Diversity-Multiplexing Trade-Off In Ad-Hoc Networks

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***Abstract*— In this paper, the diversity-multiplexing trade-off is derived for a one-dimensional equally-spaced Rayleigh fading ad-hoc Network. It is assumed that the interference from each link to the other links in the network declines exponentially with the distance, such that the attenuation between two neighbor links is ρ^{α_0}. For any given multiplexing gain r, the maximum diversity gain is achieved by utilizing a general time-sharing scheme. We obtain an explicit formula for the maximum diversity gain, and show that depending on the value of r and α_0, there is an optimum time-sharing factor which yields the maximum diversity gain.**

I. Introduction

The rapidly increasing number of wireless users and new bandwidth-consuming applications fuel the growing demand for more bandwidth and higher data rates. The trend goes to providing higher data rates in suitable areas, such as offices, homes or public places. This is the domain of Wireless Local Area Network (WLAN) systems, which are low-cost, easy to deploy and robust due to their specific design for unlicensed operation.

The primary goal of WLAN is to provide connectivity, or in other words coverage at all desired locations. There are several factors in designing WLAN systems; infrastructure density which addresses the coverage requirement, e.g., the number of access point required to cover a typical environment and the frequency reuse factor which controls the received interference level. It is of interest to find the frequency reuse factor (or equivalent time-sharing factor in TDMA schemes) which yields the highest spectral efficiency while maintaining a given minimum signal quality. In this respect, there is an inherent tradeoff between the reliability of reception and the rate of communication. We may allocate different frequency bands to different users such that all receivers are interference free, which boosts the reliability of the network. On the other hand, the rate can be increased by sharing the bandwidth among users at the cost of a considerable interference level. In this work, we investigate the optimal time-sharing factor to maximize the network spectral efficiency considering the network's infrastructure density .

Resource allocation algorithms are studied vastly for ad-hoc networks [4]–[6]. A joint scheduling and power control algorithm is proposed in the context of unicast transmissions in ad-hoc networks [4]. The scheduling algorithm eliminates strong interferers so that the remaining nodes can solve the power control problem by using the distributed algorithm in [5]. Optimal spectrum sharing for a single-hop wireless network is studied in [6] where power allocation between different users has been discussed. A prevalent medium access scheme for channel reuse in ad-hoc wireless networks is spatial time division multiple access (STDMA), in which time is divided into fixed length slots that are organized cyclically [1]–[3]. STDMA schemes (with no power control) proposed in the literature can be classified into two focal categories: link scheduling [1], [2] and node scheduling [2], [3]. In each cycle, or time frame, every time slot is allocated to different designated communication links (under link scheduling) or to different designated user nodes (under node scheduling) such that all transmissions are received successfully at their intended receivers.

In this work, we consider a one-dimensional ad-hoc network in the high signal to noise ratio (SNR) scenario, and investigate the maximum achievable reliability in decoding (diversity gain) at the receivers, assuming a fixed transmission rate (multiplexing gain) for all the links in the network. The fundamental trade-off between the diversity and multiplexing gain has been characterized for multiple input multiple output (MIMO) systems in [7] and is extended for MIMO multiple-access channel in [8]. This approach has been applied for other wireless channels and networks [9]–[11]; In [9], the trade-off between the rate and the reliability is studied for different strategies in a wireless relay network. In [11], diversity-multiplexing trade-off upper bounds are obtained for cooperative diversity protocols in a wireless network.

For characterizing the diversity-multiplexing trade-off, we consider a simple one-dimensional equally-spaced Rayleigh fading ad-hoc network utilizing a general TDMA scheme. The one-dimensional model for wireless network is used in several works (e.g. [12] [13]). It is assumed that the interference from each link to the other links in the network grows exponentially with the distance, such that the interference between two neighbor links declines as $\rho^{-\alpha_0}$. It is shown that for any given multiplexing gain r, the maximum diversity gain is achieved by utilizing a general time-sharing scheme where the active users form equal-size equally-spaced clusters (a group

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of adjacent nodes) with the size at most 3, and the distance at most $\lceil \frac{1}{\alpha_0} \rceil$. The maximum diversity gain for each value of r is obtained by taking the maximum diversity gain among all the time-sharing strategies.

