Effect of jamming on the capacity of MIMO channels

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Abstract

In this work, we study a communication system consisting of a transmitter, a receiver, and a jammer. It is assumed that the transmitter and the jammer are equipped with multiple antennas, while receiver is equipped with single antenna. Different cases are investigated, depending on the availability of Channel State Information (CSI) for the transmitter and/or the jammer. For each case, we propose the best strategy for the jammer, and for the transmitter by focusing on the ergodic capacity between the transmitter and the receiver, and derive an analytical expression for it. [1]

1 Introduction

Recently, Multiple Input Multiple Output (MIMO) antenna systems have attracted the attention of the research community. Such systems offer two key advantages: (i) spatial diversity (independent fading for different antennas), and (ii) multiplexing gain (the creation of multiple transmission channels) [2]. It has been proved in [3] and [4], that the capacity of a MIMO system grows linearly with the minimum number of transmit and receive antennas. The capacity of fading channels varies depending on the assumptions one makes about fading statistics and the knowledge of fading coefficients. References [3] and [4] consider the capacity of MIMO systems when CSI is available only at the receiver, and [5] investigates the capacity of such systems when neither the transmitter nor the receiver knows the channel coefficients. When CSI is available at both transmitter and receiver, the capacity of multiple receiver antenna systems (with a single transmit antenna) has been considered in [6], and for a general MIMO system has been examined in [7].

Many research works have addressed the problem of jamming and anti-jamming in Single-Input Single-Output (SISO) antenna systems, in military communication networks. It is well known that in an SISO antenna system, the best strategy for jamming/anti-jamming is to uniformly spread the energy over the available frequency band. On the other hand, spatial dimensionality of MIMO systems suggests a natural extension to this topic dealing with the distribution of energy over space. Due to the significant role of MIMO systems, it is of interest to investigate the effect of jamming on the throughput of a communication system where the parties can take advantage of multiple antennas. To the best of our knowledge, the only prior work that has addressed this problem is [9], where the authors

[1]More details about this work can be found in [1].

have considered an MIMO system attacked by a jammer. In [9], it is assumed that the jammer knows the transmitted signal, and there is no feedback between the transmitter and the receiver. In addition, it is assumed that the jamming signal directly affects the receiver, i.e., the effect of the propagation channel between the jammer and the receiver is not included. With these assumptions, the authors have reached the following interesting conclusions: (i) knowledge of the transmitted signal by the jammer is not beneficial in improving the jamming strategy, and (ii) the optimum jamming strategy is to allocate the power uniformly across the receiver antennas.

The rest of this paper is organized as follows. In Section 2, we introduce the system model. The best strategies for the transmitter and the jammer in different cases (in terms of the availability of the CSI for the transmitter and/or the jammer) are investigated in Section 3. In Section 4, we present closed form expressions of the capacity. Finally, in Section 5, the paper is concluded.

Throughout this paper, we denote the trace operation by tr(.), the complex conjugate operation by $(.)^*$, the transpose operation by $(.)^T$, and the determinant operation by det(.). All the logarithms are in two bases.

2 Channel Model

It is assumed that the transmitter, and the jammer are equipped with n_T, n_J antennas, respectively, while the receiver is equipped with single antenna. The channel between the transmitter and the receiver is denoted by $\mathbf{H} \in \mathbb{C}^{1\times n_T}$, and the channel between the jammer and the receiver is signified by $\breve{\mathbf{H}} \in \mathbb{C}^{1\times n_J}$. It is assumed that $\mathbf{H}$ and $\breve{\mathbf{H}}$ are MISO quasi-static uncorrelated Rayleigh fading channels. Also, we assume that the transmitted vectors by the transmitter and the jammer have Gaussian distribution. The received signal at the receiver can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + n + z, \tag{1}$$

where $\mathbf{x}$ is the transmitted signal with the covariance matrix $\mathbf{Q}$. $z \in \mathbb{C}^{1\times 1} \sim CN(0, \breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^*)$ is a Gaussian noise induced by the jammer; $\breve{\mathbf{Q}}$ is the jammer's covariance matrix, and $\mathbf{n} \in \mathbb{C}^{1\times 1} \sim N(0, \sigma^2)$ is the white Gaussian noise at the receiver. The power constraint for the transmitter is $\text{tr}(\mathbf{Q}) \leq E_T$, and for the jammer is $\text{tr}(\breve{\mathbf{Q}}) \leq E_J$.

