An Efficient Signaling Method over MIMO Broadcast Channels

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Abstract

A simple signaling method for multi-antenna broadcast channel is proposed. In the proposed method, for each user, a direction in which that user has the maximum gain is determined. The base station selects the best user in terms of the larger maximum gain, and allocates the corresponding direction to that user. In each step, the maximization is performed in the null space of the former selected coordinates. Finally, the transmitted signal is a superposition of the selected coordinates. It is shown that in this method, the later selected user has no interference on the former users. A dirty paper pre-coding on top of the system eliminates the remaining interferences. Note that, in this method, the set of active users and the power allocated to each of them is easily determined, which is a significant advantage in the case of large number of users in the system. Furthermore, this method exploits multiuser diversity.

Further, the proposed algorithm is modified to work with partial side information at the base station. Simulations show that the achieved rate is close to the sum-rate capacity. In addition, the asymptotic analysis shows that the increase of the sum-rate with number of transmit antenna is linear and with the number of users, K, is proportional to $\log\log(K)$.

I. Introduction

Recently, Multiple Input Multiple Output (MIMO) systems have received a considerable attention as a promising solution to provide reliable high data rate communication. MIMO systems have proved their ability in terms of supporting high bit rates and creating highly reliable data link [1], [2]. More recently, the work on MIMO systems has been extended to MIMO multiple access channels [3]–[5].

In [3]–[5], the capacity of the MIMO broadcast channels is investigated. Since MIMO broadcast channel is non-degraded [3], characterizing of the full capacity region leads to a non-convex non-linear optimization which is a complex problem [4]. By using a result, known as *dirty paper coding*, due to Costa [6] on known interference cancelation at the transmitter, the sum-rate capacity of the broadcast channels has been fully characterized [3]–[5]. Dirty paper coding states that in a AWGN channel, if the transmitter non-causally knows the interference, it can compensate the interference, such that the capacity of the channel with interference is exactly the same as the capacity of the channel without any interference. A method of implementing the dirty paper coding are presented in [7], [8]. Using the concept of dirty paper coding, authors in [3], [4] have introduced duality between broadcast and multiple access channel to characterize the sum-rate capacity of the broadcast channel with a convex optimization problem. It has been shown that in the optimal solution, the power is allocated to at most M^2 users, where M is the number of transmit antennas [9]. In practical systems, the number of users is large. In this case, finding the set of active users by solving mentioned optimization problem is a complex operation. In addition, all the channel state information are required at the base station which needs high data rate feedback link.

On the other hand, in the case of large number of users, we can take advantage of a special kind of diversity, so-called *multiuser diversity* [10], [11]. To exploit multiuser diversity, the channel resource is allocated to the users which results in the highest throughput at that time. In this paper, we address both problems of the selecting the set of active users which exploits multiuser diversity and signaling over the selected users.

In the context of signalling over broadcast systems, some different methods are presented . In [12], a signaling method based on QR decomposition and dirty paper coding is introduced. In [13], a greedy method for selecting active users which exploit multiuser diversity is presented. In [14], a variation method of channel inversion is introduced. The inverse of the channel matrix is regularized and the data is perturbed such that the energy of the transmitted signal is reduced. However, in this method, the pre-coding matrix depends on the data and therefore is complex. In addition, the number of users must be equal to the number of transmit antennas. All the references [12]–[14] assume that the number of antennas at the receiver is equal to one.

For the case of more than one antennas at the receivers, a zero forcing method is introduced [15], [16]. However, this method is highly restricted in the sense that the number of transmit antennas must be greater than the total number of the users antennas.

In this paper, a sub-optimum method for finding the set of active users and signaling over selected users is proposed. In the proposed method, for each user, a direction in which that user has the maximum gain is determined. The base station selects the best user in terms of the larger maximum gain, and allocates the corresponding direction to that user. In each step, the maximization is performed in the null space of the former selected coordinates. Finally, the transmitted signal is a superposition of the selected coordinates. It is shown that in this method, the later selected user has no interference on the former users. A dirty paper pre-coding on top of the system eliminates the remaining interferences. Note that, in this method, the set of active users and the power allocated to each of them is easily determined, which is a significant advantage in the case of large number of users in the system. On the other hand, this method exploits multiuser diversity.