The rest of the paper is organized as follows; In section II, the system model is described. Section III is devoted to the characterization of the diversity-multiplexing trade-off for the ad-hoc network and finally, section IV concludes the paper.

II. System Model

We consider a homogeneous one-dimensional network, consisting of n pairs of transmitters and receivers. The nodes are equally spaced on two parallel lines such that the corresponding transmitter-receiver links are parallel. The network utilizes a general TDMA scheme, such that each link i is active in δ_i portion of the times. The objective is to find the optimum $\eta(t) = (\eta_1(t), \cdots, \eta_n(t))$, where $\eta_i(t)$ is one, if the ith link is active in the tth transmission block and otherwise is zero.

The channels between each transmitter and receiver nodes is assumed to be Rayleigh fading. We consider a simplistic model of signal attenuation, $e^{-\lambda d}$ over a distance d, where $\lambda \geq 0$ is the absorption constant (high attenuation environment). The attenuation factor from each node to its neighbors is assumed to be ρ^{α_0}. Since the attenuation model is assumed to be exponentially related to the distance, the closest active link will dominate the interference. Each receiver perfectly knows its own channel, as well as the channel corresponding to the strongest interference. The received signal at the ith receiver at transmission block t can be written as

$$\mathbf{y}_i(t) = \mathbf{H}_i(t)\mathbf{x}_i(t)\eta_i(t) + \sum_{j \neq i} \sqrt{\alpha_{ji}} \mathbf{H}_{ji}(t)\mathbf{x}_j(t)\eta_j(t) + \mathbf{n}_i(t), \tag{1}$$

where $\mathbf{H}_{ji}(t) \sim \mathcal{CN}(0, 1)$ and α_{ji} denote the interference channel and the attenuation factor from the jth transmitter to the ith receiver, respectively. The power constraint for ith transmitter is $\mathbb{E}\{\|\mathbf{x}_i\|^2\} \leq \rho_i$.

III. Diversity-Multiplexing Trade-off Analysis

In this part, we derive the diversity-multiplexing trade-off curve for the system described in the previous section. For each link, we define

$$r_i = \lim_{\rho_i \to \infty} \frac{\mathcal{R}_i(\rho_i)}{\log \rho_i}, \tag{2}$$

where $\mathcal{R}_i(\rho_i)$ denotes the transmission rate of link i. The optimal diversity-multiplexing tradeoff curve for this setup is defined as an $(n+1)$-dimensional vector $(r_1, \cdots, r_n, d^*(\mathbf{r}))$, $\mathbf{r} \triangleq (r_1, \cdots, r_n)$, such that

$$d^*(\mathbf{r}) = \max \lim_{\rho \to \infty} \frac{\log \Pr\{\mathscr{B}(\mathbf{r})\}}{\log \rho}, \tag{3}$$

where $\mathscr{B}(\mathbf{r}) \triangleq \bigcup_{i=1}^{n} \mathscr{B}_i(\mathbf{r})$ and $\mathscr{B}_i(\mathbf{r})$ denotes the outage event in link i, and the maximization is taken over all time-sharing strategies. For simplicity, we assume that the multiplexing gains of all the links are the same, i.e., $r_1 = \cdots = r_n = r$. Hence, we can express $d^*(\mathbf{r})$ as $d^*(r)$. Considering this assumption and the symmetry of the network, we conclude $\Pr\{\mathscr{B}(\mathbf{r})\} = \max_i \Pr\{\mathscr{B}_i(\mathbf{r})\}$.

