

Robust Joint Source-Channel Coding for Delay-Limited Applications

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¹ **Abstract**—In this paper, we consider the problem of robust joint source-channel coding over an additive white Gaussian noise channel. We propose a new scheme which achieves the optimal slope for the signal-to-distortion (SDR) curve (unlike the previous known coding schemes). We also derive some theoretical bounds on the asymptotic performance of delay-limited hybrid digital-analog (HDA) coding schemes. We show that, unlike the delay-unlimited case, for any family of HDA codes, the asymptotic performance loss is unbounded (in terms of dB).

I. INTRODUCTION

In many applications, delay-limited transmission of analog sources over an additive white Gaussian channel is needed. Also, in many cases the exact signal-to-noise ratio is not known at the transmitter, and may vary over a large range of values. Two examples of this scenario are transmitting an analog source over a quasi-static fading channel and/or multicasting it to different users (with different channel gains).

Without considering the delay limitations, digital codes can theoretically achieve the optimal performance. Indeed, for the ergodic channels, Shannon's source-channel coding separation theorem [1] [2] ensures the optimality of separately designing source and channel codes. However, for the case of a limited delay, several articles [3] [4] [5] [6] [7] have shown that joint source-channel codes have a better performance, compared to the separately designed source and channel codes (which are called tandem codes). Also, digital coding is very sensitive to the mismatch in the estimation of the channel signal-to-noise-ratio (SNR).

To avoid the saturation effect of digital coding, in [8] and [9] analog codes, based on dynamical systems are proposed. Although these codes can provide asymptotic gains (for high SNR) over the simple repetition code, they suffer from a threshold effect. Indeed, when the SNR becomes less than a certain threshold, the performance of these systems degrades severely. Therefore, the parameters of these methods should be chosen according to the operating SNR, hence, these methods are still very sensitive to the errors in the estimation of SNR. Also, although the performance of the system is not saturated for the high SNR (unlike digital codes), the scaling of the end-to-end distortion is far from the theoretical bounds.

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Theoretical bounds on the robustness of joint source channel coding schemes (for the delay-unlimited case) are presented in [10] and [11].

In this paper, we present a delay-limited analog coding scheme which achieves the optimum slope of the signal-to-distortion curve, with just a single mapping for different SNR values. We also analyze the limits on the asymptotic performance of delay-limited source-channel coding schemes.

II. SYSTEM MODEL AND THEORETICAL LIMITS

We consider a memoryless $\{X_i\}_{i=1}^{\infty}$ uniform source with zero mean and variance $\frac{1}{12}$, i.e. $-\frac{1}{2} \leq x_i < \frac{1}{2}$. For other sources, such as the Gaussian source, we can use the standard companding techniques. Also, the samples of the source sequence are assumed independent with identical distributions (i.i.d.).

The transmitted signal is sent over an additive white Gaussian noise (AWGN) channel. The problem is to map the one-dimensional signal to the N -dimensional channel space, such that the effect of the noise is minimized. This means that the data x , $-\frac{1}{2} \leq x < \frac{1}{2}$, is mapped to the transmitted vector $\mathbf{s} = (s_1, \dots, s_N)$ (with an average power less than 1). At the receiver side, the received signal is $\mathbf{y} = \mathbf{s} + \mathbf{z}$ where $\mathbf{z} = (z_1, \dots, z_N)$ is the additive white Gaussian noise with variance σ .

As an upper bound on the performance of the system, we can consider the case of delay-unlimited. In this case, we can use Shannon's theorem on the separation of source and channel coding. By combining the lower bound on the distortion of the quantized signal (using the rate-distortion formula) and the capacity of N parallel Gaussian channels with the noise variance σ^2 , we have [9]

$$D \geq c\sigma^{2N} \quad (1)$$

where c is a constant number and D is the average distortion.