In the proposed method, the users have no interference on each other and the decoding procedure is simple. Furthermore, unlike the former methods [12]–[16], there is no restriction on the number of transmit/receive antennas. In addition, this method can easily be modified to work with partial side information at the base station.

Simulation results show that the rate of the proposed method is close to the sum-rate capacity of the system.

The rest of this paper is organized as follows: In Section II, the system model and proposed signaling method are presented. In Section III, an algorithm to select active users and corresponding coordinates is developed. Asymptotic analysis of the sum-rate achieved by the proposed method is derived in Section IV. In Section V, the simulation results and comparisons with the sum-rate capacity of the MIMO broadcast are presented. Concluding remarks are provided in the last section.

II. Preliminaries

Consider a MIMO broadcast channel with M transmit antennas and K users, each of them equipped with N receive antennas. In the flat fading environment, the baseband model of this system is defined by

$$\mathbf{y}_k = \mathbf{H_k}\mathbf{s} + \mathbf{w}_k, \quad 1 \le k \le K \tag{1}$$

where $\mathbf{H}_k \in \mathcal{C}^{N\times M}$ denotes the channel matrix from the base station to the k^{th} user, $\mathbf{s} \in \mathcal{C}^{M\times 1}$ represents the transmitted vector, and $\mathbf{y}_k \in \mathcal{C}^{N\times 1}$ signifies received vector by the k^{th} user. The vector $\mathbf{w}_k \in \mathcal{C}^{N\times 1}$ is white Gaussian noise with zero-mean and unit-variance.

In the proposed method, the transmitted vector $\mathbf{s}$ carries information for M users, defined as follows,

$$\mathbf{s} = \sum_{j=1}^{M} d_{\pi(j)}\mathbf{v}_{\pi(j)} \tag{2}$$

where $\pi(j)$, $j = 1 \ldots M$ are the indexes of a subset of users so-called *active users*, $\mathbf{v}_{\pi(j)} \in \mathcal{C}^{M\times 1}$, $j = 1 \ldots M$ are a set of orthogonal vectors, and $d_{\pi(j)}$ includes information for the user $\pi(j)$. In addition, dirty-paper pre-coding is used on top of the system such that if $i > j$, the interference of the user $\pi(i)$ over user $\pi(j)$ is zero.

For demodulation, the user $\pi(j)$ multiplies the received vector to a normal vector $\mathbf{u}'_{\pi(j)}$, where $(.)'$ denotes transpose conjugate operation.

In the next section, we specify a method to select the set of active users, modulation vectors $\mathbf{v}_{\pi(j)}$, and demodulation vectors $\mathbf{u}_{\pi(j)}$ for $j = 1 \ldots M$.

III. Determination of the active users, modulation, and demodulation vectors

In this part, it is assumed that the channel state information is available at the transmitter. Later, this algorithm is modified such that only partial channel state information is required.

Each stage of the algorithm includes two optimizing operations. First, for each user, finding a direction in which that user has maximum gain. Second, selecting the best user in terms of the larger gain. This optimization is performed in the null space of the former selected coordinates. In the following, the proposed algorithm is presented.

1) Set $j = 1$ and the condition matrix $\mathbf{G}_{eq} = \mathbf{0}_{M\times M}$.
2) Find $\sigma^2_{\pi(j)}$ where

$$\begin{array}{c} \sigma^2_{\pi(j)} = \max_r \max_x \mathbf{x}^{'}\mathbf{H}^{'}_r\mathbf{H}_r\mathbf{x}. \\ \mathbf{x}^{'}\mathbf{x} = 1 \\ \mathbf{G}'_{eq}\mathbf{x} = 0 \end{array} \tag{3}$$

Set $\pi(j)$ and $\mathbf{v}_{\pi(j)}$ be equal to the optimizing parameter r and $\mathbf{x}$, respectively.
3) Set

$$\mathbf{u}_{\pi(j)} = \frac{1}{\sigma_{\pi(j)}}\mathbf{H}_{\pi(j)}\mathbf{v}_{\pi(j)}. \tag{4}$$

4) Set $\mathbf{g}_j = \mathbf{v}_{\pi(j)}$, where $\mathbf{g}_j$ is the j^{th} column of the matrix $\mathbf{G}_{eq}$.
5) $j \leftarrow j + 1$. If $j \leq M$ go to step two, otherwise stop.