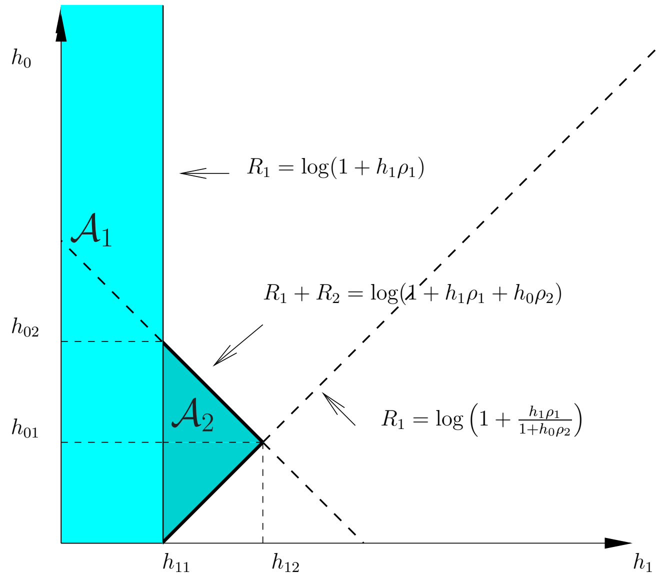

Fig. 1. Outage region

In the following, $\mathbf{X}_0(t)$ and $\mathbf{H}_0$ represent the strongest interference signal and the corresponding interference channel for the ith link, respectively. We distinguish between two scenarios:

- One strong interferer: ith link receives the dominant interference from the mth transmitter. In this case, $\mathbf{X}_0(t) = \mathbf{x}_m(t)$, $\mathbf{H}_0 = \mathbf{H}_{mi}$, $\mathcal{R}_0 = \mathcal{R}_m$ and $\alpha_\nu = \alpha_{mi} = \rho^{-|m-i|\alpha_0}$. We set $\tau_i = 1$ if ith link is in this case.
- Two strong interferers: ith link receives two equal interference from mth and lth transmitters such that $m - i = i - l$. In this case, we have $\mathbf{H}_0 = [\mathbf{H}_{li}, \mathbf{H}_{mi}]$, $\mathcal{R}_0 = \mathcal{R}_l + \mathcal{R}_m$, $\mathbf{X}_0(t) = [\mathbf{x}_l(t), \mathbf{x}_m(t)]^T$, and $\alpha_\nu = \alpha_{li} = \alpha_{mi} = \rho^{-|m-i|\alpha_0}$. We set $\tau_i = 2$ if ith link is in this case.

Lemma 1 *The probability of the outage event of the ith user is as follows:*

$$Pr\{\mathscr{B}_i\} = Pr\left\{\left[\mathcal{R}_i > \delta_i \log\left(\frac{h_i\rho}{1+\alpha_\phi\rho}\right)\right] \bigcup \left[\mathcal{R}_i + \mathcal{R}_0 > \delta_i \log\left(\frac{\alpha_\nu\rho\|\mathbf{H}_0\|^2 + h_i\rho}{1+\alpha_\phi\rho}\right)\right] \bigcap \left[\mathcal{R}_i > \delta_i \log\left(1 + \frac{\rho h_i}{1+\alpha_\nu\rho\|\mathbf{H}_0\|^2 + \alpha_\phi\rho}\right)\right]\right\} \tag{4}$$

where $\alpha_\phi = \max_{j, j \notin [2i-m, m], \eta_j = 1} \alpha_{ji}$.

Proof: Please see Appendix A. ∎

From Lemma 1, we find that the outage event is the intersection of multiple access channel outage region and interference outage region as depicted in Fig. 1. As can be observed, the outage event can be expressed as the union of the events $\mathcal{A}_1$ and $\mathcal{A}_2$. $\mathcal{A}_1$ is the outage event as if the second user does not exist. The effect of interference from the second user is captured in $\mathcal{A}_2$.

Theorem 2 *The diversity-multiplexing trade-off of ith link is given by*

$$d_i^*(r) = \begin{cases} \min(1-\beta_\phi - \frac{r}{\delta_i}, (1+\tau_i\beta_\nu - (\tau_i+1)\beta_\phi) \\ -\frac{(\tau_i+1)r}{\delta_i}(1+\tau_i\beta_\nu)) & \mathcal{F}_1 \\ 1-\beta_\phi - \frac{r}{\delta_i}(1+\tau_i\beta_\nu) & \mathcal{F}_2 \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

where $\beta_\nu \triangleq (1+\frac{\log a_\nu}{\log\rho})^+$, $\beta_\phi \triangleq (1+\frac{\log a_\phi}{\log\rho})^+$, $\mathcal{F}_1 \equiv r < \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+\tau_i\beta_\nu}$, *and* $\mathcal{F}_2 \equiv \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+\tau_i\beta_\nu} < r < \frac{\delta_i(1-\beta_\phi)}{1+\tau_i\beta_\nu}$.

