MULTILEVEL CODING STRATEGY FOR TWO-HOP SINGLE-USER NETWORKS

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Abstract—**In this paper, a two-hop network in which the information is transmitted from a source via a relay to a destination is considered. It is assumed that both channels are quasi-static fading and all nodes are equipped with a single antenna. The knowledge of the channel for each transmission hop is only available at the corresponding receiver. The relay is assumed to be simple, i.e., not capable of data buffering over multiple coding blocks or rescheduling tasks. Considering a continuum of multilevel codes at both of the source and the relay, in conjunction with decode and forward strategy, we present a scheme to optimally allocate the available source and relay powers to different levels of their multilevel codes. Assuming Rayleigh fading, the performance of this scheme is also evaluated and compared with the previously known strategies.**

1. INTRODUCTION

The increasing demand for advanced broadcasting strategies and the emergence of relay nodes motivate researchers to work on a new class of problems which is very important both from the practical and theoretical viewpoints [1, 2, 3]. For instance, transmitting over slow fading channels without channel state information (CSI) at the transmitter becomes an important problem in several applications including TV broadcasting and satellite communications. A special case of this question is to find the optimal transmission strategy for a one-hop single-user network when no CSI is available at the transmitter and the channel has slow fading characteristic.

To answer, in a pioneering work, Shamai has modeled this problem with a continuum of virtual receivers, each corresponding to a specific realization of the channel gain [4]. Relying on the resulting degraded broadcast channel, reference [4] shows that a multilevel coding achieves the capacity of the network. This scheme has also been investigated for a Multiple Input Multiple Output (MIMO) channels in [5]. Afterwards, A. Steiner *et al*, in [6], investigate the performance of multilevel codes in a two-hop network where the link between the source and the destination is extremely poor, and therefore, all data should be received via the relay node, Fig. 1. They have studied some different transmission schemes, including Decode and Forward ($\mathbf{DF}$), Amplify and Forward ($\mathbf{AF}$), and Quantize and Forward (**QF**).

As discussed in [6], due to high complexity of the infinite multilevel $\mathbf{DF}$ codes, they have only considered a finite-level $\mathbf{DF}$ strategies. Comparing the results in [6], it turn out that $\mathbf{AF}$ strategy outperforms all other investigated transmission schemes, specifically in high SNR regime. However, as concluded in [6], although $\mathbf{AF}$ has the best performance among the other strategies, the optimality of $\mathbf{AF}$ scheme is not implied. In fact, since the general infinite level $\mathbf{DF}$ broadcast strategy remains unsolved, there is always a question of whether or not there exists any proper power assignment of $\mathbf{DF}$ that can achieve a higher rate compared to $\mathbf{AF}$.

The main motivation of this work is to answer to above question. To this end, we propose a method to evaluate the optimal source and relay power distribution functions when $\mathbf{DF}$ is applied as the relaying mechanism. Numerical results are also presented and verify that the proposed scheme outperforms the $\mathbf{AF}$ strategy discussed in [6].

The organization of this paper is as follows: First, in section 2, we review some related previous results on single-hop links and then the multilevel transmission scheme is formulated for two-hop networks. This formulation is used in section 3 to evaluate the optimal power assignment for the multilevel decode and forward relaying. Next, in section 4, the achievable rate of the proposed coding scheme is evaluated and compared with that of the other existing schemes. Finally, section 5 concludes the paper.

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2. MULTILEVEL CODE TRANSMISSION SCHEMES

To enhance the lucidity of the future sections, in this part, before analyzing the two-hop network, we first review some previous results on single-hop networks. To this end, subsection 2.1 reviews the original broadcasting strategy [4]. Then, in subsection 2.2, the equivalent results is discussed for the case that the available data rate for transmission is limited [6]. At the end, we describe and formulate the multilevel coding scheme for the two-hop network configuration.

2.1. Single-Hop Broadcast Strategy

The optimum scheme for a single-hop link, in case that the transmitter knows the fading power for each block, say l, is to design a single-level code with the rate of $\log(1 + lP)$ for that block. P denotes the normalized transmission power, i.e., the equivalent transmission power if $N_0 = 1$. Therefore, the average achievable rate for this setup would be $R_{erg} = E_l[\log(1 + lP)]$ [7].