III. PREVIOUS WORKS

Previously, three related schemes, based on dynamical systems, have been proposed for the scenario of delay-limited analog coding:

- 1) Shift-map dynamical system [8], and
- 2) Spherical shift-map dynamical system [9]

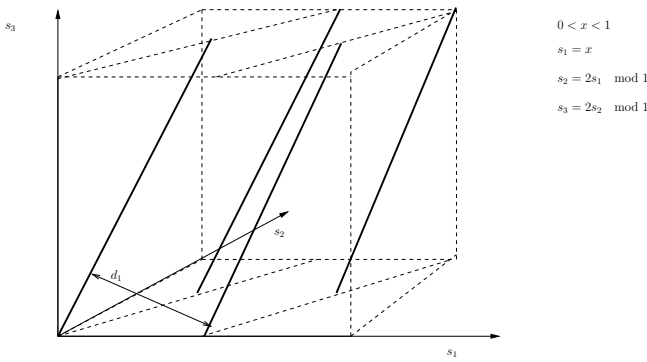


Fig. 1. The shift-map modulated signal set for $N = 3$ dimensions and $a = 2$.

In [8], an analog transmission scheme based on shift-map dynamical systems is presented. In this method, every analog data x is mapped to the modulated vector (s_1, \dots, s_N) where

$$s_1 = x \quad \text{mod } 1 \quad (2)$$

$$s_{i+1} = b_i s_i \quad \text{mod } 1, \quad \text{for } 1 \leq i \leq N-1 \quad (3)$$

where b_i is an integer number, $b_i \geq 2$. The set of modulated signals generated by the shift map consists of $b = b_1 \dots b_{N-1}$ parallel segments inside an N -dimensional unit hypercube. In [9], the authors have shown that by appropriately choosing the parameters $\{b_i\}$ for different SNR values, one can achieve the SDR scaling (versus the channel SNR) with the slope $N - \epsilon$, for any positive number ϵ . Indeed, we can have these bounds on the end-to-end distortion:

Theorem 1 Consider the the shift-map analog coding system which maps the modulating signals to N -dimensional modulated vectors,

i) For any noise variance σ^2 , we can find a number a such that for the shift-map scheme with the parameters $b_i = a^i$, the distortion of the decoded signal D is bounded as

$$D \leq c\sigma^{2N}(-\log \sigma)^{N-1}$$

ii) For any shift-map scheme, the output distortion is lower bounded as

$$D \geq c'\sigma^{2N}(-\log \sigma)^{N-1}$$

where c and c' are constant numbers (depending on only N).

Proof: see [12].

In [9], a spherical code based on the linear system $\hat{\mathbf{s}}_T = \mathbf{A}\mathbf{s}_T$ is introduced, where \mathbf{s}_T is the $2N$ -dimensional modulated signal and \mathbf{A} is a skew-symmetric matrix, i.e. $\mathbf{A}^T = -\mathbf{A}$.

This scheme is very similar to the shift-map scheme. Indeed, with an appropriate change of coordinates, the above modulated signal can be represented as

$$\mathbf{s}_T = \frac{1}{\sqrt{N}} (\cos 2\pi x, \cos 2a\pi x, \dots, \cos 2a^{N-1}\pi x,$$

$$\sin 2\pi x, \sin 2a\pi x, \dots, \sin 2a^{N-1}\pi x) \quad (4)$$

for some parameter a .

If we consider \mathbf{s}_{sm} as the modulated signal generated by the shift-map scheme with parameters $b_i = a$ in (3), then (4) can be written in the vector form as

$$\mathbf{s}_T = (\text{Re} \{e^{\pi i \mathbf{s}_{sm}}\}, \text{Im} \{e^{\pi i \mathbf{s}_{sm}}\}).$$

The relation between the spherical code and the linear shift-map code is very similar to the relation between PSK and PAM modulations. Indeed, the spherical shift-map code and PSK modulation are, respectively, the linear shift-map and PAM modulations which are transformed from the unit interval, $(\frac{-1}{2}, \frac{1}{2})$, to the unit circle.