3 Transmitter and Jammer Strategies

3.1 Case I: H unknown to the transmitter, $\breve{\mathbf{H}}$ unknown to the jammer

In this case, neither the transmitter nor the jammer have access to the CSI . However, we assume that the transmitter and the jammer know the distribution of $\mathbf{H}$ and $\breve{\mathbf{H}}$.

For the case under consideration, the ergodic capacity, as originally defined in [3], is computed from

$$C_{\text{erg}}(\mathbf{Q}, \breve{\mathbf{Q}}) = \mathbf{E}_{\mathbf{H},\breve{\mathbf{H}}} \log\det\left[1 + \mathbf{H}\mathbf{Q}\mathbf{H}^*\left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2\right)^{-1}\right], \tag{2}$$

where $\mathbf{E}_{\mathbf{H},\breve{\mathbf{H}}}$ is the expectation operation over $\mathbf{H}$ and $\breve{\mathbf{H}}$.

In this scenario, the transmitter tries to maximize the ergodic capacity, whereas the jammer attempts to minimize the ergodic capacity. In other words,

$$\mathbf{Q} = \arg \max_{\substack{\mathbf{X} \\ \mathrm{tr}(\mathbf{X}) \le E_T}} \min_{\substack{\breve{\mathbf{Q}} \\ \mathrm{tr}(\breve{\mathbf{Q}}) \le E_J}} C_{\mathrm{erg}}(\mathbf{X}, \breve{\mathbf{Q}}) \tag{3}$$

and

$$\breve{\mathbf{Q}} = \arg \min_{\substack{\breve{\mathbf{Y}} \\ \mathrm{tr}(\breve{\mathbf{Y}}) \le E_J}} \max_{\substack{\mathbf{Q} \\ \mathrm{tr}(\mathbf{Q}) \le E_T}} C_{\mathrm{erg}}(\mathbf{Q}, \breve{\mathbf{Y}}). \tag{4}$$

In (3)-(4), it is assumed that all the matrices involved are positive semi-defenite (PSD). For the simplicity of notations, this property is not mentioned explicitely hereafter.

3.1.1 Transmitter Strategy

As described earlier, the transmitter should solve the following problem:

$$\mathbf{Q} = \arg \max_{\substack{\mathbf{X} \\ \mathrm{tr}(\mathbf{X}) \le E_T}} \min_{\substack{\breve{\mathbf{Q}} \\ \mathrm{tr}(\breve{\mathbf{Q}}) \le E_J}} \mathbf{E}_{\mathbf{H},\breve{\mathbf{H}}} \log\det \left[1 + \mathbf{H}\mathbf{Q}\mathbf{H}^* \left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2 \right)^{-1} \right]. \tag{5}$$

In [1] (Theorem 1), it is proved that the solution to the above problem is $\mathbf{Q} = \frac{E_T}{n_T}\mathbf{I}$, for any $\breve{\mathbf{Q}}$ chosen by the jammer.

