|a|^2 + |b|^2 = 1$. The resulting set can be seen as the union of cosets of a full-diverse subset consisting of the following matrices:

$$\begin{bmatrix} ae^{j\theta_1} & be^{j\theta_2} \\ -b^*e^{-j\theta_2} & a^*e^{-j\theta_1} \end{bmatrix}, |a|^2 + |b|^2 = 1 \tag{7}$$

which is the same as the Hamiltonian constellation, mentioned in [9].

If we choose $e^{i\theta_3}$ from a PSK constellation withsize K_3, the minimum Frobenius distance among the codewords from different cosets is

$$4\sin^2(\pi/K_3) \geq d_{min-inter-coset} \geq 2\sin^2(\pi/K_3). \tag{8}$$

no matter how we choose the full-diverse subcode.

The full-diverse subcode corresponds to a set of points on S_3 (the unit sphere in R^4). The pairwise Euclidean distances of the points in S_3 are directly related to the pairwise determinant distances, as well as to the pairwise Frobenius distances, of the corresponding codewords [9]. Therefore, to maximize the minimum distance (Frobenius distance as well as the determinant distance) among the codewords of the subcode, in general, we must find a good packing in S_3.

However, to simplify the decoding process, as well as the encoding, we impose some restrictions on the constellation and assume that for each *a* and *b*, θ_1 and θ_2 are independent from each other. This restriction helps us to use the Alamouti decoder to find θ_1 and θ_2, conditioned on *a* and *b*.

If we choose *a* and *b* from a set of possibilities $\{(a_1, b_1), .., (a_n, b_n)\}$, the full-diverse subcode consists of *n* subsets. For the *i*'th subset, the minimum determinant distance (diversity product) and the minimum Frobenius distance are respectively equal to

$$\zeta(i) = \min\left\{\frac{1}{2}d_{i1}, \frac{1}{2}d_{i2}\right\}$$

$$d_{min}(i) = 8\zeta^2(i)$$

where d_{i1} and d_{i2} are the minimum Euclidean distances of the constellations related to θ_1 and θ_2 for each subset. For fixed sizes of these constellations, d_{i1} and d_{i2} will be maximized if θ_1 and θ_2 are chosen from a PSK constellations. In this case,

$$\zeta(i) = \min\left\{|a_i|\sin\left(\frac{\pi}{K_{i1}}\right), |b_i|\sin\left(\frac{\pi}{K_{i2}}\right)\right\}$$

$$d_{min}(i) = 8\zeta^2(i)$$

where K_{i1} and K_{i2} are the sizes of the PSK constellations related to θ_1 and θ_2 for the *i*'th subset.

For the special case of *n*=2, we assign $(a_1, b_1) = (\alpha, \beta)$ and $(a_2, b_2) = (\beta e^{j\varphi}, \alpha e^{j\varphi})$ where $0 < \alpha < \frac{1}{\sqrt{2}} < \beta$ and $\alpha^2 + \beta^2 = 1$. To have the largest minimum distance between two subsets, we must maximize the minimum phase difference among the points of two PSK constellations corresponding to α and β. If we choose PSK constellations with sizes M_1 and M_2, corresponding to the amplitudes α and β (in general, $M_1 \leq M_2$), then the minimum phase difference will be maximized when $\varphi = \frac{\pi}{l.c.m(M_1,M_2)}$ where $l.c.m(M_1, M_2)$ is the least common multiplier of M_1 and M_2. In this case:

$$d_{min-intersubset} = 2\beta^2\sin^2\varphi + 2(\beta\cos\varphi - \alpha)^2,$$

$$d_{min-intrasubset} = \min\left\{8\alpha^2\sin^2(\pi/M_1), 8\beta^2\sin^2(\pi/M_2)\right\}.$$

For fixed M_1 and M_2, when α increases, $d_{min-intersubset}$ and $8\beta^2\sin^2(\pi/M_2)$ will decrease and $8\alpha^2\sin^2(\pi/M_1)$ will increase. Thus, to maximize d_{min}, we must choose α such that

$$2\beta^2\sin^2\varphi + 2(\beta\cos\varphi - \alpha)^2 = 8\alpha^2\sin^2(\pi/M_1) \tag{9}$$

or

$$8\beta^2\sin^2(\pi/M_2) = 8\alpha^2\sin^2(\pi/M_1). \tag{10}$$

For $n = 2$, by using the above formulas, for each choice of the constellation sizes (i.e. K_3, M_1, M_2), we can compute other parameters of the code (i.e. α, β, φ). To choose the size of the constellations, corresponding to θ_1, θ_2 and θ_3, we must balance inter-coset, inter-subset and intra-subset distances. This situation is quite similar to the rate assignment for the different levels of a multilevel code or a trellis coded modulation scheme, to have balanced distances.