Although designing a single-level code is optimum in the above scenario, it can not be applied for a transmitter which does not have access to the channel state information. For this scenario, S. Shamai have introduce a technique called broadcasting strategy [4]. In this technique, the transmitter sends the data through infinite levels of a superposition code. Then, conditioned on the channel state, i.e., the fading power, the receiver decodes up to a certain level of the code. Therefore, the total receiving rate for each channel realization, say l, can be evaluated as

$$R(l) = \int_0^l dR(a), \tag{1}$$

where $dR(a)$ represents the differential rate transmitted over level 'a' of the code. Defining $\rho(l)dl$ as the power assigned for the l^{th} level, $dR(l)$ is given by

$$dR(l) = \log(1 + \frac{l\rho(l)dl}{1 + lI(l)}) \simeq \frac{l\rho(l)dl}{1 + lI(l)}, \tag{2}$$

where $I(l) = \int_l^\infty \rho(a)da$. The aim is to find a $\rho(l)$ such that the average received data rate is maximized while the source power constraint holds. Thus, we have,

$$\max_{\rho(l)} \quad \int_0^\infty dl f(l) R(l) \qquad (3)$$
$$s.t. \quad \int_0^\infty \rho(l) = P,$$

where $f(l)$ shows the probability density function (*pdf*) of the channel fading power and P represents the normalized total available power at the transmitter.

This maximization problem has been studied and solved using calculus of variations technique (see [4] for details of the proof). Here, we only mention the final solution as

$$I^*(l) = \begin{cases} P & l < l_0 \\ \frac{1-F(l)-lf(l)}{l^2 f(l)} & l_0 < l < l_1 \\ 0 & l_1 < l \end{cases}, \qquad (4)$$

where $F(l) = \int_{-\infty}^l f(a)da$. l_0 and l_1 are determined such that they satisfy $I^*(l_0) = P$ and $I^*(l_1) = 0$, respectively. Moreover, since $I(l) = \int_l^\infty \rho(a)da$, the optimum power assignment, $\rho^*(l)$, can also be determined by $\rho^*(l) = -\frac{dI^*(l)}{dl}$.

With this scheme, the rate that the transmitter feeds to the channel is equal to

$$R_F^* = \int_0^\infty dR^*(l), \qquad (5)$$

where $dR^*(l) = \frac{l\rho^*(l)dl}{1+lI^*(l)}$. Note that, this value is constant and only depends on the fading power distribution and the transmitter power. In other words, it does not depend on a specific channel realization. The optimal average rate at the receiver can also be evaluated by

$$R_{av}^* = \int_{l=0}^\infty dl f(l) R^*(l). \qquad (6)$$

2.2. Single-Hop Rate-Limited Broadcast Strategy

An interesting extension to the above original broadcast strategy is designing a multilevel code for a source with available data rate limited to R_{in}. This limitation can be implied by the user traffic model or packet dropping due to some network congestions.

This problem has been formulated in a similar way as in subsection 2.1, except it has one more condition on the total transmission rate. Thus, the modified optimization problem will be

$$\max_{\rho(l)} \quad \int_0^\infty dl f(l) R(l) \qquad (7)$$
$$s.t. \quad \int_0^\infty \rho(l)dl = P, \text{ and } \int_0^\infty dR(l) \leq R_{in},$$

where the second condition ensures that the total transmission rate remains less than the available data rate at the source. This problem can be solved using *constrained* calculus of variations and has been previously addressed in [6]. Due to the limitation of space, we will not bring the solution here and for more details we refer the readers to the technical report version of this paper [8].

One important observation is that in the case of $R_{in} \geq R_F^*$, the above rate-limited problem will be simplified to the original problem in subsection 2.1. Although this statement can be verified from the equations, its intuitive explanation could be insightful. It is obvious that if R_{in} tends to infinity, the rate condition is always satisfied and therefore, (7) relaxes to (3). Besides, we know that in the original problem (without rate limitation) to achieve the highest received data rate, the source requires to feed the channel with a rate equal to R_F^* (5). Hence, it turns out that even though the available rate at the source is more than R_F^*, the source needs to transmit only R_F^* bits of information in each block. Thus, if $R_{in} \geq R_F^*$, the solutions of the two optimization problems of (3) and (7) are equal.