The following theorem proves that if $i < j$, the interference of the user $\pi(j)$ over user $\pi(i)$ is zero.

Theorem 1 *Consider the following optimization problem,*

$$\begin{array}{c} \max_x \mathbf{x}^{'}\mathbf{H}^{'}\mathbf{H}\mathbf{x}, \\ \mathbf{x}^{'}\mathbf{x} = 1 \\ \mathbf{G}'_{eq}\mathbf{x} = 0 \end{array} \tag{5}$$

where $\mathbf{H}$ *and* $\mathbf{G}_{eq} = [\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_\varrho]$ *are complex matrices. Let* $\mathbf{v}_1$ *be the vector that maximizes (5) and* σ^2 *be the result of optimization. Define a vector* $\mathbf{u}_1$ *as follows:*

$$\mathbf{u}_1 = \frac{\mathbf{H}\mathbf{v}_1}{\sigma}. \tag{6}$$

If there exists a vector $\mathbf{v}_2$ *such that* $\mathbf{G}^{'}_{eq}\mathbf{v}_2 = 0$ *and* $\mathbf{v}^{'}_1\mathbf{v}_2 = 0$*, then*

$$\mathbf{u}^{'}_1\mathbf{H}\mathbf{v}_2 = 0. \tag{7}$$

Proof: According to (6),

$$\mathbf{u}^{'}_1\mathbf{H}\mathbf{v}_2 = (\frac{\mathbf{H}\mathbf{v}_1}{\sigma})'\mathbf{H}\mathbf{v}_2 = \frac{1}{\sigma}\mathbf{v}^{'}_1\mathbf{H}^{'}\mathbf{H}\mathbf{v}_2. \tag{8}$$

To optimize the cost function in (5), we use the Lagrange multiplier method,

$$L(\mathbf{x}, \lambda, \mathbf{\Theta}) = \mathbf{x}^{'}\mathbf{H}^{'}\mathbf{H}\mathbf{x} + \lambda(1 - \mathbf{x}^{'}\mathbf{x}) + \mathbf{\Theta}\ \mathbf{G}_{eq}^{\prime}\mathbf{x}, \tag{9}$$

where λ and $\mathbf{\Theta} = \text{diag}([\theta_1, \theta_2, \ldots, \theta_\varrho])$ are Lagrange multipliers. The gradient of $L(\mathbf{x}, \lambda, \mathbf{\Theta})$ corresponding to the vector $\mathbf{x}$ is,

$$\nabla_{\mathbf{x}} L(\mathbf{x}, \lambda, \mathbf{\Theta}) = 2\mathbf{H}^{'}\mathbf{H}\mathbf{x} - 2\lambda\mathbf{x} + \sum_{\tau=1}^{\varrho} \Theta_\tau \mathbf{g}_\tau. \tag{10}$$

Since $\mathbf{v}_1$ maximizes the cost function, it satisfies (10). Therefore, we have,

$$\nabla_{\mathbf{x}} L(\mathbf{v}_1, \lambda, \mathbf{\Theta}) = 2\mathbf{H}^{'}\mathbf{H}\mathbf{v}_1 - 2\lambda\mathbf{v}_1 + \sum_{\tau=2}^{\varrho} \theta_\tau \mathbf{g}_\tau = 0. \tag{11}$$

Multiplying both sides of (11) to $\mathbf{v}_2^{\prime}$ results in

$$\mathbf{v}_2^{'}\nabla_{\mathbf{x}} L(\mathbf{v}_1, \lambda, \mathbf{\Theta}) = 2\mathbf{v}_2^{'}\mathbf{H}^{'}\mathbf{H}\mathbf{v}_1 - 2\lambda\mathbf{v}_2^{'}\mathbf{v}_1 + \mathbf{v}_2^{'}\sum_{\tau=2}^{\varrho} \theta_\tau \mathbf{g}_\tau = 0. \tag{12}$$

