PAPR Reduction Using Integer Structures In OFDM Systems

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***Abstract*— In this work, the problem of reducing the Peak to Average Power Ratio (PAPR) in an Orthogonal Frequency Division Multiplexing (OFDM) system is considered. We design a cubic constellation, called the Hadamard constellation, whose boundary is along the bases defined by the Hadamard matrix in the transform domain. Then, we further reduce the PAPR by applying a Selective Mapping (SLM) technique. The encoding method, following the method introduced in [1], is derived from a decomposition, known as the Smith Normal Form (SNF), and has a minimal complexity. This new technique offers a PAPR that is significantly lower than that of the best known techniques without any lose in terms of energy and/or spectral efficiency and without any side information being transmitted. Moreover, it has a low computational complexity.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission technique which is widely adopted in different communication applications. OFDM prevents Inter Symbol Interference (ISI) by inserting a guard interval and mitigates the frequency selectivity of a multi-path channel by using a simple equalizer. This simplifies the design of the receiver and leads to inexpensive hardware implementations. Moreover, OFDM offers some advantages in higher order modulations and in the networking operation that position OFDM as the technique of choice for the next generation of wireless networks. However, OFDM systems have the undesirable feature of a large Peak to Average Power Ratio (PAPR) of the transmitted signals. Consequently to prevent the spectral growth of the OFDM signal, the transmit amplifier must operate in its linear regions. Therefore, power amplifiers with a large linear range are required for OFDM systems, but such amplifiers will continue to be a major cost component of OFDM systems. Consequently, reducing the PAPR is pivotal to reducing the expense of OFDM systems.

A large number of different methods for the PAPR reduction have been proposed. In [1]–[3], a constellation shaping technique is proposed to reduce the PAPR of the OFDM signals. The encoding and decoding algorithms of this method are based on the relations and generators in a free Abelian group. Due to the large complexity of this algorithm, its practical implementation, in the case of Fourier basis, is very challenging. In this paper, we propose a constellation as a shaping method in an OFDM system with a low complexity encoding method, based on [1]–[3], and a considerable PAPR reduction. A Selective Mapping (SLM) method is applied in conjunction with our constellation to further reduce the PAPR in the OFDM signals.

The rest of this paper is organized as follows. After the preliminary section, constellation shaping is introduced in Section III. A brief description of the work in [1] is also given. Section IV describes the Hadamard constellation as a shaping method in OFDM systems. Some issues of the encoding and decoding algorithms are also investigated. An SLM method is applied to the Hadamard constellation in Section V. Section VI is devoted to some numerical results and a comparison of our method with some recent works. The paper is concluded in Section VII.

II. Preliminary

Let $\mathbf{x} = (x_0, x_1, \cdots, x_{N-1})^T$ denote a vector of $2N$ Dimensional ($2N$-D) constellation point selected from a set of N identical 2-D sub-constellations, $\{x_0, x_1, \cdots, x_{N-1}\}$, to be transmitted by using one OFDM vector of size N; namely, $\mathbf{y}$. The discrete time samples of the OFDM signal can be expressed as

$$y_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{j2\pi \frac{nk}{N}}. \quad (1)$$

The matrix representation of this signal is

$$\mathbf{y} = \mathbf{F}_N \mathbf{x}, \quad (2)$$

where $\mathbf{y} = (y_0 \cdots y_{N-1})^T$, $\mathbf{x} = (x_0 \cdots x_{N-1})^T$, and $\mathbf{F}_N$ is the known **IFFT** matrix.

The 2-D constellation points, $\{x_0, x_1, \cdots, x_{N-1}\}$, may add constructively and produce a time domain signal with a large amplitude. Thus, the output signal $\mathbf{y}$ can have high output levels, which leads to the requirement of an expensive analog front end.