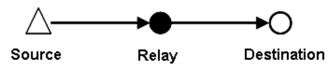

Fig. 1. Two-hop Network Model

2.3. Two-hop Broadcasting Strategy

Let us first restate the two-hop network model. As Fig. 1 shows, the destination can solely receive data via the relay and there is no direct link between the source and the destination. It is assumed that the source has no information about neither of the channels, the relay knows only the channel between itself and the source, and the destination only knows its channel gain to the relay. Moreover, we assume that the source always has as much as data that it requires and there is no constraint on the data rate. One transmission block consists of two phases:

A. In the first phase, the source allocates its power among different code levels with the power distribution function $\rho_s(l)$. $\rho_s(l)$ should satisfy the power constraint $\int_0^\infty \rho_s(a)da = P_s$. P_s is the total source power. Then, based on the source to the relay fading channel power, say x, the relay is able to decode up to the level x of the transmitted data. Thus, the relay received data rate is

$$R_r(x) = \int_0^x \log\left(1 + \frac{a\rho_s(a)da}{1 + aI_s(a)}\right) \simeq \int_0^x \frac{a\rho_s(a)}{1 + aI_s(a)}da, \qquad (8)$$

where $I_s(a) = \int_a^\infty P_s(a)da$.

B. In the second phase, the relay should transmit the data to the destination. As noted earlier, in this work, we only focus on *simple relays* which *can neither buffer any of the previously received data nor do any scheduling tasks*. As a results, these relays have two features which seem obvious but have important effect on the code design. To illustrate, consider the previous example in which the relay succeeded to decode $R_r(x)$ bits of the transmitted data. It turns out that, firstly, the relay can not transmit with the rate greater than $R_r(x)$. Secondly, if the relay transmits with the rate R_2, $R_2 < R_r(x)$, the rest of the data $(R_r(x) - R_2)$ can not be stored and should be discarded. Consequently, after receiving the source data, the relay should choose an optimum power distribution that satisfies both the relay total power constraint (P_r) and at the same time, does not require to transmit more than its received data rate in that block $(R_r(x))$. Defining γ and $\rho_r(l|\gamma = x)$ as the fading power of the source-relay link, and the relay power distribution of the code level l conditioned on $\gamma = x$, respectively, we can summarize these conditions as

   (a) Power constraint at the relay: $\forall x \in \gamma$: $\int_0^\infty \rho_r(a|\gamma = x)da = P_r$.

   (b) Available rate constraint at the relay: $\forall x \in \gamma$: $\int_0^\infty \frac{a\rho_r(a|\gamma=x)da}{1+aI_r(a|\gamma=x)} \leq R_r(x)$, where $R_r(x)$ is defined by (8).

Clearly, the relay requires to know $\rho_r(a|\gamma = x)$ for all possible values of 'γ' in order that it can optimally assign its power depending on different realizations of the first hop channel.

Transmitting the multilevel code on the relay to the destination link, the destination is able to decode up to a certain level 'y'. Here, 'y' denotes the fading power of the second link. Therefore, the received data rate at the destination can be written as

$$R_d(y|\gamma) = \int_0^y \log\left(1 + \frac{a\rho_r(a|\gamma)da}{1 + aI_r(a|\gamma)}\right) \simeq \int_0^y \frac{a\rho_r(a|\gamma)}{1 + aI_r(a|\gamma)}da. \quad (9)$$

Similar to the single-hop scenario, we want to maximize the average received data rate at the destination. Assuming $f_\gamma(x)$ and $f_\mu(y)$ as the probability density functions of the fading power in the source-relay and relay-destination links, respectively, the average destination rate can be written as

$$\begin{aligned} E\{R_d\} &= E_\gamma\left\{E_\mu\{R_d\left(\mu = y|\gamma = x\right)\}\right\} \quad (10) \\ &= \int_0^\infty \int_0^\infty f_\gamma(x) f_\mu(y) \int_0^y \frac{a\rho_r(a|\gamma = x)}{1 + aI_r(a|\gamma = x)} dadydx. \end{aligned}$$

Therefore, we come up to the final optimization problem as follows

$$\begin{aligned} \max_{\rho_s(l),\ \rho_r(l|\gamma)} \quad & \int_0^\infty \int_0^\infty f_\gamma(x) f_\mu(y) R_d(y|\gamma = x) dydx \quad (11) \\ s.t. \quad & \int_0^\infty \rho_s(a)da = P_s, \\ & \forall x \in \gamma : \int_0^\infty \rho_r(a|\gamma = x)da = P_r, \\ & \forall x \in \gamma : \int_0^\infty \frac{a\rho_r(a|\gamma = x)da}{1 + aI_r(a|\gamma = x)} \le R_r(x). \end{aligned}$$

The above optimization problem has the same form of the one derived in [6]. The only difference is in the rate limitation constraint, i.e., the last constraint, which is written as an equality constraint in [6]. In fact, we have relaxed the rate limitation constraint by letting the relay to discard some of its received data if it wants to do so.

