Optimizing the Encoder/Decoder Structures in a Discrete Communication System

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Abstract — **The problem of optimizing the structure of the encoder/decoder pair in a discrete communication system (with an additive distortion measure) is expressed in terms of a Bilinear Programming Problem (BLP Problem). An efficient method, based on the simplex search in conjunction with the Generalized Upper Bounding Technique is presented for the solution. The special features of the problem are exploited to reduce the computational complexity of the proposed algorithm.**

I. Introduction

Consider a discrete communication system composed of a source $\boldsymbol{S}$, a channel $\boldsymbol{C}$, an encoder ξ and a decoder η. The source $\boldsymbol{S}$ is composed of N_s symbols s_i, $i=0,\dots,N_s-1$. The symbol $s_i \in \boldsymbol{S}$ occurs with probability $P_s(i)$. A measure of distortion is defined between each pair of the source symbols. The distortion between the symbols $s_i, s_j \in \boldsymbol{S}$ is denoted as $D_s(i,j)$, $i,j=0,\dots,N_s-1$. The channel $\boldsymbol{C}$ is composed of N_c symbols c_i, $i=0,\dots,N_c-1$. The symbol $c_i \in \boldsymbol{C}$ occurs with probability $P_c(i)$ and has an energy of $E_c(i)$. This results in an average energy of $\sum_{i=0}^{N_c-1} P_c(i)E_c(i)$ at the channel input. The transition probabilities of the channel are denoted as $T_c(j|i)$.

The encoder provides a mapping, denoted as ξ, from the set of source symbols to the set of channel symbols such that the ith source symbol, $i=0,\dots,N_s-1$, is mapped to the channel symbol indexed by $\xi(i)\in[0,N_c-1]$. Each source symbol is encoded to a specific channel symbol, however, (i) several source symbols may be encoded to the same channel symbol, and (ii) some of the channel symbols may not be used. The decoder provides a mapping, denoted as η, from the set of channel symbols to the set of source symbols such that the ith channel symbol, $i=0,\dots,N_c-1$, is mapped to the source symbol indexed by $\eta(i)\in[0,N_s-1]$. Each channel symbol is decoded to a specific source symbol, however, several channel symbols may be decoded to the same source symbol.

Our objective is to optimize the two mappings, namely ξ, η, to minimize the average distortion between the encoder input and the decoder output. The introduced formulation optimizes the combined effects of source quantization and channel coding on the end-to-end distortion. Quantization of the source symbols occurs when several source symbols are encoded to the same channel symbol. Channel coding occurs when some of the channel symbols are not used at all. In the following, this optimization problem is formulated as a zero-one program.

II. Zero-one formulation of the problem

We assign an N_c dimensional binary vector to each symbol of the source at the channel input. The vector corresponding to the ith source symbol, $i=0,\dots,N_s-1$, is denoted as $\mathbf{e}_i=[e_{ij}, j=0,\dots,N_c-1]$. We impose the constraints that $e_{ij}\in\{0,1\}$ and $\sum_{j=0}^{N_c-1} e_{ij}=1$, $\forall i$. If the ith source symbol is encoded to the ℓth channel symbol, we set, $e_{ij}=1$, $j=\ell$ and $e_{ij}=0$, $j\neq\ell$. Similarly, we assign an N_s dimensional binary vector to each channel symbol at the decoder side. The vector corresponding to the jth channel symbol, $j=0,\dots,N_c-1$, is denoted as $\mathbf{d}_j=[d_{ij}, i=0,\dots,N_s-1]$. We impose the constraints that $d_{ij}\in\{0,1\}$ and $\sum_{i=0}^{N_s-1} d_{ij}=1$, $\forall j$. If the jth channel symbol is decoded to the ℓth source symbol, we set $d_{ij}=1$, $i=\ell$ and $d_{ij}=0$, $i\neq\ell$. Using these notations, the optimization problem is formulated as:

$$\begin{aligned}
&\text{Minimize} \sum_{i=0}^{N_s-1}\sum_{j=0}^{N_c-1}\sum_{k=0}^{N_s-1}\sum_{l=0}^{N_c-1} P_s(i)T_c(l|j)D_s(i,k)e_{ij}d_{kl}\\
&\text{Subject to:} \sum_{i=0}^{N_s-1}\sum_{j=0}^{N_c-1} P_s(i)E_c(j)e_{ij} \le \overline{E}\\
&\qquad e_{ij}\in\{0,1\} \quad \text{and} \quad \sum_{j=0}^{N_c-1} e_{ij}=1, \ \ \forall i\\
&\qquad d_{ij}\in\{0,1\} \quad \text{and} \quad \sum_{i=0}^{N_s-1} d_{ij}=1, \ \ \forall j
\end{aligned} \tag{1}$$

This optimization problem is transformed into a *Bilinear Programming Problem* (BLP Problem) [1]. The problem has some special features which substantially facilitates its solution. These features are: (i) Existence of the Generalized Upper Bounding (GUB) constraints for both encoder and decoder. (ii) The encoder structure has only one extra constraint in addition to the GUB's, namely the energy constrain. (iii) The decoder constraints are all GUB's and consequently the linear program involved in the optimization of the decoder is decomposable. Using these features, an efficient method based on a variant of the simplex search is presented for the solution.

References

[1] F. A. Al-Khayyal, "Jointly Constrained Bilinear Programs and Related Problems: An Overview," *Computers Math. Applic.*, vol. 19, no. 11, pp. 53—62, 1990.

[1]This work was supported by Natural Sciences and Engineering Research Council of Canada (NSERC).