Usually, the level of the amplitude fluctuation of the discrete time OFDM signal is measured in terms of the peak factors that indicate the ratio of the peak power to the average envelop power of the signal as

$$\text{PAPR}(\mathbf{y}) = \frac{\|\mathbf{y}\|_\infty^2}{E_y\left[\frac{1}{N}\|\mathbf{y}\|^2\right]}. \quad (3)$$

III. Constellation Shaping

In constellation shaping, a constellation in the frequency domain must be found such that the resulting shaping region in the time domain has a low PAPR. A new constellation shaping method is introduced in [1]–[3] by Kwok and Jones. Based on the encoding algorithm introduced in [1]–[3], we propose a cubic constellation, along with an SLM method to reduce the PAPR in an OFDM system.

In a PAPR reduction problem, the peak value of the signal vector should be bounded by a specified value ($\|\mathbf{y}\|_\infty \leq \beta$). This constraint on the time domain boundary translates to a parallelotope in the frequency domain. The points inside this parallelotope are used as constellation points in transmitting the OFDM signals. The main challenge in constellation shaping is to find a unique way to map the input data to the constellation points such that the mapping (encoding) and its inverse (decoding) can be implemented by a reasonable complexity. In [1], it is shown that the parallelotope boundary is along the columns of $\mathbf{Q}_N$, which is based on rounding off the scaled version of the IFFT matrix. The encoding of this constellation is performed by the decomposition of the matrix $\mathbf{Q}_N$, based on column and row operations. Indeed, in the mathematical literature this decomposition is known as Smith Normal Form (SNF) of an integer matrix [4].

Theorem 1: [4] Any integer matrix $\mathbf{Q}_N$ can be decomposed into $\mathbf{Q}_N = \mathbf{UDV}$, where $\mathbf{D}$ is diagonal with the entries $\{\sigma_i\}_{i=1}^N$ such that $\sigma_1 \mid \sigma_2 \mid \cdots \mid \sigma_N$, and $\mathbf{U}$ and $\mathbf{V}$ are unimodular matrices. The matrix $\mathbf{D}$ is called the SNF of the matrix $\mathbf{Q}_N$.
The condition $\sigma_1 \mid \sigma_2 \mid \cdots \mid \sigma_N$ in Theorem 1 is defined for finding a unique decomposition and can be ignored in the encoding procedure.

The complexity of the encoding algorithm is the result of the computation of the SNF decomposition for the matrix $\mathbf{Q}_N$. We can use the SNF decomposition methods for the encoding procedure; however, the computational complexity for OFDM systems that are defined by the IFFT matrix remains very high.

In [2], it is shown that in a multi-carrier modulation system whose relation between time and frequency domain signals in equation (2) is based on the Hadamard matrix[1], the encoding and decoding algorithm can be implemented by a butterfly structure that uses only bit shifting and logical AND. This simplicity is hidden in the following recursive formula for the Hadamard matrix:

$$\mathbf{H}_{2^n} = \begin{bmatrix} \mathbf{H}_{2^{n-1}} & \mathbf{H}_{2^{n-1}} \\ \mathbf{H}_{2^{n-1}} & -\mathbf{H}_{2^{n-1}} \end{bmatrix}, \text{ where } \mathbf{H}_1 = [1]. \quad (4)$$

The SNF of (4) can be easily computed as follows:

$$\mathbf{U}_{2^n} = \begin{bmatrix} \mathbf{U}_{2^{n-1}} & 0 \\ \mathbf{U}_{2^{n-1}} & \mathbf{U}_{2^{n-1}} \end{bmatrix} \mathbf{D}_{2^n} = \begin{bmatrix} \mathbf{D}_{2^{n-1}} & 0 \\ 0 & 2\mathbf{D}_{2^{n-1}} \end{bmatrix}$$
$$\mathbf{V}_{2^n} = \begin{bmatrix} \mathbf{V}_{2^{n-1}} & \mathbf{V}_{2^{n-1}} \\ 0 & -\mathbf{V}_{2^{n-1}} \end{bmatrix} \mathbf{U}_{2^n}^{-1} = \begin{bmatrix} \mathbf{U}_{2^{n-1}} & 0 \\ -\mathbf{U}_{2^{n-1}} & \mathbf{U}_{2^{n-1}} \end{bmatrix}, \quad (5)$$