3.1.2 Jammer Strategy

The jammer should solve the following problem:

$$\breve{\mathbf{Q}} = \arg \min_{\substack{\breve{\mathbf{Y}} \\ \mathrm{tr}(\breve{\mathbf{Y}}) \le E_J}} \max_{\substack{\mathbf{Q} \\ \mathrm{tr}(\mathbf{Q}) \le E_T}} \mathbf{E}_{\mathbf{H},\breve{\mathbf{H}}} \log\det \left[1 + \mathbf{H}\mathbf{Q}\mathbf{H}^* \left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2 \right)^{-1} \right]. \tag{6}$$

Similarly, it can be proved that the best strategy for the jammer is to choose $\breve{\mathbf{Q}} = \frac{E_J}{n_J}\mathbf{I}$, for any $\mathbf{Q}$ chosen by the transmitter.
In this case, the ergodic capacity can be written as

$$C_{\mathrm{erg}} = E_{\mathbf{H},\breve{\mathbf{H}}} \log \left[1 + \frac{E_T}{n_T} \|\mathbf{H}\|^2 \left(\frac{\breve{E}_J}{n_J} \|\breve{\mathbf{H}}\|^2 + 1 \right)^{-1} \right]. \tag{7}$$

3.2 Case II: H unknown to the transmitter, $\breve{\mathbf{H}}$ known to the jammer

In this case, $\breve{\mathbf{H}}$ can be considered as a deterministic matrix (for the jammer), and $\mathbf{H}$ can be treated as a random complex Gaussian matrix whose distribution is known for both the transmitter and the jammer.

3.2.1 Transmitter Strategy

In this scenario, the transmitter should solve the following max-min problem:

$$\mathbf{Q} = \arg \max_{\substack{\mathbf{X} \\ \mathrm{tr}(\mathbf{X}) \le E_T}} \min_{\substack{\breve{\mathbf{Q}} \\ \mathrm{tr}(\breve{\mathbf{Q}}) \le E_J}} E_{\mathbf{H},\breve{\mathbf{H}}} \log\det \left[1 + \mathbf{H}\mathbf{X}\mathbf{H}^* \left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2 \right)^{-1} \right]. \tag{8}$$

The best strategy for the transmitter here is again to choose $\mathbf{Q} = \frac{E_T}{n_T}\mathbf{I}$. Note that this result does not depend on the $\breve{\mathbf{Q}}$.

3.2.2 Jammer Strategy

The jammer should solve the following problem:

$$\breve{\mathbf{Q}} = \arg \min_{\substack{\breve{\mathbf{Y}} \\ \mathrm{tr}(\breve{\mathbf{Y}}) \le E_J}} \max_{\substack{\mathbf{Q} \\ \mathrm{tr}(\mathbf{Q}) \le E_T}} E_{\mathbf{H}} \log\det \left[1 + \mathbf{H}\mathbf{Q}\mathbf{H}^* \left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2 \right)^{-1} \right]. \tag{9}$$

Since $\breve{\mathbf{H}}$ is known to the jammer, for any channel realization $\mathbf{H}$, it should maximize the term $\breve{\mathbf{H}}\breve{\mathbf{Y}}\breve{\mathbf{H}}^* + \sigma^2$. So, the jammer strategy reduces to the following simple max problem:

$$\breve{\mathbf{Q}} = \arg \max_{\substack{\breve{\mathbf{Y}} \\ \mathrm{tr}(\breve{\mathbf{Y}}) \le E_J}} \breve{\mathbf{H}}\breve{\mathbf{Y}}\breve{\mathbf{H}}^*. \tag{10}$$

This problem is a special case of "water-filling" in which the matrix $\breve{\mathbf{H}}^*\breve{\mathbf{H}}$ is of rank one, with the singular value of $\|\breve{\mathbf{H}}\|^2$. It can be easily shown that the solution to (10) is $\breve{\mathbf{Q}} = \frac{E_J}{\|\breve{\mathbf{H}}\|^2}\breve{\mathbf{H}}^*\breve{\mathbf{H}}$, and the maximum value is $E_J\|\breve{\mathbf{H}}\|^2$.