Substituting $\mathbf{v}_1^{1}\mathbf{v}_2 = 0$ and $\mathbf{v}_2^{\prime}\mathbf{g}_\tau = 0$ ($\tau = 1, \ldots, \varrho$) in (12), we have

$$\mathbf{v}_2^{'}\mathbf{H}^{'}\mathbf{H}\mathbf{v}_1 = 0. \tag{13}$$

Finally, (8) and (13) result in,

$$\mathbf{u}_1^{'}\mathbf{H}\mathbf{v}_2 = 0. \tag{14}$$

■

The interference of the user $\pi(i)$ over user $\pi(j)$ is equal to $\mathbf{u}_{\pi(j)}^{\prime}\mathbf{H}_{\pi(i)}\mathbf{v}_{\pi(i)}$. According to the theorem 1, if $j > i$, the interference of the user $\pi(i)$ over user $\pi(j)$ is zero. Recall that if $i > j$, the interference of the user $\pi(i)$ over user $\pi(j)$ is already canceled with dirty paper pre-coding. Therefore, the broadcast channel is reduced to a set of independent parallel channels with gain $\sigma_{\pi(j)}$, $j = 1, \ldots, M$. As a result, the sum-rate of the system is,

$$R = \sum_{j=1}^{M} \log_2(1 + \sigma_{\pi(j)}^2 P_j), \tag{15}$$

where P_j is the power allocated to the user $\pi(j)$ and $\sum_{j=1}^{M} P_j \leq P$. (Note that in the channel model, the power of the noise is normalized.) The power can optimally allocated by using water-filling method [17].

A. Modified Algorithm

As it has been mentioned before, in this algorithm, a major part of the processing can be accomplished at the receivers. Therefore the perfect channel state information is not required at the transmitter which results a significant decreasing in the rate of feedback.

The modified algorithm is as follows:

1) Set $j = 1$ and $\mathbf{G}_{eq} = \mathbf{0}_{M\times M}$.
2) Each user calculates $\sigma_{r(j)}^2$, defined as follows

$$\begin{aligned} \sigma_{r(j)}^2 = \ & \max\nolimits_x \mathbf{x}^{'}\mathbf{H}_r^{'}\mathbf{H}_r\mathbf{x}. \\ & \mathbf{x}^{'}\mathbf{x} = 1 \\ & \mathbf{G}_{eq}^{\prime}\mathbf{x} = 0 \end{aligned} \tag{16}$$

$\mathbf{v}_{r(j)}$ represents the optimizing parameter $\mathbf{x}$.

3) Each user calculates

$$\mathbf{u}_{r(j)} = \frac{1}{\sigma_{r(j)}} \mathbf{H}_r \mathbf{v}_{r(j)}. \tag{17}$$

4) Each user sends $\sigma^2_{r(j)}$ and $\mathbf{v}_{r(j)}$ to the base station, if $\sigma^2_{r(j)} \geq \text{th}(j)$. $\text{th}(j)$ is a threshold which is predetermined by the base station.
5) Base station selects the user with the largest $\sigma^2_{r(j)}$. Let $\pi(j)$ be the index of the selected user. The corresponding gain and coordinate of that user are $\sigma^2_{\pi(j)}$ and $\mathbf{v}_{\pi(j)}$, respectively.
6) The $\pi(j)^{th}$ user sends $\mathbf{u}_{\pi(j)} \mathbf{H}_{\pi(j)} \mathbf{v}_{\pi(i)}$, $i = 1, \ldots, j-1$, to the base station.
7) Base station sends $\mathbf{v}_{\pi(j)}$ to all users. All users include $\mathbf{v}_{\pi(j)}$ in $\mathbf{G}_{eq}$ as the j^{th} column.
8) $j \leftarrow j+1$. If $j \leq M$ go to step two, otherwise stop.

Note that the performance of this method is exactly the same as the first algorithm. However, the rate of the feedback required in the modified algorithm is less than that of the first algorithm. Also for dirty paper coding, the partial channel state information is required. In step 6, required information for dirty paper coding is sent to the receiver.

IV. Performance analysis

The distribution of the $\sigma^2_{\pi(j)}$, $j = 1, \ldots, M$ is required to analyze the performance of the system. The following lemma leads us to these distributions.