[1]The matrix $\mathbf{Q_N}$ is based on the Hadamard matrix.

where $\mathbf{U}_1 = \mathbf{U}_1^{-1} = \mathbf{D}_1 = \mathbf{V}_1 = [1]$. Therefore, the encoding algorithm for this constellation can be represented by

$$\begin{aligned} \hat{\mathbf{x}} &= \mathbf{U}_N \boldsymbol{\lambda} \\ \boldsymbol{\gamma} &= \left\lfloor \frac{\mathbf{H}_N^T \hat{\mathbf{x}}}{N} \right\rfloor \\ \mathbf{x} &= \hat{\mathbf{x}} - \mathbf{H}_N \boldsymbol{\gamma}, \end{aligned} \quad (6)$$

where $N = 2^n$, and $\boldsymbol{\lambda}$ is the canonical representation of integers I representing the constellation points. The time domain signal is computed using the inverse of the Hadamard matrix. The canonical representation of any integer can be calculated by the recursive modulo operations; namely,

$$\begin{aligned} \lambda_1 &= I \bmod \sigma_1 \\ I_1 &= \frac{I - \lambda_1}{\sigma_1} \\ \lambda_i &= I_{i-1} \bmod \sigma_i \\ I_i &= \frac{I_{i-1} - \lambda_i}{\sigma_i}, \end{aligned} \quad (7)$$

where $1 \leq i \leq N$.
The reverse operation for finding I from the N-D vector $\mathbf{x}$ is [1]

$$\begin{aligned} &\boldsymbol{\lambda} = \mathbf{U}_N^{-1} \mathbf{x} = (\lambda_1, \lambda_2, \cdots, \lambda_N)^T, \\ &\tilde{\lambda}_i = \lambda_i \bmod \sigma_i, \\ &I = \tilde{\lambda}_1 + \sigma_1(\tilde{\lambda}_2 + \sigma_2(\cdots(\tilde{\lambda}_{N-1} + \sigma_{N-1}\tilde{\lambda}_N)\cdots)). \end{aligned} \quad (8)$$

IV. Hadamard Constellation in OFDM Systems

As mentioned in Section III, if the IFFT operation in OFDM multicarrier modulation could be changed by the Hadamard matrix, a very simple encoding algorithm would result. However, this type of multicarrier modulation is not very popular because it does not offer all the advantages of conventional OFDM systems [5]. The constellation that should be used in an OFDM system has a boundary along the bases of the IFFT matrix, but the encoding of containing constellation points cannot be easily implemented. The boundary of this constellation is shown by a solid line in Fig. 1. In this paper, we propose a cubic constellation, called the Hadamard constellation, for an OFDM system whose boundary is along the bases defined by the Hadamard matrix in the transform domain. The boundary of the Hadamard constellation is depicted by a dashed line in Fig. 1. The IFFT and Hadamard matrices are both orthogonal matrices, and therefore, the constellation boundaries along these orthogonal bases are a rotated version of each other. However, a large number of points within these boundaries are the same, as shown in Fig. 1. Therefore, by substituting the proper constellation along the IFFT matrix by a constellation along the Hadamard matrix in an OFDM system, the resulting PAPR is reduced and the encoding of this constellation, based on the SNF decomposition of the Hadamard matrix, is simple and practical. Moreover, the encoding algorithm can be implemented by a butterfly structure that uses bit shifting and logical AND structures [1].

Note that in this work, the time domain signal, $\mathbf{y}$, is obtained by the IFFT transformation of the constellation point, $\mathbf{x}$ (i.e., we do not compute it by the inverse of the Hadamard transform). This results in a traditional OFDM signal based on IFFT/FFT.