Proof: Considering the result of Lemma 1 and noting Fig. 1, we define $h_{11} \triangleq \frac{1+a_\phi\rho}{\rho}(e^{\frac{\mathcal{R}_i}{\delta_i}}-1)$, $h_{01} \triangleq \frac{1+a_\phi\rho}{a_\nu\rho}(e^{\frac{\mathcal{R}_0}{\delta_i}} - 1 + e^{-\frac{\mathcal{R}_i}{\delta_i}})$, $h_{12} \triangleq \frac{1+a_\phi\rho}{\rho}(e^{\frac{\mathcal{R}_i+\mathcal{R}_0}{\delta_i}} - e^{\frac{\mathcal{R}_0}{\delta_i}} + e^{-\frac{\mathcal{R}_i}{\delta_i}}-1)$, $h_{02} \triangleq \frac{1+a_\phi\rho}{a_\nu\rho}(e^{\frac{\mathcal{R}_i+\mathcal{R}_0}{\delta_i}} - e^{\frac{\mathcal{R}_i}{\delta_i}}+1)$, $m_1 \triangleq \frac{h_{01}}{h_{12}-h_{11}}$, and $m_2 \triangleq \frac{h_{02}-h_{01}}{h_{12}-h_{11}}$. Considering Fig. 1, the outage probability in (4) can be written as

$$\begin{aligned}\Pr\{\mathscr{B}_i\} &= \Pr\{\mathcal{A}_1\} + \Pr\{\mathcal{A}_2\} \\ &= 1-e^{-h_{11}} \\ &+ \int_{h_{11}}^{h_{12}}\int_{m_1(h_1-h_{11})}^{h_{02}+m_2(h_{11}-h_1)} f(h_1)f(h_0)\mathrm{d}h_0\mathrm{d}h_1, \end{aligned} \tag{6}$$

where $h_0 \triangleq \|\mathbf{H}_0\|^2$. We consider the following two scenarios:

i) One strong interference: In this case, $f(h_0) = e^{-h_0}$ and $f(h_1) = e^{-h_1}$. From (6), we derive $\Pr\{\mathcal{A}_2\}$ as follows:

$$\begin{aligned}\Pr\{\mathcal{A}_2\} = &\frac{e^{-h_{11}}}{m_1+1}\left(1-e^{-(1+m_1)(h_{12}-h_{11})}\right) - \\ &\frac{e^{-(h_{12}+h_{01})}}{m_2-1}\left(1-e^{-(m_2-1)(h_{12}-h_{11})}\right) \\ \overset{(a)}{\doteq}\, & e^{-h_{11}}(h_{12}-h_{11}) - \\ &\frac{e^{-(h_{12}+h_{01})}}{m_2-1}\left(1-e^{-(m_2-1)(h_{12}-h_{11})}\right), \end{aligned} \tag{7}$$

where (a) comes from applying the approximation $1-e^{-(1+m_1)(h_{12}-h_{11})} \doteq (1+m_1)(h_{12}-h_{11})$, since $(1+m_1)(h_{12}-h_{11}) \to 0$. For the case that $r < \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+\beta_\nu}$, rewriting (7) as

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &\doteq e^{-h_{11}}(h_{12}-h_{11}) - \\ &\quad \frac{1}{m_2-1}e^{-(h_{11}+h_{02})}\left(e^{(m_2-1)(h_{12}-h_{11})}-1\right), \\ &\overset{(a)}{\doteq} e^{-h_{11}}\left(1-e^{-h_{02}}\right)(h_{12}-h_{11}) \\ &\overset{(b)}{\doteq} h_{02}(h_{12}-h_{11}) \end{aligned} \tag{8}$$

where (a) comes from applying the approximation $1-e^{(m_2-1)(h_{12}-h_{11})} \doteq (m_2-1)(h_{12}-h_{11})$ since in this case, we have $(m_2-1)(h_{12}-h_{11}) \to 0$ and (b) comes from the facts that $e^{-h_{11}} \doteq 1$ and $e^{-h_{02}} \doteq 1-h_{02}$, since we have $h_{11}, h_{02} \to 0$. For the case that $r > \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+\beta_\nu}$, noting that $(m_2-1)(h_{12}-h_{11}) \to \infty$, (7) is approximated as

$$\Pr\{\mathcal{A}_2\} \doteq h_{12} - h_{11}. \tag{9}$$

Using (8) and (9), we derive the diversity gain in (5).