IV. Decoding

For the *t*'th block (consisting of *M* consecutive channel uses), the transmitted signal can be represented by

$$\mathbf{X}_t = \mathbf{S}_t\mathbf{S}_{t-1}...\mathbf{S}_1\mathbf{S}_0 \tag{11}$$

where $\mathbf{S}_0$ is the initial matrix and can be any unitary matrix (with a proper scaling). At the receiver, we have

$$\mathbf{R}_t = \mathbf{S}_t\mathbf{S}_{t-1}...\mathbf{S}_1\mathbf{S}_0\mathbf{H} + \mathbf{W}_t \tag{12}$$

where $\mathbf{W}_t$ is the additive noise at the receiver and $\mathbf{H}_{M\times N}$ is the matrix of fading coefficients. Received signal can be written as

$$\mathbf{R}_t = \mathbf{S}_t\mathbf{R}_{t-1} + \mathbf{W}_t - \mathbf{S}_t\mathbf{W}_{t-1}. \tag{13}$$

In this case, if the elements of the noise matrices are i.i.d. with $CN(0, \sigma^2)$ distribution, elements of $\mathbf{S}_t\mathbf{W}_{t-1}$ will be i.i.d. with the same distribution (because $\mathbf{S}_t$ is unitary). Thus, $\mathbf{W}' = \mathbf{W}_t - \mathbf{S}_t\mathbf{W}_{t-1}$ will have i.i.d. complex Gaussian elements with the variance $2\sigma^2$:

$$\mathbf{R}_t = \mathbf{S}_t\mathbf{R}_{t-1} + \mathbf{W}' \tag{14}$$

To decode the received signal, receiver must find $\hat{\mathbf{S}}$ such that $\hat{\mathbf{S}}\mathbf{R}_{t-1}$ has the minimum Euclidean distance to $\mathbf{R}_t$.

For the proposed code, for fixed values of θ_3 and a and b, we can easily use a decoding method similar to the Alamouti scheme:

$$\hat{\theta}_1 = \arg\min_{\theta_1} |\mathbf{r}_{t,1}\mathbf{r}^*_{t-1,1}e^{j\theta_3} - \mathbf{r}_{t,2}\mathbf{r}^*_{t-1,2} - \|\mathbf{R}_{t-1}\|ae^{j\theta_1}|$$

$$\hat{\theta}_2 = \arg\min_{\theta_2} |\mathbf{r}_{t,1}\mathbf{r}^*_{t-1,2} + \mathbf{r}_{t,2}\mathbf{r}^*_{t-1,1}e^{j\theta_3} - \|\mathbf{R}_{t-1}\|be^{j\theta_2}|,$$

where $\mathbf{r}_{t,i}$ is the *i*'th row of $\mathbf{R}_t$. By using the above method, decoding of the proposed code is equivalent to nK_3 parallel Alamouti decoder for PSK signals.

For very high rates and large values of n and K_3, it is more appropriate to use bucket algorithms [16] to find *a*, *b* and θ_3 (in the process of decoding). This approach helps us to reduce the number of parallel Alamouti decoders. In general, bucket algorithms operate on the data which are partitioned into *d*-dimensional hyper-rectangles, called cells or buckets. For our problem, the idea is similar to the Kannan strategy to find the closest lattice point [17]. Indeed, we can restrict the search to a neighborhood of the starting point. Complexity of these algorithms is essentially independent of K_3 and *n* (indeed, independent of the rate), because the number of the points in the search neighborhood is almost independent of the overall size of the constellation [17]. However, for practical rates and practical values of K_3 and *n*, the first approach is quite simple and bucket algorithms are not helpful.

V. Differential codes for more than two transmit antennas

To generalize the proposed double-antenna system for more than two transmit antennas, we can consider the cosets of a good full-diverse (or a code which has a partial diversity) for more than two transmit antennas. Also, we can use the proposed double-antenna construction instead of the Alamouti code in the codes which use the Alamouti scheme as the building block.

As an example, for 4 transmit antennas, we can use the cosets of a code from $Sp(2)$ (the Lie group consisting of 4 by 4 symplectic unitary matrices). The set of 2*N* by 2*N* symplectic matrices consists of the matrices which can be presented as the following:

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\bar{\mathbf{B}} & \bar{\mathbf{A}} \end{bmatrix}$$

where **A** and **B** are two *N* by *N* matrices. Codes obtained from $Sp(2)$ have a diversity of order 2 [15].