The advantage of using the Hadamard constellation is not only a simple encoding algorithm with a low PAPR, but also the possibility of concatenating it with other methods for PAPR reduction. This motivates us to apply a SLM technique [6] to the Hadamard constellation in an OFDM system. In typical SLM methods [6], the major PAPR reduction is achieved by the first few redundant bits. Employing more redundant bits necessitates a high level of complexity to obtain modest improvements in the PAPR value. However, in the proposed SLM method, employing the Hadamard constellation causes a considerable PAPR reduction by itself. As a result, this method, by just using one or two redundant bits in SLM, significantly outperforms the other PAPR reduction techniques, reported in the literature. The details of this method will be explained in the next section and will be confirmed by simulation results. In [7], we have investigated some issues that have emerged regarding the use of the Hadamard constellation in an OFDM system.

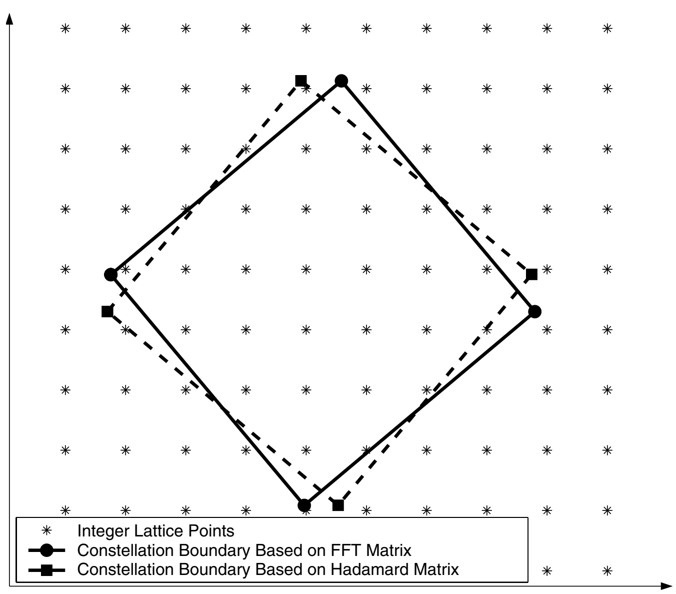

Fig. 1. N-D signal constellation for IFFT and Hadamard matrix.

A. Complex Representation

Generally, we can distinguish between two classes of boundaries [8]: 1) Cartesian boundary that is resulted by viewing the real and imaginary parts of the signal as two separate real signals, and 2) polar boundary that considers the envelope and phase of the OFDM signal in a complex plain. Cartesian boundary limits each component of the complex signal within a square, while the polar boundary limits this component within a circle. In this paper, we avoid the complex representation of the OFDM signal by treating the real and imaginary parts of the signal separately, which is equivalent to using a Cartesian boundary.

B. Encoding Procedure

All the points inside the Hadamard constellation should be mapped by the encoding procedure, introduced in (6) to (8). The number of points inside the shaped constellation is determined by the determinant of the Hadamard matrix, $\det(\mathbf{H}_{2^n})$ [9].

Theorem 2: [7] The constellation size for a $2^n \times 2^n$ Hadamard matrix is $\det(\mathbf{H}_{2^n}) = 2^{n2^{n-1}}$.

According to the large Hadamard constellation size, the canonical representation of the large numbers should be computed in the transmission of the OFDM signals. The canonical representation can be simplified by using the fact that digital communication systems deal with binary input streams. Also, according to (5), for a $2^n \times 2^n$ Hadamard matrix, all $\{\sigma_i\}_{i=1}^{N}$ are powers of 2. Considering these facts, we can represent each λ_i by $k_i = \log_2 \sigma_i$ bits of the input binary data [7] (Fig. 2). This representation will simplify the encoding algorithm. Moreover, the problem of using large numbers in the encoding procedure will be avoided.