ii)Two strong interference: In this case, $f(h_0) = h_0e^{-h_0}$ and $f(h_1) = e^{-h_1}$. In (6), $\mathscr{I} \triangleq \Pr\{A_2\}$ can be upper-bounded and lower-bounded as follows:

$$\begin{aligned}\mathscr{I} \le \mathscr{I}^U &\triangleq \int_{h_{11}}^{h_{12}}\int_0^{h_{02}} h_0e^{-h_0}e^{-h_1}\mathrm{d}h_0\mathrm{d}h_1 \\ &= \left(e^{-h_{11}}-e^{-h_{12}}\right)\left(1-(1+h_{02})e^{-h_{02}}\right), \end{aligned} \tag{10}$$

$$\begin{aligned}\mathscr{I} \ge \mathscr{I}^L &\triangleq \int_{h_{11}}^{h_{12}}[e^{-m_1(h_1-h_{11})} \\ &\quad - (h_{02}+1)e^{-(h_{02}+m_2(h_{11}-h_1))}]e^{-h_1}\mathrm{d}h_1 \\ &= e^{-h_{11}}[\frac{1}{m_1+1}\left(1-e^{-(1+m_1)(h_{12}-h_{11})}\right) \\ &\quad - e^{-h_{02}}\frac{(h_{02}+1)}{m_2-1}\left(e^{(m_2-1)(h_{12}-h_{11})}-1\right)]. \end{aligned} \tag{11}$$

For $r < \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+2\beta_\nu}$, the first and the second terms in (10) can be approximated by $(h_{12}-h_{11})$ and h_{02}^2, respectively. For $r > \frac{\delta_i(\beta_\nu-\beta_\phi)}{1+2\beta_\nu}$, (10) can be approximated by $h_{12}-h_{11}$. It can be easily shown that the same result is true for the lower-bound. As a result, the final diversity gain is obtained as in (5). ■

In the above, we have derived the maximum diversity gain for the ith link, conditioned on having a fixed δ_i and $\eta(t)$. Now, we want to obtain the optimum values for $\delta = (\delta_1, \delta_2, \cdots, \delta_n)$ and $\eta(t)$, based on α_0 and r. Let us consider the following special cases:

- $\alpha_0 > 1$: In this case, it is easy to see that the interference from all the links are negligible with respect to the noise. Therefore, we can consider this case as a parallel non-interfering ad-hoc Network, where the optimal values of η and δ are equal to $\mathbf{1}$ and $\mathbf{1}$, respectively. The maximum diversity gain of the network can be obtained as

$$d^*(r) = 1-r, \quad 0 \le r < 1. \tag{12}$$

- $\alpha_0 = 0$: In this case, the attenuation of all interference channels is 1. Assuming that all the receiver nodes know all their corresponding channels (direct channel and interference channels), similar to (13), the outage probability for all the links can be written as

$$\begin{aligned}\Pr\{\mathscr{B}_i\} = \Pr\Big\{&[r\log\rho > \delta_i\log(1+h_i\rho)]\bigcup \\ &\left[n'r\log\rho > \delta_i\log(1+\rho\|\mathbf{H}_0\|^2+h_i\rho)\right]\bigcap \\ &\left[r\log\rho > \delta_i\log\left(1+\frac{\rho h_i}{\rho\|\mathbf{H}_0\|^2+1}\right)\right]\Big\}, \end{aligned} \tag{13}$$

where $\mathbf{H}_0 \triangleq [h_{1i}, \cdots, h_{(i-1)i}, h_{(i+1)i}, \cdots, h_{ni}]$ and n' denotes the number of active links in the network. Due to the symmetry between the links, we have $\delta_i = \delta$, $\forall i$. As a consequence, $n' = n\delta$. Following the equations (6), (10), and (11), and noting that $f(h_0) = \frac{h_0^{n'-1} e^{-h_0}}{(n'-1)!}$, we have

$$d_i^*(r) = \begin{cases} \min\left(1 - \frac{r}{\delta}, n\delta(1 - nr)\right) & r < \frac{1}{n} \\ 0 & \text{Otherwise} \end{cases} \quad (14)$$