Finally, the ergodic capacity can be witten as

$$C_{\text{erg}} = E_{\mathbf{H},\breve{\mathbf{H}}} \log \left[1 + \frac{E_T}{n_T} \|\mathbf{H}\|^2 \left(\breve{E}_J \|\breve{\mathbf{H}}\|^2 + \sigma^2 \right)^{-1} \right]. \tag{11}$$

3.3 Case III: H known to the transmitter, $\breve{\mathbf{H}}$ unknown to the jammer

In this case, we assume that the transmitter has perfect information about the channel matrix between itself and the receiver, $\mathbf{H}$. So, $\mathbf{H}$ can be considered as a deterministic matrix (for the transmitter), and $\breve{\mathbf{H}}$ can be treated as a random complex Gaussian matrix whose distribution is known for both the transmitter and the jammer.

3.3.1 Transmitter Strategy

The transmitter should solve the following problem:

$$\mathbf{Q} = \arg \max_{\substack{\mathbf{X} \\ \mathrm{tr}(\mathbf{X}) \le E_T}} \min_{\substack{\breve{\mathbf{Q}} \\ \mathrm{tr}(\breve{\mathbf{Q}}) \le E_J}} E_{\breve{\mathbf{H}}} \log\det \left[\mathbf{I} + \mathbf{H}\mathbf{X}\mathbf{H}^* \left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2\mathbf{I} \right)^{-1} \right]. \tag{12}$$

Since $\mathbf{H}$ is known to the transmitter, for any channel realization $\breve{\mathbf{H}}$, it should maximize the term $\mathbf{HXH}^*$. Therefore,

$$\mathbf{Q} = \arg \max_{\substack{\mathbf{X} \\ \mathrm{tr}(\mathbf{X}) \leq E_T}} \mathbf{HXH}^*. \tag{13}$$

Similar to the previous case, we have $\mathbf{Q} = \frac{E_T}{\|\mathbf{H}\|^2}\mathbf{H}^*\mathbf{H}$, and $\mathbf{HQH}^* = E_T\|\mathbf{H}\|^2$.

3.3.2 Jammer Strategy

The jammer should solve the following problem:

$$\breve{\mathbf{Q}} = \arg \min_{\substack{\breve{\mathbf{Y}} \\ \mathrm{tr}(\breve{\mathbf{Y}}) \leq E_J}} \max_{\substack{\mathbf{Q} \\ \mathrm{tr}(\mathbf{Q}) \leq E_T}} \mathbf{E}_{\mathbf{H},\breve{\mathbf{H}}} \log\det\left[1 + \mathbf{HQH}^*\left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2\right)^{-1}\right]. \tag{14}$$

Similar to case I, it can be proved that the best strategy for the jammer is to choose $\breve{\mathbf{Q}} = \frac{E_J}{n_J}\mathbf{I}$.

Assuming the best strategies for the transmitter and the jammer, the ergodic capacity in this case can be written as

$$C_{\text{erg}} = E_{\mathbf{H},\breve{\mathbf{H}}} \log\left[1 + E_T\|\mathbf{H}\|^2\left(\frac{E_J}{n_J}\|\breve{\mathbf{H}}\|^2 + \sigma^2\right)^{-1}\right]. \tag{15}$$

3.4 Case IV: H known to the transmitter, $\breve{\mathbf{H}}$ known to the jammer

In this case, we assume that the transmitter has full information about the channel matrix between the transmitter and the receiver, $\mathbf{H}$, and the jammer has the complete information about its channel to the receiver, $\breve{\mathbf{H}}$.

3.4.1 Transmitter Strategy

The transmitter should solve the following problem:

$$\mathbf{Q} = \arg \max_{\substack{\mathbf{X} \\ \mathrm{tr}(\mathbf{X}) \leq E_T}} \min_{\substack{\breve{\mathbf{Q}} \\ \mathrm{tr}(\breve{\mathbf{Q}}) \leq E_J}} E_{\breve{\mathbf{H}}} \log\det\left[\mathbf{I} + \mathbf{HXH}^*\left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2\mathbf{I}\right)^{-1}\right]. \tag{16}$$

Similar to case III, the best strategy for the transmitter is to choose $\mathbf{Q} = \frac{E_T}{\|\mathbf{H}\|^2}\mathbf{H}^*\mathbf{H}$, and as a result $\mathbf{HQH}^* = E_T\|\mathbf{H}\|^2$.