It is shown that a matrix **S** belongs to $Sp(2)$ iff there exist 2 by 2 unitary matrices **U** and **V** and diagonal matrices $\mathbf{D}_1$ and $\mathbf{D}_2$ such that [15]:

$$\mathbf{S} = \begin{bmatrix} \mathbf{U}\mathbf{D}_1\mathbf{V} & \mathbf{U}\mathbf{D}_2\bar{\mathbf{V}} \\ -\bar{\mathbf{U}}\mathbf{D}_2\mathbf{V} & \bar{\mathbf{U}}\mathbf{D}_1\bar{\mathbf{V}} \end{bmatrix} \tag{15}$$

where $\mathbf{D}_1\mathbf{D}_1^* + \mathbf{D}_2\mathbf{D}_2^* = \mathbf{I}_2$ and $\bar{\mathbf{U}}$ means the complex conjugate of $\mathbf{U}$. In [15], authors have chosen $\mathbf{U}$ and $\mathbf{V}$ which have Alamouti structures with PSK signals and they have considered $\mathbf{D}_1$ and $\mathbf{D}_2$ as the identity matrix. Also, the size of the PSK constellations are chosen such that the resulting code is full-diverse (the constellation sizes must be relatively prime). Instead, we can exploit more degrees of freedom to construct the subcode from $Sp(2)$ in order to obtain codes with greater spectral-efficiencies. For this purpose, we can consider $\mathbf{U}$ and $\mathbf{V}$ as two independent 2 by 2 unitary matrices. These matrices can be constructed by the method which we used for 2 transmit antennas. As another parametrization for $Sp(2)$, we can use the following structure:

$$\begin{bmatrix} e^{j\theta_1}\mathbf{U}\mathbf{D}_1\mathbf{V} & e^{j\theta_2}\mathbf{U}\mathbf{D}_2\bar{\mathbf{V}} \\ -e^{-j\theta_2}\bar{\mathbf{U}}\mathbf{D}_2\mathbf{V} & e^{-j\theta_1}\bar{\mathbf{U}}\mathbf{D}_1\bar{\mathbf{V}} \end{bmatrix} \tag{16}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices from *SU(2)* (Hamiltonian constellation [9]). The problem is that these two parameterizations are not one-to-one. Therefore, to construct codewords by using these parameterizations, we must impose some restrictions to avoid the overlap of the codewords.

We consider $\mathbf{D}_1 = \mathbf{D}_2 = \mathbf{I}$ (the identity matrix) and $\mathbf{U}$ and $\mathbf{V}$ as the Alamouti structures. If we assume that $0 \leq \theta_1, \theta_2, \theta_3, ...\theta_6 < \pi$ (where $\theta_3, ...\theta_6$ are the parameters of the Alamouti structures, $\mathbf{U}$ and $\mathbf{V}$) and $\theta_3 \pm \theta_5 \neq \theta_4 \mp \theta_6$, then the resulting codewords from (16) will be distinct and we can use them as the codewords of the basic subcode.

Cosets of this subcode are obtained by multiplying the first two columns of the codewords of the subcode by arbitrary unit-norm scalars.

VI. Simulation Results

Fig. 1 compares the proposed unitary space-time code and the differential methods based on orthogonal designs [6], the Cayley transform [11] and the TAST code [18]. We see that for the same spectral efficiency (same as used in [11] and [18]), the proposed method has a considerably better performance. We see that exploiting the maximum degrees of freedom (for example, using two choices for a and b in this case, i. e. $n = 2$) can be very useful in high rates. For the proposed code, 4, 4 and 3 bits are transmitted by θ_1, θ_2 and θ_3 and one bit corresponds to the choice of the layer (choosing a and b). ML decoding of the proposed codes is reasonably simple (only $16 = 2 \times 8$ linear processing) and compared to TAST, we have about 2 dB improvement with a simpler decoder (even as compared to the suboptimal decoder for TAST which is based on sphere decoder). In this case, $K_3 = 8$ and $M_1 = M_2 = 16$. We have used $\varphi = \pi/l.c.m(16, 16) = \pi/16$ and based on (9), we have chosen $\alpha = 0.895$, $\beta = 1.095$ to have relatively balance distances (among $d_{min-intersubset}$, $d_{min-intrasubset}$ and $d_{min-intercoset}$).