3. DESIGN OF THE TWO-HOP OPTIMAL MULTILEVEL CODE

Unlike the two previous scenarios, the two-hop optimization problem (optimization problem in (11)) can not be directly solved by variations methods. It is due to the fact that the constraint on the second hop rate does not have a fixed value on the right side, i.e., it does not have a form of isoperimetric problem. For complete discussion on isoperimetric problem, refer to the reference [9].

Here, we choose an alternative approach. In fact, to find the solution, we first rearrange the problem as a two-step optimization problem. It should be noted that with this alternation, each of the subproblems can be solved by Euler-Lagrange method. For convenience in presentation, let us denote $\rho_r(a|\gamma = x)$ by $\rho_r(a|R_r(x) = i)$, where $i = \int_0^x \frac{a\rho_s(a)da}{1+aI_s(a)}$. In other words, we represent the first hop fading condition by the associated received data rate of the relay for that channel state. Using this presentation and noting that $f_\gamma(x) \ge 0, \forall x$, we can rewrite $E\{R_d\}$ in (11) as the following

$$\begin{aligned} &\max E\{R_d\} = \quad (12) \\ &\max_{\rho_s(l)} \int_0^\infty dx f_\gamma(x) \times \\ &\max_{\rho_r(l|R_r(x))} \int_0^\infty dy f_\mu(y) \int_0^y \frac{a\rho_r(a|R_r(x))}{1 + aI_r(a|R_r(x))}da, \end{aligned}$$

where the outer maximization is subject to

$$\int_0^\infty \rho_s(a)da = P_s, \quad (13)$$

and the constraints of the inner problem are as follows

$$\forall x \in \gamma : \int_0^\infty \rho_r(a|R_r(x) = i)da = P_r, \quad (14)$$

$$\forall x \in \gamma : \int_0^\infty \frac{a\rho_r(a|R_r(x)=i)}{1+aI_r(a|R_r(x)=i)}da \le i. \quad (15)$$

Given (12), we can now explain the procedure of finding the optimum $\rho_s(l)$ and $\rho_r(l)$.

3.1. Relay-Destination Link Optimization Problem

Receiving $R_r(x)$ bits from the first hop, the aim of the relay is to maximize the average received data rate received by the destination. Note that, if the input rate changes, the relay should modify its power distribution $\rho_r(l)$, accordingly. However, the knowledge of the input rate ($R_r(x)$), the relay total power, and the *pdf* of the second hop fading power is sufficient for determining the optimum distribution function, $\rho_r^*(l|R_r(x) = i)$. It is evident that the optimum power distribution function, $\rho_r^*(l)$, can be completely determined by evaluating the $\rho_r^*(l|R_r(x) = i)$ for all values of $R_r(x)$. The $\rho_r^*(l|R_r(x) = i)$, itself, is the solution of the following problem

$$\begin{aligned} h(i) \quad = \quad & \max_{\rho_r(l|R_r(x)=i)} \int_0^\infty dy\ f_\mu(y) \int_0^y \frac{a\rho_r(a|R_r(x) = i)da}{1 + aI_r(a|R_r(x) = i)} \quad (16) \\ s.t. \quad & \int_0^\infty \rho_r(a|R_r(x) = i)da = P_r, \\ & \int_0^\infty \frac{a\rho_r(a|R_r(x) = i)}{1 + aI_r(a|R_r(x) = i)}da \le i. \end{aligned}$$