3.4.2 Jammer strategy

The jammer should solve the following problem:

$$\breve{\mathbf{Q}} = \arg \min_{\substack{\breve{\mathbf{Y}} \\ \mathrm{tr}(\breve{\mathbf{Y}}) \leq E_J}} \max_{\substack{\mathbf{Q} \\ \mathrm{tr}(\mathbf{Q}) \leq E_T}} E_{\mathbf{H}} \log\det\left[1 + \mathbf{HQH}^*\left(\breve{\mathbf{H}}\breve{\mathbf{Q}}\breve{\mathbf{H}}^* + \sigma^2\right)^{-1}\right], \tag{17}$$

which results to $\breve{\mathbf{Q}} = \frac{E_J}{\|\breve{\mathbf{H}}\|^2}\breve{\mathbf{H}}^*\breve{\mathbf{H}}$.

Substituting the optimum covariance matrices for the transmitter and the jammer, the ergodic capacity will be equal to

$$C_{\text{erg}} = E_{\mathbf{H},\breve{\mathbf{H}}} \log \left[1 + E_T\|\mathbf{H}\|^2 \left(E_J\|\breve{\mathbf{H}}\|^2 + \sigma^2\right)^{-1}\right]. \tag{18}$$

4 Analytical Results

4.1 Capacity Computation

In the previous section, we figured out the best strategies (in terms of the covariance matrices) for the transmitter to maximize the ergodic capacity and for the jammer to minimize it. we assumed 4 different cases in terms of the availability of CSI at the transmitter and the jammer, and obtained an expression for the ergodic capacity. Unifying all the expressions (7), (11), (15), and (18) in one, we have

$$C_{\text{erg}} = E_{\mathbf{H},\breve{\mathbf{H}}} \log \left[1 + \rho\|\mathbf{H}\|^2 \left(\breve{\rho}\|\breve{\mathbf{H}}\|^2 + 1\right)^{-1}\right], \tag{19}$$

where

$$\rho = \begin{cases} \dfrac{E_T}{n_T\sigma^2} & \text{No feedback present between transmitter and receiver} \\ \\ \dfrac{E_T}{\sigma^2} & \text{Perfect feedback present between transmitter and receiver} \end{cases}, \tag{20}$$

$$\breve{\rho} = \begin{cases} \dfrac{E_J}{n_J\sigma^2} & \text{No feedback present between jammer and receiver} \\ \\ \dfrac{E_J}{\sigma^2} & \text{Perfect feedback present between jammer and receiver} \end{cases}. \tag{21}$$

Noting the fact that $\mathbf{H}$ and $\breve{\mathbf{H}}$ are complex Gaussian vectors with i.i.d entries, their Square norms, i.e., $Z = \|\mathbf{H}\|^2$, and $Y = \|\breve{\mathbf{H}}\|^2$ have χ^2 distribution with parameters $2n_T$ and $2n_J$, respectively. So, their pdfs (probability density functions) are expressed as follows:

$$f_Z(z) = \frac{z^{n_T-1}\exp(-z)}{\Gamma(n_T)}, \tag{22}$$

$$f_Y(y) = \frac{y^{n_J-1}\exp(-y)}{\Gamma(n_J)}, \tag{23}$$

To calculate the capacity, we should first derive the pdf of $X = \frac{Z}{\breve{\rho}Y+1}$, which can be computed as follows:

$$\begin{aligned} f_X(x) &= \int_0^\infty f_{X/Y}(x/y) f_Y(y)dy \\ &= \int_0^\infty (\breve{\rho}\, y + 1)\, f_Z\left[x\left(\breve{\rho}\, y + 1\right)\right] f_Y(y)dy \end{aligned}$$