For the performance of the spherical codes, the same result as Theorem 1 is valid. Indeed, for any parameters a and N , the spherical code asymptotically has a saving of $\frac{(2\pi)^2}{12}$ or 5.17 dB in the power. This asymptotic gain results from transforming the unit-interval signal set (with length 1 and power $\frac{1}{12}$) to the unit-circle signal set (with length 2π and power 1). However, the spherical code uses $2N$ dimensions (compared to N dimensions for the linear shift-map scheme).

For both these methods, for any fixed parameter a , the output SDR asymptotically has linear scaling with the channel SNR. The asymptotic gain (over the simple repetition code) is proportional to $a^{2(N-1)}$ (because the modulated signal is stretched approximately a^{N-1} times)². Therefore, larger scaling parameters a results in higher asymptotic gains. However, by increasing a , the distance between the parallel segments of the modulated signal set decreases. This distance is approximately $\frac{1}{a}$ and for the low SNRs (when the noise variance is larger than or comparable to $\frac{1}{a}$), jumping from one segment of the modulated signal set to another one becomes the dominant factor in the distortion of the decoded signal which results in a poor performance in this SNR region. Thus, there is a trade-off between the gain in the high-SNR region and the critical noise level which is fatal for the system. By increasing the scaling parameter a , the asymptotic gain increases, but at the same time, a higher SNR threshold is needed to achieve that gain. In [13], the authors have combined the dynamical-system schemes with LDPC and iterative decoding to slightly reduce the critical SNR threshold. However, overall behavior of the output distortion is the same for all these methods.

The shift-map analog coding system can be seen as a slight variation of a Hybrid Digital-Analog (HDA) joint source-channel code. Various types of these hybrid schemes are investigated in [10] and [14]. Indeed, for the shift-map system, we can rotate the modulated signal set such that all the parallel segments of it become aligned in the direction of one of the dimensions. In this case, by slightly changing the support region of the modulated set (which is a rotated N -dimensional

²The exact asymptotic gain is equal to the scaling factor of the signal set, i.e. $a^{2(N-1)} \left(1 + \frac{1}{a^2} + \dots + \frac{1}{a^{N-1}}\right)$ for the shift map and $\frac{(2\pi)^2}{12} a^{2(N-1)} \left(1 + \frac{1}{a^2} + \dots + \frac{1}{a^{N-1}}\right)$ for the spherical shift map.

cube) to the standard cube, we obtain a new similar modulation with almost the same performance. In the new modulation, the information signal is quantized by a^{N-1} points which are sent over $(N-1)$ dimensions, and the quantization error is sent over the remaining dimension.

Regarding the scaling of the output distortion, the performance of the shift-map scheme, with appropriate choice of parameters for each SNR, is very close to the theoretical limit. In fact, the output distortion scales as $\sigma^{2N}(-\log \sigma)^{N-1}$, instead of being proportional to σ^{2N} . However, for any fixed set of parameters, the output SNR (versus the input SNR) is saturated by the unit slope (instead of N). This shortcoming is an inherent drawback of schemes like the shift-map code or spherical code (which are based on dynamical systems). Indeed, in [15], it is shown that no single differentiable mapping can achieve an asymptotic slope better than 1. This article addresses this shortcoming.

IV. A NEW APPROACH

We propose a scheme for analog coding using a bandwidth expansion factor of N . In this technique, the mapping is not differentiable and it can achieve the optimal slope of N .