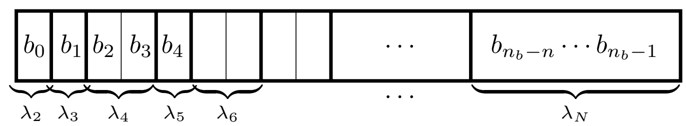

Fig. 2. Mapping between binary representation of the information and $\{\lambda_i\}$.

Theorem 2 shows the size of the Hadamard constellation for a $2^n \times 2^n$ Hadamard matrix is $2^{n2^{n-1}}$. Therefore, the transmission rate is related to the number of subcarriers $N = 2^n$ in the OFDM system[2]. This rate is unacceptable not only because of the dependency on N but also because the value is usually much higher than the required rate. A subgroup of the constellation points is chosen in [7] such that the constellation has the desired rate, and the constellation points have a uniform distribution in the cubic constellation. Relying on the continuous approximation, such a uniform distribution affects neither the probabilistic behavior of the PAPR nor the average energy of the constellation points. The Hadamard constellation is called the root constellation for the aforementioned set of the uniformly distributed points in the sequel.

C. Decoding Procedure

A conventional Fast Fourier Transform (FFT) based receiver is considered for the OFDM signal. At the receiver end, the time domain signal is filtered by a low pass filter and sampled at the Nyquist rate. The samples are processed by an FFT to recover the constellation point in the frequency domain. For an Additive White Gaussian Noise (AWGN) channel, the received vector is given by

$$\mathbf{z} = \mathbf{y} + \mathbf{n}, \tag{9}$$

where $\mathbf{y}$ is the transmitted time domain signal in (6) and $\mathbf{n}$ is a zero-mean complex AWGN. The approximated constellation

[2]For $N = 2^n$, the rate for each real component is $\log_2(2^{n2^{n-1}})/N = \frac{n}{2}$.

point is written as

$$\hat{\mathbf{x}} = \text{FFT}(\mathbf{z}) = \mathbf{x} + \text{FFT}(\mathbf{n}) = \mathbf{x} + \mathbf{n}', \tag{10}$$

where $\mathbf{x}$ is the transmitted constellation point, and $\mathbf{n}'$ is a zero-mean complex AWGN. The maximum likelihood decoder simply rounds off the received constellation point $\hat{\mathbf{x}}$ in the integer domain. Then, the resulting constellation point is employed in (8) to decode the transmitted information.

V. Selective Mapping

As mentioned in Section III, the complexity of using the Hadamard constellation in an OFDM system is very low, and this shaping method can be concatenated by other methods. In the following, we propose an SLM technique, applied to the Hadamard constellation to further reduce the PAPR.

SLM is a method to reduce the PAPR in an OFDM system which involves generating a large set of data vectors that represent the same information, where the data vector with the lowest PAPR is used for the transmission. Here, we present a method to apply the SLM technique to further reduce the PAPR in the constellation developed earlier.

Assume that the data rate to be transmitted is r bits per block of length-N FFT symbol. Let r_s denote the number of redundant bits of r bits specified for SLM ($r_s \ll r$ and $r = \log_2(\text{Hadamard constellation size})$). Consequently, $N_s = 2^{r_s}$ constellation points should represent the same information. In this method, the input integers I are mapped to the Hadamard constellation points, and the output integers are classified by the sets with the same r_s Most Significant Bits (MSBs). All the corresponding constellation points in each set represent the same information. The IFFT operation for all these constellation points in each set is computed, and the constellation point with the lowest PAPR is transmitted.