$$= \int_0^\infty (\breve{\rho}\, y+1)\, \frac{1}{\Gamma(n_T)} \left[x\left(\breve{\rho}\, y+1\right)\right]^{n_T-1} \exp\left[-x\left(\breve{\rho}\, y+1\right)\right] \frac{1}{\Gamma(n_J)} y^{n_J-1} \exp(-y) dy$$
$$= \frac{x^{n_T-1}\exp(-x)}{\Gamma(n_T)\Gamma(n_J)} \int_0^\infty \left(\breve{\rho}\, y+1\right)^{n_T} y^{n_J-1} \exp\left[-y\left(1+x\breve{\rho}\right)\right] dy$$
$$= \frac{x^{n_T-1}\exp(-x)}{\Gamma(n_T)\Gamma(n_J)} \left(1+x\breve{\rho}\right)^{-n_J} \int_0^\infty \left(\frac{\breve{\rho}\, u}{1+x\breve{\rho}}+1\right)^{n_T} u^{n_J-1}\exp(-u) du$$
$$= \frac{x^{n_T-1}\exp(-x)}{\Gamma(n_T)\Gamma(n_J)} \left(1+x\breve{\rho}\right)^{-n_J} \sum_{k=0}^{n_T} \binom{n_T}{k} \left(\frac{\breve{\rho}}{1+x\breve{\rho}}\right)^k \int_0^\infty u^{k+n_J-1}\exp(-u) du$$
$$= \frac{x^{n_T-1}\exp(-x)}{\Gamma(n_T)\Gamma(n_J)} \sum_{k=0}^{n_T} \binom{n_T}{k} \breve{\rho}^k \left(1+x\breve{\rho}\right)^{-(n_J+k)} \Gamma(n_J+k). \tag{24}$$

Thus, the unified ergodic capacity which is defined in (19), can be written as

$$C_{\text{erg}} = \int_0^\infty f_X(x)\log(1+\rho\, x) dx$$
$$= \frac{1}{\Gamma(n_T)\Gamma(n_J)} \sum_{k=0}^{n_T} \binom{n_T}{k} \breve{\rho}^k \Gamma(n_J+k) \int_0^\infty x^{n_T-1}\exp(-x)\left(1+x\breve{\rho}\right)^{-(n_J+k)} \log(1+\rho\, x) dx. \tag{25}$$

Figs. 1-4 illustrate the plot of the capacity versus $\frac{E_T}{\sigma^2}$ for $\frac{E_J}{\sigma^2} = 10dB$, $n_T = 4$, and for the different numbers of n_J, and finally, Figs. 5-6 present comparisons for the capacity of the four cases for the different values of n_T and n_J.

4.2 Discussion

It is evident in Figs. 1-4 that, when the jammer does not have information about its channel to the receiver (case I, and case III), with a fixed transmit power, it is not beneficial for the jammer to use more antennas. However, if it knows $\breve{\mathbf{H}}$, it can take advantage of using more antennas to reduce the capacity more significantly. Figs. 5-6 show that knowing the channel is very useful for both the transmitter and the jammer, especially when the number of transmit antennas is large. For the situation $\rho \gg \breve{\rho} \gg 1$, we can write

$$C_{\text{erg}} \approx E_{\mathbf{H},\breve{\mathbf{H}}} \log \frac{\rho \|\mathbf{H}\|^2}{\breve{\rho}\|\breve{\mathbf{H}}\|^2}. \tag{26}$$

Consequently, for a fixed number of transmitter and jammer antennas,

$$\text{Case I} \rightarrow C_{\text{erg}} \approx \log\frac{E_T}{E_J} + \log n_J - \log n_T + E_{\mathbf{H}} \log\|\mathbf{H}\|^2 - E_{\breve{\mathbf{H}}} \log\|\breve{\mathbf{H}}\|^2, \tag{27}$$

$$\text{Case II} \rightarrow C_{\text{erg}} \approx \log\frac{E_T}{E_J} - \log n_T + E_{\mathbf{H}} \log\|\mathbf{H}\|^2 - E_{\breve{\mathbf{H}}} \log\|\breve{\mathbf{H}}\|^2, \tag{28}$$