Lemma 1 *Consider a vector space Ω defined by,*

$$\Omega = \{\mathbf{x} \quad | \quad \mathbf{x} \in \mathcal{C}^{M \times 1}, \quad \mathbf{G}'_{eq} \mathbf{x} = 0\}, \tag{18}$$

where $\mathbf{G}_{eq}$ is a complex matrix. Assume that Ω is spanned by a set of orthogonal vectors $\{\phi_1, \phi_2, \ldots, \phi_\nu\}$, $\nu \leq M$, then for a complex matrix $\mathbf{H}$,

$$\begin{array}{c} \max_x \mathbf{x}'\mathbf{H}'\mathbf{H}\mathbf{x} = \sigma^2 \\ \mathbf{x}'\mathbf{x} = 1 \\ \mathbf{x} \in \Omega \end{array} \tag{19}$$

where σ is the maximum singular value of the matrix $\widehat{\mathbf{H}}$ [1], where

$$\widehat{\mathbf{H}} = \mathbf{H}\mathbf{\Phi} \tag{20}$$

and

$$\mathbf{\Phi} = [\phi_1, \phi_2, \ldots, \phi_\nu]. \tag{21}$$

Proof: σ^2, the square of the maximum singular value of the matrix $\widehat{\mathbf{H}}$ is equal to [18],

$$\begin{array}{c} \sigma^2 = \max_y \mathbf{y}'\widehat{\mathbf{H}}'\widehat{\mathbf{H}}\mathbf{y} \\ \mathbf{y}'\mathbf{y} = 1 \end{array} \tag{22}$$

By substituting (20) in (22), we have,

$$\begin{array}{c} \sigma^2 = \max_y \mathbf{y}'\mathbf{\Phi}'\mathbf{H}'\mathbf{H}\mathbf{\Phi}\mathbf{y}. \\ \mathbf{y}'\mathbf{y} = 1 \end{array} \tag{23}$$

Let $\mathbf{x} = \mathbf{\Phi}\mathbf{y} = \sum_{\nu=1}^{\nu} y_\nu \phi_\nu$. Since $\{\phi_1, \phi_2, \ldots, \phi_\nu\}$ are an orthogonal vector set, $\mathbf{y}'\mathbf{y} = \mathbf{x}'\mathbf{x}$. In addition, $\mathbf{x}$ is a linear combination of vectors $\{\phi_1, \phi_2, \ldots, \phi_\nu\}$ and $\mathbf{x} \in \Omega$. Consequently,

$$\begin{array}{c} \sigma^2 = \max_x \mathbf{x}'\mathbf{H}'\mathbf{H}\mathbf{x}. \\ \mathbf{x}'\mathbf{x} = 1 \\ \mathbf{x} \in \Omega \end{array} \tag{24}$$

[1]Square of the maximum singular value of a matrix A is equal to the maximum eigenvalue of the matrix $A'A$ [18].

■

According to lemma 1, $\sigma^2_{\pi(j)}$ in (3) is equal to,

$$\sigma^2_{\pi(j)} = \max_r \sigma^2_{(r,j)}, \tag{25}$$

where $\sigma^2_{(r,j)}$ is the square of maximum singular value of $\widehat{\mathbf{H}}_{(r,j)} = \mathbf{H}_r \mathbf{\Phi}_j$ (or maximum eigenvalue of $\widehat{\mathbf{H}}_{(r,j)} \widehat{\mathbf{H}}'_{(r,j)}$) and $\mathbf{\Phi}_j$ is a matrix with orthogonal columns which span the the complex vector space $\Omega = \{\mathbf{x} | \quad \mathbf{x} \in \mathcal{C}^{M \times 1} \quad \mathbf{G}'_{eq}\mathbf{x} = 0\}$.

Note that in (3), $\mathbf{G}'_{eq}$ has $j-1$ non-zero orthogonal rows. Therefore, the dimension of the complex vector space Ω is $M-(j-1)$, resulting in $\mathbf{\Phi}_j \in \mathcal{C}^{M \times (M-j+1)}$. Since $\mathbf{\Phi}_j$ has orthogonal columns and $\mathbf{H}_r$ has Gaussian distribution, the distribution of $\widehat{\mathbf{H}}_{(r,j)} \in \mathcal{C}^{N \times (M-j+1)}$ is Gaussian as well. Therefor, $\widehat{\mathbf{H}}'_{(r,j)} \widehat{\mathbf{H}}_{(r,j)}$ has Wishart distribution. The following lemma formulates the distribution of the maximum eigenvalue of a Wishart matrix.