For the modulating signal x , $-\frac{1}{2} \leq x < \frac{1}{2}$, we consider the binary expansion of $x + \frac{1}{2}$:

$$x + \frac{1}{2} = (0 \cdot b_1 b_2 b_3 \dots)_2 \quad (5)$$

Now, we construct s_1, s_2, \dots, s_N as

$$\begin{aligned} s_1 &= (0 \cdot \overbrace{b_1 0 b_{\frac{N(N+1)}{2}+1} \dots b_{\frac{N(N+1)}{2}+N+1}} \dots 0 \overbrace{b_{(2N)\frac{(2N+1)}{2}+1} \dots})_2 \\ s_2 &= (0 \cdot \overbrace{b_2 b_3 0 b_{\frac{(N+1)(N+2)}{2}+1} \dots b_{\frac{(N+1)(N+2)}{2}+N+2}} \dots 0 \dots)_2 \\ &\quad \dots \\ &\quad \dots \\ s_N &= (0 \cdot \overbrace{b_{\frac{N(N-1)}{2}+1} \dots b_{\frac{N(N-1)}{2}} 0 \dots})_2 \end{aligned} \quad (6)$$

In summary, the bits of the binary expansion of $x + \frac{1}{2}$ are grouped such that the l th group ($l = kN + i$) consists of l bits and is assigned to the i th user.

Theorem 2³ Using the mapping constructed by the proposed method, the output distortion D is upper bounded by

$$D \leq c_1 \sigma^{2N} 2^{c_2 \sqrt{-\log \sigma}} \quad (7)$$

where c_1 and c_2 are constant⁴.

Proof: Consider w_i as the Gaussian noise on the i th channel and assume that n is selected such that

$$\sum_{k=1}^n kN + i \leq -\log_2 \sigma < \sum_{k=1}^{n+1} kN + i \quad (8)$$

³This theorem is also presented in [16].

⁴Troughout this paper c_1, c_2, c_3, \dots are constant numbers, independent of σ (they may depend on the dimensions).

The probability that $|w_i| \geq \frac{2^{-\sum_{k=1}^{n-1} kN+i}}{2}$ is negligible. Indeed,

$$\Pr \left\{ |w_i| \geq 2^{-\frac{\sum_{k=1}^{n-1} kN+i}{2}} \mid -\log_2 \sigma \geq \sum_{k=1}^n kN + i \right\} \leq Q(2^{nN-1}). \quad (9)$$

On the other hand, when $|w_i| < 2^{-\frac{\sum_{k=1}^{n-1} kN+i}{2}}$, the first $\sum_{k=1}^{n-1} kN + i$ bits of s_i can be decoded error-free. The same is true for all $1 \leq i' \leq i$, and for $i < i' \leq N$, the first $\sum_{k=1}^{n-2} kN + i'$ can be decoded error-free. Thus, the first $\sum_{j=1}^{(n-1)N+i} j$ bits of x can be decoded error-free. Now,

$$\sum_{j=1}^{(n-1)N+i} j \geq \quad (10)$$

$$N \sum_{k=1}^{n-2} kN + i \geq \quad (11)$$

$$N \sum_{k=1}^{n+1} kN + i - N(nN + i + (n+1)N + i) \geq \quad (12)$$

$$N \sum_{k=1}^{n+1} kN + i - N^2(2n+3) \geq \quad (13)$$

$$N \sum_{k=1}^{n+1} kN + i - c_2 \sqrt{\sum_{k=1}^{n+1} kN + i} \quad (14)$$

where c_2 depends only on N . Therefore, by using the assumption (8),

$$\sum_{j=1}^{(n-1)N+i} j \geq \quad (15)$$

$$-N \log_2 \sigma - c_2 \sqrt{-\log_2 \sigma} \quad (16)$$

Therefore the output distortion is bounded by

$$D \leq 2^{-2 \sum_{j=1}^{(n-1)N+i} j} \quad (17)$$

$$\leq 2^{2N \log_2 \sigma + 2c_2 \sqrt{-\log_2 \sigma}} \quad (18)$$

$$\implies D \leq c_1 \sigma^{2N} 2^{c_2 \sqrt{-\log \sigma}}. \quad (19)$$

Although the proposed scheme achieves the same SDR slope as the theoretical limit, there is an unbounded asymptotic gap between these two (when $\text{SNR} \rightarrow \infty$). In [11], the authors have shown that no single coding scheme can achieve the optimum SDR curve for all ranges of SNR. Indeed, the authors have shown that no coding scheme can touch the optimum curve more than once. However the question about

whether a bounded performance gap can be achieved by a single coding scheme or not, remains open.