$$\text{Case III} \rightarrow C_{\text{erg}} \approx \log\frac{E_T}{E_J} + \log n_J + E_{\mathbf{H}} \log\|\mathbf{H}\|^2 - E_{\breve{\mathbf{H}}} \log\|\breve{\mathbf{H}}\|^2, \tag{29}$$

and

$$\text{Case IV} \rightarrow C_{\text{erg}} \approx \log\frac{E_T}{E_J} + E_{\mathbf{H}} \log\|\mathbf{H}\|^2 - E_{\breve{\mathbf{H}}} \log\|\breve{\mathbf{H}}\|^2. \tag{30}$$

In other words, if the transmitter knows $\mathbf{H}$, it can increase the capacity by the term $\log n_T$, and if the jammer knows $\breve{\mathbf{H}}$, it can decrease the capacity by the term $\log n_J$.

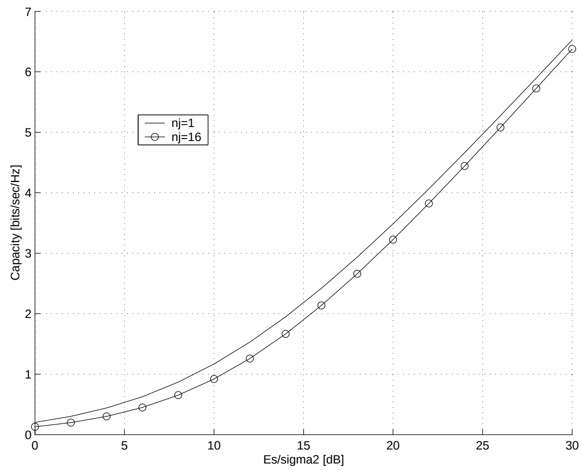

Figure 1: Case I, $n_T = 4, n_R = 1, \frac{E_J}{\sigma^2} = 10dB$

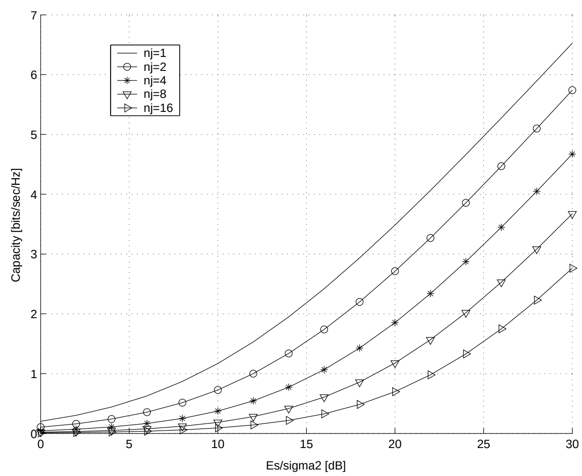

Figure 2: Case II, $n_T = 4, n_R = 1, \frac{E_J}{\sigma^2} = 10dB$

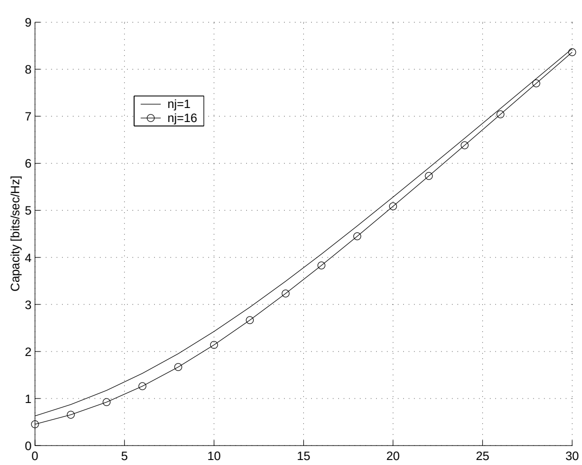

Figure 3: Case III, $n_T = 4, n_R = 1, \frac{E_J}{\sigma^2} = 10dB$

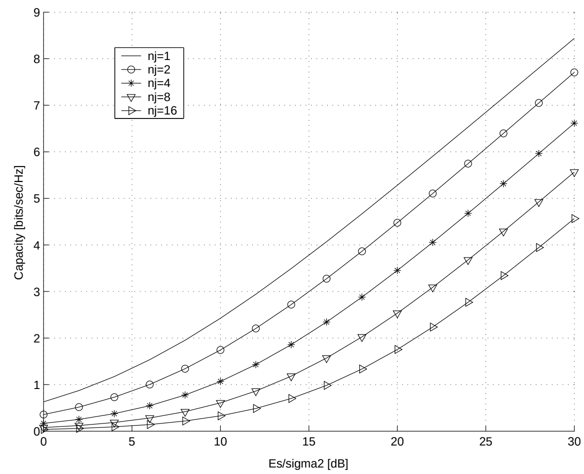

Figure 4: Case IV, $n_T = 4, n_R = 1, \dfrac{E_J}{\sigma^2} = 10dB$

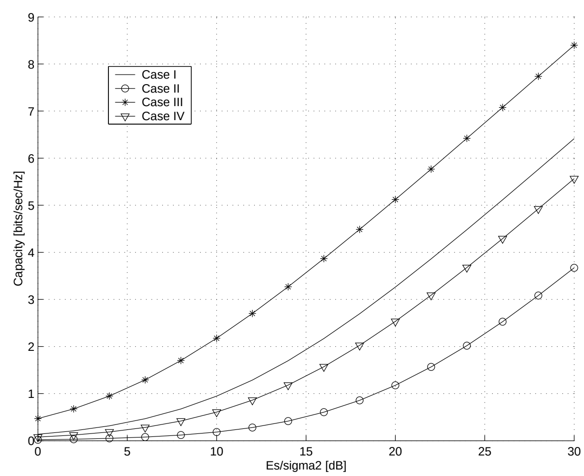

Figure 5: $n_T = 4, n_J = 8, n_R = 1, \dfrac{E_J}{\sigma^2} = 10dB$