Lemma 2 *[19] Assume that the entries of $A \in \mathcal{C}^{m \times n}$ has zero mean, unit variance Gaussian distribution, then, the Cumulative Distribution Function (CDF) of the maximum eigenvalue of the matrix $A'A$ is equal to,*

$$F(z) = Pr(\lambda_{max} \leq z) = \frac{1}{\prod_{k=1}^{a} \Gamma(b-k+1)\Gamma(a-k+1)} \det(\mathbf{\Psi}) \tag{26}$$

where $a = \min\{m,n\}$, $b = \max\{m,n\}$, and $\mathbf{\Psi}$ is an $a \times a$ Hankel matrix of $z \in (0,\infty)$, with entries given by

$$\mathbf{\Psi}(p,q) = \gamma(b-a+p+q-1, z), \qquad p,q = 1,\ldots,s \tag{27}$$

and γ is incomplete gamma function.

By using lemma 1 and lemma 2, $F_j(z)$, the CDF of the $\sigma^2_{(r,j)}$ is determined, by substituting

$$\begin{aligned} a &= \min\{M-j+1, N\} \\ b &= \max\{M-j+1, N\}, \end{aligned} \tag{28}$$

in (26).

Since $\sigma^2_{(r,j)}$, $r = 1,\ldots,K$ are independent for different r, according to (25) [20]

$$P(\sigma^2_{\pi(j)} < z) = [F_j(z)]^K. \tag{29}$$

By using (15) and (29), we can calculate the average sum-rate of the proposed method,

$$\overline{R} = E\{R\} = \sum_{j=1}^{M} E\left\{\log_2(1 + \sigma^2_{\pi(j)} P_j\right\}. \tag{30}$$

Analysis of the the asymptotic behavior ($K \longrightarrow \infty$) of the average sum-rate provides a good insight about the performance of the proposed method. For simplicity, we assume that the allocated powers to active users are the same (no water-filling). Note that in high Signal-to-Noise-Ratio (SNR), the optimal allocated powers are almost equal for different coordinates. Therefore, in high SNR regime the investigation of the sum-rate without using water-filling provides a tight lower bound for sum-rate with optimal power allocation.

When $K \longrightarrow \infty$, the behavior of $\sigma^2_{\pi(j)}$ depends on the tail of the distribution function $F_j(z)$ [10]. The following lemma allows us to use this fact to derive limiting distribution.

Lemma 3 *[20] Let $z_1, z_2, \ldots, z_K$ be i.i.d random variable with with a common CDF $F(.)$ and probability density function $f(.)$ satisfying that $F(.)$ is less than one for all z and is twice differentiable, and is such that*

$$\lim_{z \longrightarrow \infty} \frac{1 - F(z)}{f(z)} = c > 0 \tag{31}$$

for some constant c. Then,

$$\max_{1<r<K} z_r - l_K \tag{32}$$

converge in distribution to a limiting random variable with CDF

$$\exp(-e^{-\frac{u}{c}}). \tag{33}$$

In the above, l_K is given by $F(l_k) = 1 - \frac{1}{K}$.

Lemma 3 states that the maximum of such i.i.d random variables grows like l_K [10].

It is easy to see that $F_j(z)$ satisfies all conditions required in lemma 3.

By substituting the expansion [21],

$$\gamma(n+1, z) = n! \left(1 - \sum_{m=1}^{n} \frac{x^m}{m!}\right) \tag{34}$$

in (26), we have

$$F_j(z) = Pr(\lambda_{max} \leq z) = 1 - \frac{e^{-z} z^{a+b-2}}{(a-1)!(b-1)!} \left(1 + O(z^{-1} e^{-z})\right). \tag{35}$$

According to the lemma 3, $\sigma^2_{\pi(j)}$ grows like $l_{K,j}$ which is given by $1 - F_j(l_{K,j}) = \frac{1}{K}$. Considering (35), $l_{K,j}$ is defined by

$$l_{K,j} = \log(K) + (a+b-2)\log\log(K) - \log[(a-1)!(b-1)!] + O(\log\log(K)). \tag{36}$$