This problem can be solved in a similar fashion as in the rate-limited broadcast strategy problem, subsection 2.2. Therefore, the optimum solution is

$$I_r^*(l|R_r(x) = i) = \begin{cases} P_r & l < l_0 \\ \frac{1-F_\mu(l)+\lambda-lf_\mu(l)}{f_\mu(l)l^2} & l_0 < l < l_1 \\ 0 & l_1 < l \end{cases}, \quad (17)$$

where $F_\mu(l) = \int_{-\infty}^{l} f_\mu(a)da$. l_0 and l_1 are determined as a function of λ to satisfy $I_r^*(l_0) = P_r$ and $I_r^*(l_1) = 0$, respectively. The optimum multilevel power distribution at the relay can be found by $\rho_r^*(l|R_r(x) = i) = -\frac{dI_r^*(l|R_r(x)=i)}{dl}$. Finally, λ is computed to satisfy

$$\int_0^\infty \frac{a\rho_r(a|R_r(x) = i)}{1 + aI_r(a|R_r(x) = i)}da = \min(i, R_F^*), \quad (18)$$

where R_F^* is defined by (5). This condition comes from the fact that achieving the maximum average rate at the destination requires the relay not to transmit more than R_F^* bits (refer to the discussion in the end of subsection 2.2).

As an example, we have solved (16) for a network in which the second hop can be modeled as a Rayleigh fading channel, i.e., $F_\mu(x) = 1 - e^{-x}$. Fig. 2 shows the maximum average received rate at the destination, $h^*(i)$, for different relay input rates ($i = R_r(x)$) and different relay powers (P_r).

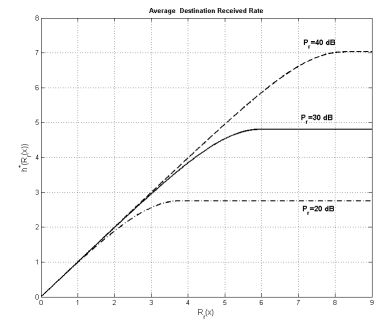

Fig. 2. Maximum Average Received Rate at the Destination

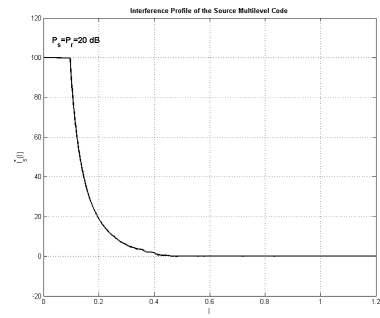

Fig. 3. Interference Profile of the Source Multilevel Code

3.2. Source-Relay Link Optimization Problem

Knowing the optimum value for the inner integration, $h^*(R_r(x))$, (12) can be written as follows

$$E\{R_d\} = \max_{\rho_s(l)} \int_0^\infty dx\, f_\gamma(x) h^*(R_r(x)) \qquad (19)$$

$$s.t. \quad \int_0^\infty \rho_s(a) da = P_s,$$

where

$$R_r(x) = \begin{cases} 0 & x < l_0 \\ \int_{l_0}^{x} \frac{a\rho_s(a)}{1+aI_s(a)} da & l_0 < x < l_1 \\ \int_{l_0}^{l_1} \frac{a\rho_s(a)}{1+aI_s(a)} da & l_1 < x \end{cases}, \qquad (20)$$

and $I_s(l) = \int_l^\infty \rho_s(a)da$. l_0 and l_1 satisfy $I_s(l_0) = P_s$ and $I_s(l_1) = 0$, respectively. Moreover, since $\rho_s(l) = -I'_s(l)$, we can define the integrand in (19) as $G(l, I_s, I'_s) = f_\gamma(l)h^*(R_r(l, I_s, I'_s))$. With this notation, (19) takes the form of a fixed end-point calculus of variations problem and can be solved using Euler-Lagrange equation, [10]:

$$\zeta = G_{I_s} - \frac{\partial G_{I'_s}}{\partial l} = 0, \qquad (21)$$

where $G_{I_s} = \frac{\partial G}{\partial R_r}\frac{\partial R_r}{\partial I_s}$, $G_{I'_s} = \frac{\partial G}{\partial R_r}\frac{\partial R_r}{\partial I'_s}$, and $\frac{\partial G_{I'_s}}{\partial l}$ is the partial derivative of $G_{I'_s}$ with respect to l. For details of the math please refer to [8].