The value of δ which maximizes the diversity gain in (14) is 1. Hence,

$$d^*(r) = \begin{cases} \min\left(1 - r, n(1 - nr)\right) & r < \frac{1}{n} \\ 0 & \text{Otherwise} \end{cases} \quad (15)$$

For $0 < \alpha_0 < 1$, we make the following observations:

- Assuming large number of links in the network, almost all the links have the same situation (except the very end ones). Hence, as a result of the symmetry, we have $\delta_i = \delta$, $\forall i$. This suggests that we only need to derive the diversity-multiplexing trade-off for one link.
- We can categorize the links into *clusters*, where each cluster consists of some neighbor links, which are active simultaneously. Because of the symmetry in the network, all the clusters have the same number of links.
- The number of links in each cluster must be less than or equal to 3. The reason is that having more than 3 links in a clusters makes the outage probability the same as when all the links are active at the same time. Therefore, the diversity gain will be strictly less than that of the all-active case, and as a result, we can disregard these cases.
- Defining the distance between two clusters as

$$\mathscr{D}(\mathcal{C}_1, \mathcal{C}_2) \triangleq \min_{l_1 \in \mathcal{C}_1, l_2 \in \mathcal{C}_2} \mathscr{D}(l_1, l_2), \quad (16)$$

where l_1 and l_2 are two links in the clusters $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively, and $\mathscr{D}(l_1, l_2)$ denotes the normalized distance between these links (in terms of the distance between two neighbor links), we have

$$\mathscr{D}(\mathcal{C}_i, \mathcal{C}_j) \leq \lceil 1/\alpha_0 \rceil, \quad (17)$$

for any active neighbor clusters $\mathcal{C}_i$ and $\mathcal{C}_j$. The reason is that for $\mathscr{D}(\mathcal{C}_i, \mathcal{C}_j) > \lceil 1/\alpha_0 \rceil$, the inter-cluster interference is negligible with respect to the noise, and since increasing the distance between the clusters reduces the diversity gain, there is no point to consider $\mathscr{D}(\mathcal{C}_i, \mathcal{C}_j) > \lceil 1/\alpha_0 \rceil$.

From all the above observations, it follows that the diversity gain is a function of k, the number of links in a cluster, and s, the distance between two neighbor clusters. We denote the diversity by $d_{k,s}^*(r)$ and define $\mathfrak{F}_\tau(\frac{r}{\delta}, \beta_\nu, \beta_\phi)$ as the diversity gain in (5). We determine $d_{k,s}^*(r)$ in terms of function $\mathfrak{F}$ as follows:

$$d_{1,s}^*(r) = \mathfrak{F}_2(rs, (1 - s\alpha_0)^+, (1 - 2s\alpha_0)^+). \quad (18)$$

$$d_{2,s}^*(r) = \mathfrak{F}_1(\frac{r(s+1)}{2}, (1 - \alpha_0)^+, (1 - s\alpha_0)^+). \quad (19)$$

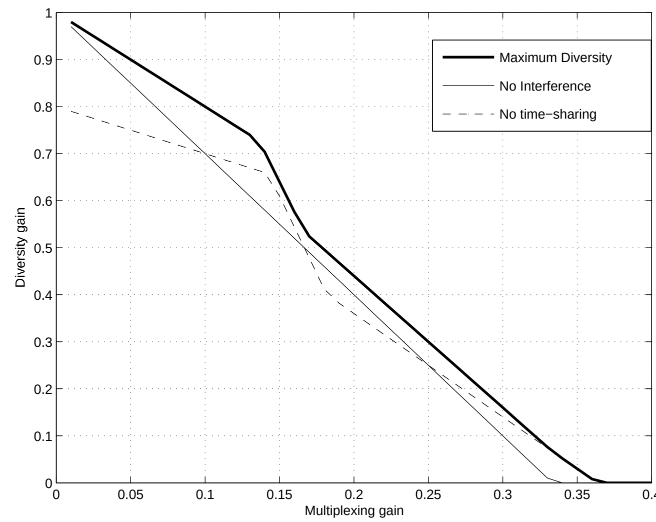

Fig. 2. The diversity gain of a one-dimensional network vs. multiplexing gain for $\alpha_0 = .4$.