Fig. 2 shows the performance of the proposed code for 4 transmit antennas for 1, 2 and 4 receive antennas, compared to the performance of the code presented in [15]. For the proposed code, we have used (16) with Alamouti structure for U and V. To have a fair comparison, we have considered a rate of 3.25 bits per channel use for the proposed code which is more than the rate for the code in [15] which is 3.13 bits per channel use. To transmit at the rate of 3.25 bits per channel use, 13 bits per matrix must be transmitted. We have considered $\theta_1, \theta_2, \theta_4, \theta_5, \theta_6 \in \{0, \pi/4, 2\pi/4, 3\pi/4\}$ and $\theta_3 \in \{\pi/8, 3\pi/8, 5\pi/8, 7\pi/8\}$, to transmit 12 bits by choosing the codeword from the subcode. These sets are chosen such that the overlap among the codewordes is avoided (i.e. $\theta_3 \pm \theta_5 \neq \theta_4 \mp \theta_6$ and $0 \leq \theta_1, \theta_2, \theta_3, ...\theta_6 < \pi$). To have two cosets to transmit one extra bit, the first two columns of the unitary codewords are multiplied by $e^{j\theta_7}$ where $\theta_7 \in \{0, \pi\}$. The resulting code still has a diversity of order two. It is observed that for two and four receive antennas, the proposed code considerably outperforms the full-diverse code in [15]. Also, for reasonable probability of errors, the proposed code has a better performance, even for one receive antenna.

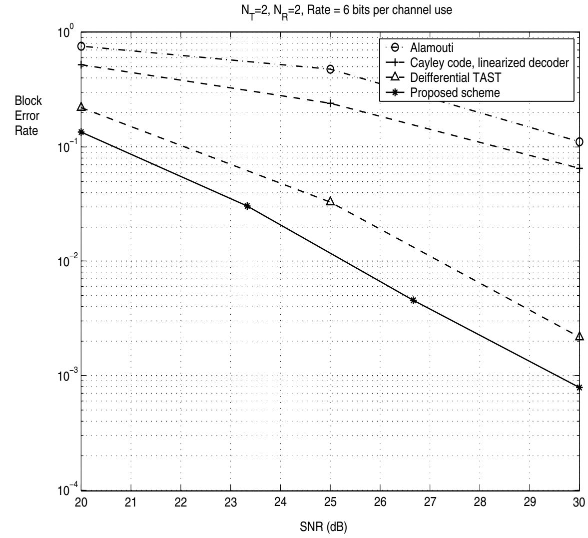

Fig. 1. Block Error Rate of the proposed codes (with one and two layers), orthogonal code, Cayley code and TAST code for $M = 2$ tranmit and $N = 2$ receive antennas with rate=6 bits per channel use.

Figure 3 compares the performance of the proposed code, the full-diverse code based on $Sp(2)$ [15] and the modified diagonal code [14] for the rates around 2 bits per channel use [1]. The modified diagonal code of [14] is designed to have a larger diversity sum as compared to the original diagonal code [7] [8], to be more appropriate for a large number of receive antennas. We see that the proposed code has a better performance, even as compared to the modified diagonal code of [14] which is obtained by computer search over various diagonal codes. It must be noted that for higher rates, the approaches based on computer exhaustive search (such as [14]) are not feasible.

[1]Due to the size constraints for the underlying PSK constellations in the code in [15], comparison with exactly the same rate is not possible. To have a fare comparison, we have chosen a higher rate for the proposed code

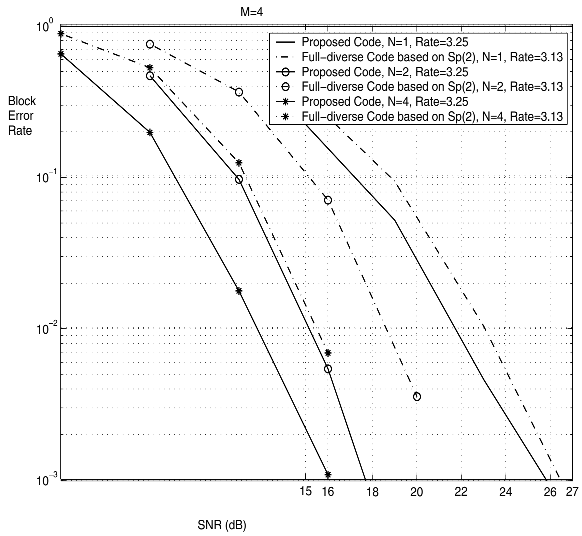

Fig. 2. Block Error Rate of the proposed code and the Full-diverse code (based on $Sp(2)$ [15]) for $M = 4$ transmit and $N = 1$, 2 and 4 receive antennas.