As an example, in the scenario where both source-relay and relay-destination links are modeled with a Rayleigh fading channel, i.e., $F_\gamma(l) = F_\mu(l) = 1 - e^{-l}$, (21) can be simplified to

$$\zeta(l, I_s, I'_s) = h^{*'}(i)\left[\int_0^l \frac{1 - a - a^2 I_s(a)}{(1 + aI_s(a))^2} da\right] - h^{*''}(i)\left[\frac{-lI'_s(l)}{1 + lI_s(l)} \int_0^l \frac{-a}{1 + aI_s(a)} da\right] = 0, \qquad (22)$$

where $i = R_r(l, I_s, I'_s)$.

As can be observed, the solution depends on the first and second derivatives of the optimal achievable rate of the second hop, $h^*(i)$. These functions can be evaluated numerically using the results of subsection 3.1.

To find the optimal source power distribution ($\rho_s^*(l)$), using numerical methods, we first replace I_s by $[I_s(1), I_s(2), ..., I_s(N)]$, corresponding to the amount of interference in each level. [1] $I_s(m)$'s are in descending order, such that $I_s(1) = P_s$ and $I_s(N) = 0$. Then, we have a nonlinear system of N equations, i.e., $\zeta(m, I_s, I'_s) = 0, \quad m = \{1, 2, ..., N\}$ which can be solved numerically. The final solution for these N variables shows the (approximately) optimal interference function, $I_s^*(l)$. As an example, Fig. 3 presents $I_s^*(l)$ in the case of Rayleigh fading model for both hops and $P_s = P_r = 20dB$. Having $I_s^*(l)$, evaluating $\frac{-dI_s^*(l)}{dl}$ is sufficient in order to find the amount of power that should be assigned to each code level, $\rho_s^*(l)$.

As a summary, assuming that the relay will allocate power optimally, i.e., $h(i) = h^*(i)$, first, the source determines its power distribution function such that it maximizes (19) (as described in subsection 3.2). Using this optimal power distribution function ($\rho_s^*(l)$), the source transmits the data to the relay. Depending on the source-relay channel realization, the relay successfully decodes the rate of $R_r(x)$. Then, conditioned on the available data rate limitation, the relay chooses the optimum $\rho_r^*(l)$ such that it maximizes the average received data rate at the destination, (subsection 3.1).

4. NETWORK PERFORMANCE ANALYSIS

In the previous section, we have determined how the source and relay should distribute their available power through different levels of a multilevel code. To analyze the results, in this section, we compare the performance of the proposed scheme with an upper-bound and two other achievable rates proposed in [6] and briefly explained in the following.

A. **Broadcasting Cutset Bound**, $\mathbf{C_{cutset}}$:
This bound simply says that the achievable average data rate of a two-hop network can not exceed the achievable average rate of any of the single-hop links, i.e., the source-relay and the relay-destination links. Therefore, this bound can be written as

$$R_{cutset} = \min\left[\int_0^\infty da\, f_\gamma(a)R_1(a), \int_0^\infty da\, f_\mu(a)R_2(a)\right], \qquad (23)$$

where $R_1(a)$ ($R_2(a)$) denotes the rate that the relay (destination) can successfully decode when the source (relay) transmits over a channel with the fading power equal to 'a'.

B. **Amplify and Forward**, $\mathbf{AF}$:
This is the achievable rate of a two-hop network in which the relay performs the amplify and forward ($\mathbf{AF}$) on the received source signal. To design the optimum multilevel power distribution, first, the total equivalent channel should be evaluated. In other words, the source-relay and relay-destination channels combined with $\mathbf{AF}$ relaying can be modeled as one channel with a new probability density function. Having this new *pdf*

[1] In fact, we have approximated a continuous variable $I_s(l)$ with a discrete N-level function, which becomes precise as N tends to infinity.