$$d_{3,s}^*(r) = \min(d_{3,s}'(r), d_{3,s}''(r)), \quad (20)$$

where

$$d_{3,s}'(r) = \mathfrak{F}_1(\frac{r(s+2)}{3}, (1 - \alpha_0)^+, (1 - s\alpha_0)^+), \quad (21)$$

and

$$d_{3,s}''(r) = \mathfrak{F}_2(\frac{r(s+2)}{3}, (1 - \alpha_0)^+, (1 - (s+1)\alpha_0)^+). \quad (22)$$

The maximum diversity gain can be obtained as

$$d^*(r) = \max_{k,s} d_{k,s}^*(r). \quad (23)$$

The diversity gain $d^*(r)$ is depicted in Fig. 2 for $\alpha_0 = 0.4$. It is compared with the diversity gain of the network when all the links are active simultaneously (No-time-sharing), and also when the time-sharing is applied such that active links do not project interference on each other (No-interference). Fig. 2 clearly shows that the optimum scheme depends on the rate of transmission, e.g. for low multiplexing gains, the performance of No-interference scheme is close to optimum while for high multiplexing gains, the performance of No-time-sharing scheme is optimum.

IV. Conclusion and Future Works

This paper introduces a measure for optimally allocating bandwidth among users considering network's infrastructure density in a one-dimensional ad-hoc network. The diversity-multiplexing trade-off curve is characterized for one-dimensional equally-spaced Rayleigh fading ad-hoc network utilizing a general time-sharing scheme. We have shown that the diversity gain for each strategy depends only on the size of the clusters as well as the distance between two neighboring clusters. Moreover, the maximum diversity gain for each value of r is obtained by taking the maximum diversity gain among all the strategies at r.

V. Appendix A

Noting (1), for all the links (except the first and last ones), we can write

$$\mathbf{y}_i(t) = \mathbf{H}_i(t)\mathbf{x}_i(t)\eta_i(t) + \sqrt{\alpha_\nu}\mathbf{H}_0(t)\mathbf{X}_0(t) + \sum_{j, j\notin[2i-m,m]} \sqrt{\alpha_{ji}}\mathbf{H}_{ji}(t)\mathbf{x}_j(t) + \mathbf{n}_i(t), \quad (24)$$

Let us define $\mathscr{B}_i^L$ and $\mathscr{B}_i^U$ as follows:

$$\mathscr{B}_i^L \triangleq \Big\{ (\mathcal{R}_i > I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0, \mathbf{V}_0)) \bigcup (\mathcal{R}_i + \mathcal{R}_0 > I(\mathbf{x}_i, \mathbf{X}_0; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0, \mathbf{V}_0)) \bigcap (\mathcal{R}_i > I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0, \mathbf{V}_0)) \Big\}, \quad (25)$$

where $\mathbf{V}_0$ includes all the transmitted signals except $\mathbf{x}_i$ and $\mathbf{X}_0$.

$$\mathscr{B}_i^U \triangleq \Big\{ (\mathcal{R}_i > I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0)) \bigcup (\mathcal{R}_i + \mathcal{R}_0 > I(\mathbf{x}_i, \mathbf{X}_0; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0)) \bigcap (\mathcal{R}_i > I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0)) \Big\}. \quad (26)$$

In fact, $\mathscr{B}_i^L$ denotes the outage event for the ith link, when the receiver has full access to the other users data, and $\mathscr{B}_i^U$ stands for the outage event in the ith link, when the receiver treats all the users data, except the dominant interference, as noise. It is clear that

$$\Pr\{\mathscr{B}_i^L\} \le \Pr\{\mathscr{B}_i\} \le \Pr\{\mathscr{B}_i^U\}. \quad (27)$$