The operation of our scheme can be described as follows. In the first step, a binary information sequence is divided into blocks of $r - r_s$ bits. r_s bits of zeros are added to each information block, and then it is divided into subblocks of lengths equal to $\log_2 \sigma_i$, $i = 1, \cdots, N$, bits (refer to Fig. 2). The binary representations of these subblocks form the vector $\boldsymbol{\lambda}$ in (6). The other multiples of this vector are obtained by changing all the possible values for r_s MSBs of the binary information sequence. Then, N_s different Hadamard constellation points are produced by (6). The corresponding time domain OFDM signals result in various values for the PAPR. Finally, the constellation point with the lowest PAPR is selected for transmission.

All the different constellation points that represent the same information have the same $r - r_s$ bits. Thus, at the receiver end, the constellation point is decoded by (8), and the r_s extra bits are discarded, since the transmitted information is in the remaining $r - r_s$ bits. Therefore, this method can be expressed as a variant of SLM in which no side information on the choice of the transmit signal needs to be transmitted. The degradation in the data rate can be ignored, since by using only one or two redundant bits a significant PAPR reduction is obtained. To be fair in viewing the potential loss in the data rate, we have to include the impact of using the SLM method on the average energy of the constellation as well. The Hadamard constellation has a zero shaping gain[3], due to its cubic boundary. Numerical results show applying the SLM method to the resulting cubic constellation and selecting the point with the lowest PAPR result in a reduction in the average energy, reflected in a small, however positive shaping gain. This justifies our earlier claim that the reduction in the PAPR is achieved at no extra cost in terms of a reduction in the spectral efficiency and/or an increase in the average energy of the constellation.

VI. Simulation Results

In this section, we present simulations for a complex baseband OFDM system with $N = 128$ subchannels employing 16-QAM by using 10^7 randomly generated OFDM symbols. Our simulation results are presented as the Complementary Cumulative Density Function (CCDF) of the PAPR of the OFDM signals, expressed as follows:

$$\text{CCDF}\{\text{PAPR}(\mathbf{y})\} = P\{\text{PAPR}(\mathbf{y}) > \gamma\}. \tag{11}$$

This equation can be interpreted as the probability that the PAPR of a symbol block exceeds some clip level γ (it is referred as symbol clip probability [8]).

According to our simulations, the use of the Hadamard constellation in OFDM systems as a constellation shaping method considerably reduces the PAPR with a low complexity encoding and decoding algorithm.

Fig. 3 shows the simulation results of implementing our SLM technique, applied to the Hadamard constellation in the simulated OFDM system. The PAPR probability for $r_s = 1, 2,$ and 4 redundant bits is depicted. As it is illustrated in Fig. 3, using only one bit in 4×128 bits per block of length 128 FFT symbol [4] results in a 5.6dB improvement in the PAPR. Simulations show that by employing more redundant bits the PAPR can be close to that of a cubic constellation, , namely $10 \log_{10}(3)$[5].

In [7], it is shown that the slope of the CCDF vs. PAPR graph increases in the SLM method. However, there is a saturation effect on the PAPR reduction by the successively doubling of N_s. As mentioned in Section V, the method employing only the Hadamard constellation considerably reduces the PAPR. By adopting the Hadamard constellation in the proposed SLM method, not only can we lower the PAPR considerably, but also we can approximately maintain the slope of the CCDF vs. PAPR curve, i.e., we gain a considerably lower PAPR by a few number of redundant bits before saturation effect. Therefore, by using just one or two redundant bits, we can significantly reduce the PAPR.

[3]Shaping gain is defined as the relative reduction in the required average energy for a given number of constellation points with respect to a cubic constellation [10].

[4]By using 16-QAM in a 128 channel OFDM system, there are $16^{128} = 2^{4 \times 128}$ constellation points.

[5]The PAPR of a cubic constellation is computed using continuous approximation.