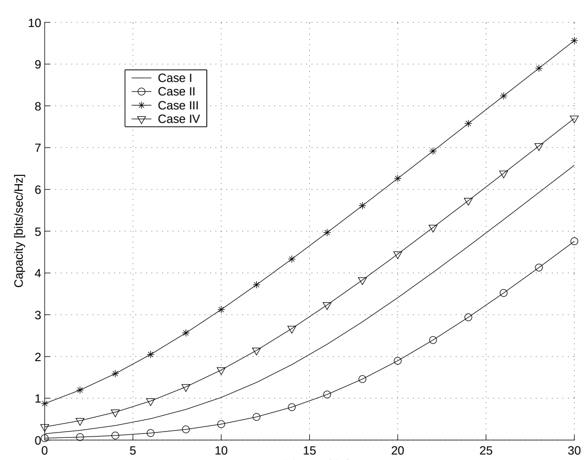

Figure 6: $n_T = 8, n_J = 4, n_R = 1, \dfrac{E_J}{\sigma^2} = 10dB$

5 Conclusion

In this work, we have examined a communication system that consists of a transmitter, a receiver, and a jammer. It is assumed that the transmitter and the jammer are equipped with multiple antennas, while receiver is equipped with single antenna. We have investigated different cases according to the availability of the Channel State Information (CSI) for the transmitter and/or for the jammer, resulting in four different cases. For each case, we have proposed the best strategy for the jammer, as well as for the transmitter, focusing on the ergodic capacity between the transmitter and the receiver. Indeed, we consider the effect of jamming on the ergodic capacity between the transmitter and the jammer from an analytical perspective. We conclude that, if each party knows the CSI, there is a significant effect on the capacity, especially when the number of transmit antennas is large.

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