In the following, $h(.)$ denotes the entropy function. We compute the mutual information in (25) and (26) as follows:

$$\begin{aligned} & I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0) \\ &= \Pr\{\eta_i = 1\} I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0, \eta_i = 1) \\ &\ge \delta_i h(\mathbf{H}_i\mathbf{x}_i + \sqrt{\alpha_\nu}\mathbf{H}_0\mathbf{X}_0 + \mathbf{n}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0) \\ &- \delta_i \log\left(2\pi e \mathrm{Var}\left(\sum_{j, j\notin[2i-m,m]} \sqrt{\alpha_{ji}}\mathbf{H}_{ji}\mathbf{x}_j\eta_j + \mathbf{n}_i\right)\right) \\ &= \delta_i \log(2\pi e(\rho h_i + 1)) \\ &- \delta_i \log\left(2\pi e\left(\sum_{j, j\notin[2i-m,m]} \alpha_{ji}\rho\eta_j + 1\right)\right) \\ &= \delta_i \log\left(\frac{h_i\rho + 1}{\sum_{j, j\notin[2i-m,m]} \alpha_{ji}\rho\eta_j + 1}\right). \end{aligned} \quad (28)$$

$$\begin{aligned} & I(\mathbf{x}_i, \mathbf{X}_0; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0) \\ &\ge \delta_i h(\mathbf{H}_i\mathbf{x}_i + \sqrt{\alpha_\nu}\mathbf{H}_0\mathbf{X}_0 + \mathbf{n}_i | \mathbf{H}_i, \mathbf{H}_0) \\ &- \delta_i \log\left(2\pi e \mathrm{Var}\left(\sum_{j, j\notin[2i-m,m]} \sqrt{\alpha_{ji}}\mathbf{H}_{ji}\mathbf{x}_j\eta_j + \mathbf{n}_i\right)\right) \\ &= \delta_i \log\left(\frac{h_i\rho + \alpha_\nu\rho\|\mathbf{H}_0\|^2 + 1}{\sum_{j, j\notin[2i-m,m]} \alpha_{ji}\rho\eta_j + 1}\right). \end{aligned} \quad (29)$$

$$I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0) \ge \delta_i \log\left(\frac{h_i\rho + \alpha_\nu\rho\|\mathbf{H}_0\|^2 + 1}{\alpha_\nu\rho\|\mathbf{H}_0\|^2 + \sum_{j, j\notin[2i-m,m]} \alpha_{ji}\rho\eta_j + 1}\right). \quad (30)$$

$$I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{X}_0, \mathbf{H}_0, \mathbf{V}_0) \ge \delta_i \log\left(1 + \frac{\rho h_i}{1 + \sum_{j, j\notin[2i-m,m]} \alpha_{ji}\rho\eta_j}\right). \quad (31)$$

$$I(\mathbf{x}_i, \mathbf{X}_0; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0, \mathbf{V}_0) \ge \delta_i \log\left(1 + \frac{\rho h_i + \alpha_\nu\rho\|\mathbf{H}_0\|^2}{1 + \sum_{j, j\notin[2i-m,m]} \alpha_{ji}\rho\eta_j}\right). \quad (32)$$

$$I(\mathbf{x}_i; \mathbf{y}_i | \mathbf{H}_i, \mathbf{H}_0, \mathbf{V}_0) \ge \delta_i \log\left(1 + \frac{\rho h_i}{1 + \alpha_\nu\rho\|\mathbf{H}_0\|^2 + \sum_{j, j\notin[2i-m,m]} \alpha_{ji}\rho\eta_j}\right) \quad (33)$$

We define $\alpha_\phi \triangleq \max_{j, j\notin[2i-m,m], \eta_j=1} \alpha_{ji}$. As $\rho \to \infty$, we have $\sum_{j, j\notin[2i-m,m]} \alpha_{ji}\rho\eta_j \doteq \alpha_\phi\rho$. From the equations (28)-(33), we can see that $\Pr\{\mathscr{B}_i^L\} \doteq \Pr\{\mathscr{B}_i^U\}$ and the result of the lemma follows.

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