Finally, by substituting (28) in (36), we have,

$$l_{K,j} = \log(K) + (N+M-j-1)\log\log(K) - \log[(N-1)!(M-j)!] + O(\log\log(K)). \tag{37}$$

It can be shown that [22],

$$Pr\left\{l_{K,j} - c\log\log(k) \leq \sigma^2_{\pi(j)} \leq l_{K,j} + c\log\log(k)\right\} \geq 1 - O\left(\frac{1}{\log K}\right). \tag{38}$$

Since $\log(.)$ is an increasing function and according to (38), we have

$$Pr\left\{\log_2\left(1 + \rho[l_{K,j} - c\log\log(k)]\right) \leq \log_2(1 + \rho\sigma^2_{\pi(j)}) \leq \log_2\left(1 + \rho[l_{K,j} - c\log\log(k)]\right)\right\} \geq 1 - O\left(\frac{1}{\log K}\right), \tag{39}$$

and

$$Pr\left\{\sum_{j=1}^{M} \log_2\left(1 + \rho[l_{K,j} - c\log\log(k)]\right) \leq R \leq \sum_{j=1}^{M} \log_2\left(1 + \rho[l_{K,j} - c\log\log(k)]\right)\right\} \geq 1 - MO\left(\frac{1}{\log K}\right). \tag{40}$$

where ρ is the SNR of the active users, if the allocated power to the active users are the same.

Equation (40) shows that the average sum-rate of the proposed method linearly increases with the number of transmit antennas. Furthermore, its increase with the number of users in the system is proportional with $\log\log(K)$.

V. SIMULATION RESULTS

In this section, the sum-rate of the proposed method is compared with the sum-rate capacity. In these simulations, the perfect channel state information is assumed to be available. In addition, the total power is equal to 15 which is optimally allocated to active users by using water-filling method. To simulate sum-rate capacity, the algorithm represented in [23] is used.

Figure 1 depicts the average sum-rate of the proposed method and average sum-capacity versus the number of users in the system for different number of receive antennas. In Fig. 1, the number of transmit antennas, M, is equal to 4. It is apparent that the average sum-rate increases with the number of receive antennas. In addition, the sum-rate of the proposed method is very close to the sum-capacity.

Figure 2 shows the sum-rate of the proposed method versus the number of transmit antennas for different number of receive antennas. In Fig. 2, the number of users is equal to 128. It can be seen that the average sum-rate linearly increases with the number of transmit antennas.

VI. CONCLUSION

In this paper, a simple signaling method for multi-antenna broadcast channel is proposed. In the proposed method, for each user, a direction in which that user has the maximum gain is determined. The base station selects the best user in terms of the larger maximum gain, and allocates the corresponding direction to that user. In each step, the maximization is performed in the null space of the former selected coordinates. In the proposed method, for each user, a direction in which that user has maximum gain is determined. The base station selects the user with the maximum gain, and allocates the corresponding coordinate to that user. In each step, the optimization is performed in the null space of the former selected coordinates. Finally, the transmitted signal is a superposition of the selected coordinates. It is shown that in this method the later selected user has no interference on the former users. A dirty paper pre-coding on top of the system eliminates the remaining interferences.

Note that, in this method, the set of active users and the power allocated to each of them is easily determined, which is a significant advantage in the case of large number of users in the system. Furthermore, this method exploits multiuser diversity.

Further, the proposed algorithm is modified to work with partial side information at the base station. In the modified algorithm, the rate of the required feedback is low. However, the performance of this algorithm is the same with the performance of the main algorithm.

Simulations show that the achieved rate is close to the sum-rate capacity. In addition, the asymptotic analysis shows that the increases of the sum-rate with number of transmit antenna is linear and with the number of users, K, is proportional to $\log\log(K)$.