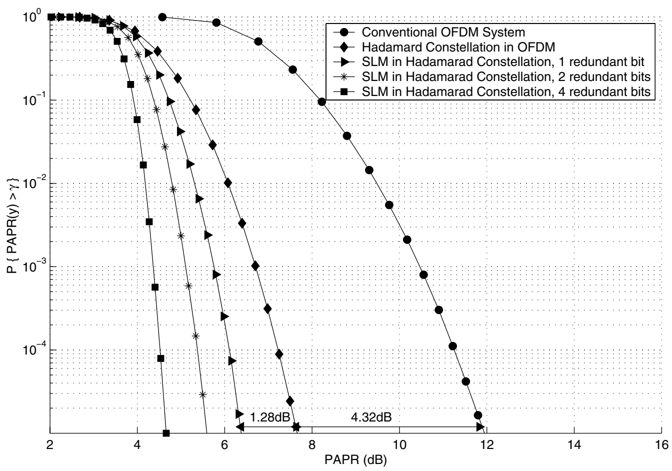

Fig. 3. CCDF of PAPR by SLM method based on Hadamard constellation in a 128 channel OFDM system employing 16-QAM constellation.

A. Comparison

In numerical simulations, we have selected the system parameters to be compatible with some recent works on PAPR reduction reported in [6], [8], [11].

As a complexity measurement, the main complexity of our method comes from the encoding algorithm and the multi-IFFT computations in the SLM technique. The complexity of the encoding algorithm is in the matrix multiplications of (6). As mentioned in section III, all the elements of the Hadamard matrix and its SNF decomposition matrices are $+1, -1$, or 0, and we just have additions which can be easily implemented by a butterfly structure. In SLM technique, for each of the N_s time domain signals we shall compute one IFFT and as we showed, for $N_s = 4$ to 16, we can significantly lower PAPR.

In [6], an SLM method based on multiplying the constellation point by N_s different and pseudo-random but fixed vectors is introduced. For the same system as ours, with $N_s = 4$ different vectors, the PAPR reduction of 3dB close to 10^{-5} symbol clip probability is gained; however, at the same symbol clip rate and N_s, we have a 6dB reduction by using our SLM method. Also, note that in [6] some side information needs to be sent, and receiving accurate side information is important.

The tone reservation [11] is a well known method for PAPR reduction in multicarrier systems, provided it can converge quickly to a good PAPR solution. In [8], an efficient approximation for the the tone reservation approach is developed with a faster convergence. However, we have about 3dB lower PAPR than that in [11] or [8] for the similar system parameters. Note that in the tone reservation method, some tones are reserved for the PAPR reduction and some of the tones are not used for transmitting, implying a loss in data rate.

In [8], a minimax problem is solved by an interior-point method which requires a descent direction and a constraint to find the solution recursively. The complexity of solving this optimization problem shall be compared with that in our encoding algorithm. As mentioned in [8], the complexity per iterations for finding the descent direction will increase with each iteration and the complexity for finding the next constraint for the next iteration is computationally intensive [6].

Recently, we became aware of the work by Ochiai [12]. For a 256 complex channel OFDM system employing 256-QAM a 4.5dB reduction in the PAPR is obtained using trellis shaping technique. In our method, for a 128 complex channel OFDM system employing up to 128-QAM a 6dB reduction is gained.

In [12], the main complexity is in finding the path with minimum cost through a trellis diagram, where the complexity of finding such a path is considerably higher than that of a Viterbi decoder. However, the author investigates methods to reduce this complexity by window truncation and sacrificing PAPR reduction, but still the overall complexity in [12] is significantly higher as compared to the method proposed here.

VII. Conclusion

We have proposed a constellation shaping method that achieves a substantial reduction in the PAPR in an OFDM system with a low complexity. An SLM technique is applied to this constellation to further reduce the PAPR of the OFDM signal. The proposed scheme significantly outperforms other PAPR reduction techniques reported in the literature .

It is also possible to apply a PTS method [6] to our Hadamard constellation, especially since the PTS is considerably better with respect to PAPR reduction vs. additional system complexity (the number of IFFTs) [6] as compared to the SLM method.

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[6] four adds and two divides for each N sample per iteration.