In [16], a lattice-based approach is introduced to improve the performance of the proposed scheme, by using larger delays (by mapping from the n -dimensional source to a nN -dimensional signal set). However, it can only result in a finite gain (in terms of dB) and can not fill the unbounded gap.

V. APPROACHING A NEAR-OPTIMUM SDR BY DELAY-LIMITED CODES

In [10], a family of hybrid digital-analog (HDA) source-channel codes are proposed which together can achieve the optimum SDR curve and each of them only suffers from the mild saturation effect (the asymptotic unit slope for the curve of SDR versus SNR). However, their approach is based on using capacity-approaching digital codes as a component of their scheme.

In this section, we consider the problem of finding a family of delay-limited analog codes which together can achieve a near-optimum SDR curve and have a bounded asymptotic loss in the SDR performance (in terms of dB). Results of Section III show that none the previous analog coding schemes (based on dynamical systems) can construct such a family of codes. In this section, we also show that no HDA source-channel coding scheme can be used to achieve it. In the HDA source-channel coding, in general, to map an M dimensional source to an N dimensional signal set, the source is quantized by k points which are sent over $N - M$ dimensions and the residual noise is transmitted over the remaining M dimensions. In other words, the region of the source (which is a hypercube for the case of a uniform source) is divided to k subregions $\mathcal{A}_1, \dots, \mathcal{A}_k$. These subregions are mapped to k parallel subsets of the N dimensional Euclidean space, $\mathcal{B}_1, \dots, \mathcal{B}_k$, where \mathcal{B}_i is a scaled version of \mathcal{A}_i with a factor of a .

Theorem 3 Consider a HDA joint source-channel code which maps an M -dimensional uniform source (inside the unit cube) to k parallel M -dimensional subsets of an N dimensional Euclidean space ($N > M$), with a power constraint of P . For any noise variance σ^2 and any integer k , the output distortion is bounded by

$$D \geq c\sigma^{\frac{2N}{M}}(-\log\sigma)^{\frac{N-M}{M}} \quad (20)$$

where c is a constant number (independent of SNR).

Sketch of the proof: By considering the volumes of $\mathcal{A}_1, \dots, \mathcal{A}_k$ and their scaled versions, to satisfy the power constraint, a , the scaling factor, can not be greater than $c_3 k^{\frac{1}{M}}$, where c_3 depends on P .

We consider three cases for k :

Case 1) $k \leq \sigma^{-(N-M)}(-\log\sigma)^{\frac{-(N-M)}{2}}$:

Each subset of the modulated signal set is the scaled version of a segment of the source signal set by a factor of a , hence, we can lower bound the distortion by only considering the case the subset is decoded correctly and there is no jump to adjacent subsets,

$$D \geq E\{|\tilde{x} - x|^2 | no\ jump\} \quad (21)$$

$$= \frac{\sigma^2}{a^2} \quad (22)$$

$$\geq \frac{\sigma^2}{c_3^2 k^{\frac{2}{M}}} \quad (23)$$

$$\geq c_4 \sigma^{\frac{2N}{M}} (-\log\sigma)^{\frac{N-M}{M}} \quad (24)$$

Case 2) $2^l < \frac{k}{\sigma^{-(N-M)}(-\log\sigma)^{\frac{-(N-M)}{2}}} \leq 2^{l+1}$ for $l \geq 0$:

In this case, we bound the output distortion by the average distortion caused by a jump to another subset, during the decoding. By considering the power constraint (and using arguments based on Dirichlet's box principle) we can conclude that there are two constants c_5 and c_6 such that probability of an squared error of at least $c_5 (2^{-l}k)^{-\frac{2}{M}}$ is lower bounded by

$$\Pr(jump) \geq c_6 Q\left(\sqrt{-\log\sigma}\right) \geq c_6 \sigma$$

By considering the lower bound on the distortion caused by this jump,

$$\begin{aligned} D &\geq c_7 (2^{-l}k)^{-\frac{2}{M}} \sigma \geq \\ &c_8 \left(\sigma^{N-M}(-\log\sigma)^{\frac{(N-M)}{2}}\right)^{\frac{2}{M}} \sigma = \\ &c_8 \sigma^{\frac{2N}{M}} (-\log\sigma)^{\frac{N-M}{M}}. \end{aligned}$$

■

Now, we construct a family of delay-limited analog codes which by a proper choice of parameters (according to the channel SNR) have only a bounded asymptotic loss in the SDR performance (in terms of dB). For any $2^{-k+1} < \sigma \leq 2^{-k}$, for $k \geq 0$, we construct and analog code as the following:

For $x + \frac{1}{2} = (\overline{0 \cdot b_1 b_2 \dots b_{Nk-1}})_2 + \frac{\{2^{Nk-1}x\}}{2^{Nk-1}}$, where $\{\cdot\}$ represents the fractional part, we construct s_1, s_2, \dots, s_N as

$$s_1 = \sum_{i=1}^k (2^{-i} + 2^{-k}(k-i)) b_{(i-1)N+1}$$

$$s_2 = \sum_{i=1}^k (2^{-i} + 2^{-k}(k-i)) b_{(i-1)N+2}$$

$$s_{N-1} = \sum_{i=1}^k (2^{-i} + 2^{-k}(k-i)) b_{(i-1)N+N-1}$$

$$s_N = \sum_{i=1}^{k-1} (2^{-i} + 2^{-k}(k-i)) b_{(i-1)N+N} + 2^{Nk-k-2} \frac{\{2^{Nk-1}x\}}{2^{Nk-1}}$$

Theorem 4 In the proposed scheme, the output distortion D is upper bounded by

$$D \leq c\sigma^{2N} \quad (25)$$

where c is a constant, independent of σ .

Sketch of the proof: The signal set consists of 2^{Nk-1} segments of length 2^{-k-1} , where each of them is a scaled version of a subsegment of the source region (the unit interval), by a factor of 2^{Nk-k-2} .

The probability that the first error occurs in the $(i-1)N+j$ th bit of x is bounded by $Q(k-i)$ and it results in an output deviation of at most $2^{(i-1)N-j+1}$. Therefore, by considering the union bound over all possible errors:

$$D \leq \sum_{l=1}^{Nk-1} P_l \cdot D_l + D_{no-bit-error}$$

$$\leq \sum_{l=1}^{Nk-1} 2^{-l+1} Q(k-l/n) + 2^{-(Nk-k-2)} \sigma$$

Now, by using $Q(x) < e^{-\frac{x^2}{2}}$ and $2^{-k+1} < \sigma$ we have

$$D \leq c\sigma^{2N}.$$

■

It is worth noting that in the proposed family of codes, for each code, the asymptotic slope of the SDR curve is 1 (as we expected from the fact that for each code, the mapping is partially differentiable).

VI. CONCLUSIONS

To avoid the mild saturation effect in analog transmission systems and achieving the optimum scaling of the output distortion, we need to use nondifferentiable mappings (more precisely, mappings which are not differentiable on any interval). A nondifferentiable scheme is introduced in this paper which achieves the optimum slope for the curve of SDR versus the channel SNR, with a simple mapping. Also, we have introduced a family of delay-limited analog codes which have only a bounded asymptotic loss (in terms of dB), compared to the optimum performance. We have shown that no family of hybrid digital-analog coding schemes can have this property.

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