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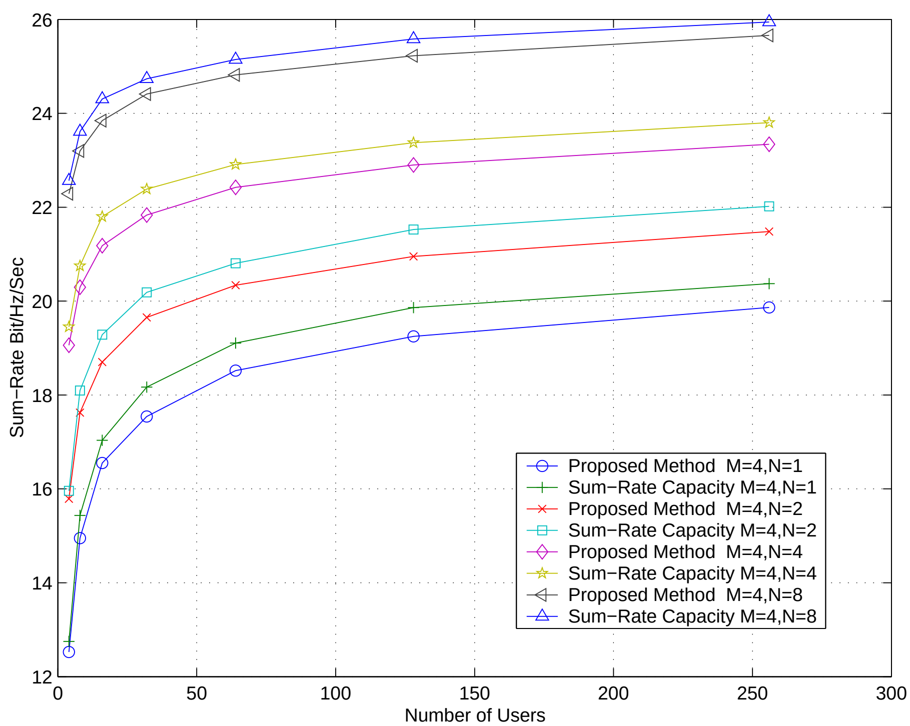

Fig. 1. Average sum rate of the proposed method and average sum capacity versus number of users

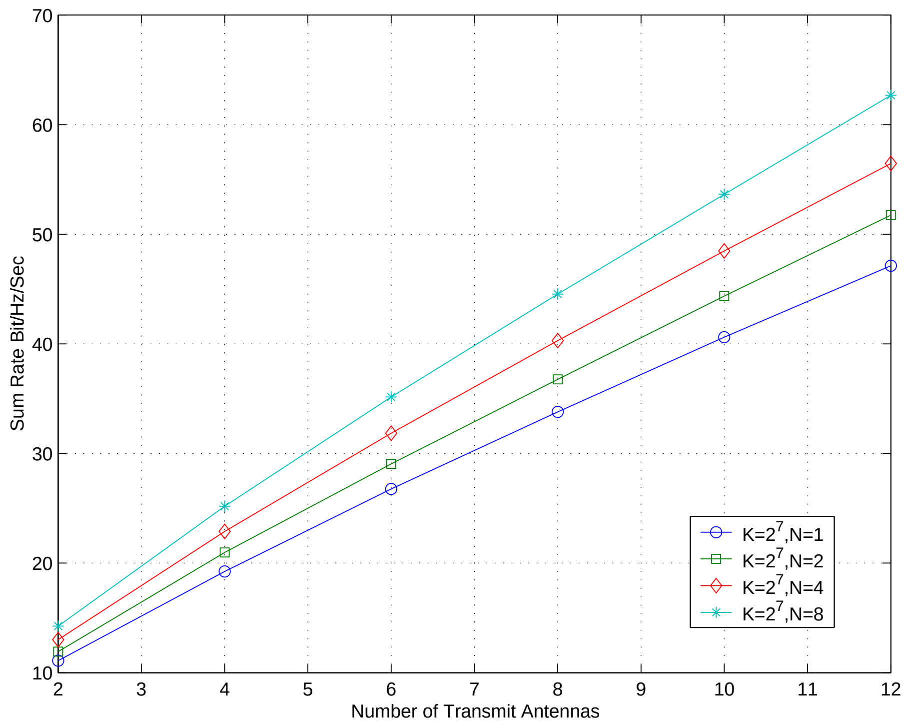

Fig. 2. Average sum rate of the proposed method versus the number of transmit antennas