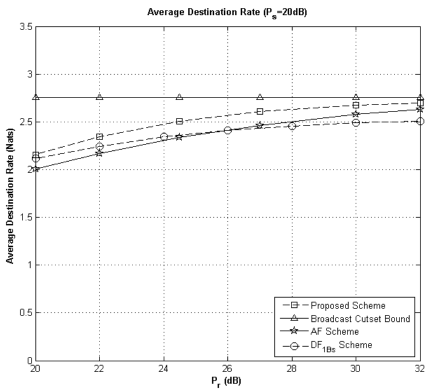

Fig. 4. Average Destination Rate P_s=20dB, P_r=20 - 32

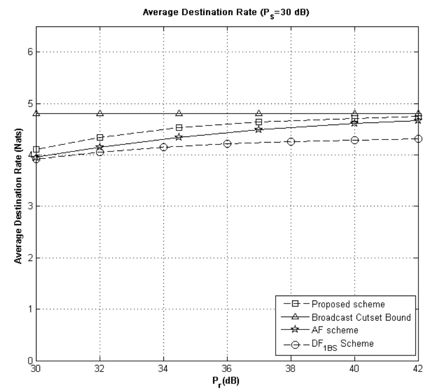

Fig. 5. Average Destination Rate P_s=30dB, P_r=30 - 42

the optimum power distribution can be evaluated. Details of the proof can be found in [6]. The final result can be written as

$$R_{AF} = \int_{x_0}^{x_1} dx \left[\frac{2(1 - F_{s_b}(x))}{x} + \frac{(1 - F_{s_b}(x)) f'_{s_b}(x)}{f_{s_b}(x)} \right], \quad (24)$$

where $I_{opt}(x_0) = P_s$, $I_{opt}(x_1) = 0$ and

$$F_{s_b}(x) = 1 - \int_{\frac{P_s}{P_r}x}^{\infty} dx_r \, e^{-x_r - \frac{x(1+P_r x_r)}{x_r P_r - x P_s}}, \quad (25)$$

and $f_{s_b}(x) = \frac{d}{dx} F_{s_b}(x)$. $I_{opt}(x)$ is also evaluated in [6].

C. **Outage At Source, Broadcasting At Relay**, $\mathbf{DF_{1-bs}}$: This scheme is another suboptimal strategy that has been studied in [6]. In this case, the source uses a one-level code, known as the *outage approach*, and the relay uses the optimal multilevel code. Clearly, this approach is a special case of the the proposed scheme. The achievable average rate of this scheme can be computed by

$$R_{DF,1-bs} = \quad (26)$$

$$\max_{s_s, I_r(y|\gamma=s_s)} \quad (1 - F_\gamma(s_s)) \int_0^\infty da (1 - F_\mu(a)) \frac{a\rho_r(a)}{1 + a I_r(a)}$$

$$s.t. \quad \int_0^\infty \frac{u\rho_r(u)du}{1 + u I_r(u)} = \log(1 + P_s s_s).$$

Figures (4) and (5) represent the average received data rate at the destination versus the relay power P_r for the proposed scheme, as well as the $\mathbf{AF}$ and $\mathbf{DF_{1-bs}}$ schemes, when $P_s = 20$dB and $P_s = 30$dB, respectively. The upper-bound, $\mathbf{C_{cutset}}$, is also depicted in both figures.

As seen in the figures, the proposed $\mathbf{DF}$ strategy outperforms the $\mathbf{AF}$ and $\mathbf{DF_{1-bs}}$ approaches. Note that, the superiority of the proposed scheme over $\mathbf{DF_{1-bs}}$ is obvious since it uses an optimal multilevel code at the source while $\mathbf{DF_{1-bs}}$ uses only one level of code. In fact, as mentioned before, $\mathbf{DF_{1-bs}}$ is a special case of the proposed scheme and hence, its performance is always below the performance of our scheme. The important observation in these figures is that the infinite multilevel $\mathbf{DF}$ strategy is superior to the $\mathbf{AF}$ strategy, which was previously the best known scheme for this setup at high SNR [6]. As a final remark, note that the proposed scheme can be implemented with low complexity as well. It is due to the possibility of offline computation of the optimum power assignments for different received relay rates, which can be stored as a look-up table. Then, during the network operations, corresponding to the relay received data rate, the relay can reload the optimum multilevel power assignment and use it in the second hop code design.

5. CONCLUSION

In this paper, a two-hop network in which the data is transmitted from the source node via a single relay to a destination node is considered. It is assumed that the knowledge of the channel for each transmission hop is only available at the corresponding receiver. For this network setup and assuming Decode and Forward ($\mathbf{DF}$) strategy at the relay, we have proposed a multilevel coding scheme at the source and the relay which achieves the maximum average received data rate at the destination. This problem had been remained unsolved before, due to its high complexity. Moreover, it is demonstrated that the $\mathbf{DF}$ multilevel coding scheme outperforms the Amplify and Forward ($\mathbf{AF}$) scheme, which was previously the best known scheme at the high SNR.

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