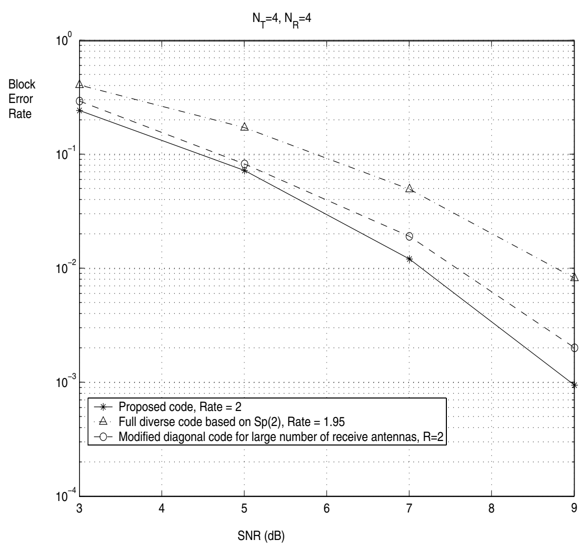

Fig. 3. Block Error Rate of the proposed code, the full-diverse code (based on $Sp(2)$ [15]) and the modified diagonal code (modified for large number of receive antennas) for $M = 4$ transmit and $N = 4$ receive antennas.

VII. CONCLUSIONS

A new method to construct unitary space-time codes have been presented. Instead of having maximum diversity, these codes are designed to have high rates with appropriate Euclidean distance. For two transmit antennas, the proposed structure allows a simple addressing and encoding method and helps us to have an efficient ML decoding. We see that by relaxing the full diversity restriction, we have a substantial improvement compared to the best differential schemes in the literature, for more than one receive antenna. Also, a similar structure is proposed for four transmit antennas. simulation results show that this structure can be very useful to construct spectral-efficient differential space-time codes.

REFERENCES

[1] I. E. Telater, "Capacity of multi-antenna gaussian channels," *Europ. Trans. Telecommun.*, pp. 585–595, Nov. 1999.

[2] G. J. Foschini, "Layered space-time architechture for wireless communication in a fading environment when using multi-element antennas," *Bell labs. Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.

[3] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Info. Theory*, vol. 44, pp. 744–765, Mar. 1998.

[4] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Selec. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[5] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Info. Theory*, vol. 45, pp. 1456–1467, July 1999.

[6] V. Tarokh and H. Jafarkhani, "A differential detection space-time block coding for time-selective fading channels," *IEEE J. Selec. Areas Commun.*, vol. 18, pp. 1169–1174, July 2000.

[7] B. Hochwald and W. Sweldens, "Differential unitary space-time modulation," *IEEE Trans. Commun.*, vol. 48, pp. 2041–2052, Dec. 2000.

[8] B. Hughes, "Differential space-time modulation," *IEEE Trans. Info Theory*, vol. 46, pp. 2567–2578, Nov. 2000.

[9] A. Shokrollahi, B. Hassibi, B. Hochwald, and W. Swelden, "Representation theory for high-rate multiple antenna code design," *IEEE Trans. Info. Theory*, pp. 2335–2367, Sep. 2001.

[10] H. Jafarkhani and V. Tarokh, "Multiple transmit antenna differential detection from generalized orthogonal designs," *IEEE Trans. Info. Theory*, pp. 2626–2631, Sep. 2001.

[11] B. Hassibi and B. Hochwald, "Cayley differential unitary space-time codes," *IEEE Trans. Info. Theory*, vol. 48, pp. 1485–1502, June 2002.

[12] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Info. Theory*, pp. 1073–1096, May 2003.

[13] X.-B. Liang and X.-G. Xia, "Unitary signal constellations for differential space-time modulation with two transmit antennas: parametric codes, optimal designs, and bounds," *IEEE Transactions on Information Theory*, pp. 2291–2322, Aug. 2002.

[14] J. Wang, M. P. Fitz, and K. Yao, "Differential unitary space-time modulation for a large number of receive antennas," in *Conference Record of the Thirty-Sixth Asilomar Conference on Signals, Systems and Computers, 2002*.

[15] Y. Jing and B. Hassibi, "Design of fully-diverse multiple-antenna codes based on *Sp(2)*," *submitted to IEEE Trans. Info. Theory*, 2003.

[16] L. Devroye, *Lecture Notes on Bucket Algorithms*. Boston, MA: Birkhauser, 1986.

[17] R. Kannan, "Improved algorithms for integer programming and related lattice problems," in *Proc. ACM Symp. Theory of Computing*, (Boston, MA), pp. 193–206, Apr. 1983.

[18] H. ElGamal and M. O. Damen, "Universal space-time coding," *IEEE Trans. Info. Theory*, vol. 49, pp. 1097–1119, May 2003.