

# Sum-Rate Maximization in Single-Hop Wireless Networks with the On-Off Power Scheme

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**Abstract**—A single-hop wireless network with  $K$  links is considered, where the links are partitioned into  $M$  clusters, each operating in a subchannel with bandwidth  $\frac{W}{M}$ . We assume that the links in each cluster perform the on-off power allocation strategy proposed in [1]. The problem is to analyze the average sum-rate of the network in terms of  $M$  and under the shadow-fading effect with probability  $\alpha$ . It is demonstrated that for  $M \sim o(K)$  and  $0 < \alpha \leq 1$ , where  $\alpha$  is a fixed parameter, the average sum-rate of the network scales as  $\frac{W}{\alpha} \log K$ . For  $M \sim \Theta(K)$ , we present an upper bound for the average sum-rate. It is proved that the maximum average sum-rate of the network for every value of  $0 < \alpha \leq 1$  is achieved at  $M = 1$ . In fact, in the proposed model, partitioning the bandwidth  $W$  into  $M$  subchannels has no gain in terms of enhancing the throughput.

## I. INTRODUCTION

The main challenge in multi-user wireless networks originates from the sharing of a common transmission bandwidth by users such that the throughput of the network is maximized. Since the throughput of the network is limited by the interference, several techniques are proposed to mitigate the interference. Effective spectral usage and power control in transmitters have long been regarded as efficient tools to reduce the interference and improve the throughput of the network with limited bandwidth. In recent years, various power control and spectrum sharing schemes have been extensively studied in cellular and multihop wireless networks [2]–[5]. Much of these works rely on centralized and cooperative algorithms. Due to significant challenges in using the centralized approaches, the attention of the researchers have drawn to the decentralized resource allocation schemes [6]–[10]. Etkin and Tse [7] develop power and spectrum allocation strategies in multiple wireless systems. Under assumptions of the omniscient nodes and strong interference, they show that frequency division multiplexing (FDM) is the optimal scheme in the throughput maximization.

In this paper, we study the performance of a single-hop wireless network, in which  $K$  links are partitioned into  $M$  clusters, each operating in a subchannel with bandwidth  $\frac{W}{M}$ . The users utilize the decentralized on-off power allocation

scheme proposed in [1]. In [1], the authors study the performance of the network only for  $M = 1$  and under a Rayleigh-fading channel model. It is well-known, however, that the wireless channel can be modeled in a more realistic manner. Here, we consider the shadow-fading effect that are caused by obstacles. The main contribution of this work is to determine the maximum throughput of the network in terms of different values of  $M$  and the probability of the shadowing effect,  $\alpha$ . Our strategy differs from the model studied in [8] and [11]; primarily we use a decentralized on-off power allocation scheme for a single-hop wireless network with  $M$  subchannels, while [8] and [11] present a model with random connections for  $M = 1$  and using relay nodes.

Under the assumption of Rayleigh fading, an asymptotic analysis is carried out to show that for  $M \sim o(K)$  and  $0 < \alpha \leq 1$ , where  $\alpha$  is a fixed value, the average sum-rate of the network is of order  $\frac{W}{\alpha} \log K$ . It is shown that the maximum average sum-rate of the network for every value of  $M$  and  $0 < \alpha \leq 1$  is achieved at  $M = 1$ .

The rest of the paper is organized as follows. In Section II, the network model and objectives are described. We analyze the average sum-rate of the network in Section III. Finally, in Section IV, an overview of the results is presented.

*Knuth's notation* [12]: For any functions  $f(n)$  and  $g(n)$ :

- $f(n) = O(g(n))$  means that  $\lim_{n \rightarrow \infty} |f(n)/g(n)| < \infty$ .
- $f(n) = o(g(n))$  means that  $\lim_{n \rightarrow \infty} |f(n)/g(n)| = 0$ .
- $f(n) = \Theta(g(n))$  means that  $\lim_{n \rightarrow \infty} |f(n)/g(n)| = c$ , where  $0 < c < \infty$ .

Also,  $\log(\cdot)$  is the natural logarithm function and  $\mathcal{N} = \{1, 2, \dots, n\}$ .

## II. NETWORK MODEL AND OBJECTIVES

In this work, we consider a single-hop wireless network consisting of  $K$  pairs of nodes, operating in a bandwidth  $W$ . The links are assumed to be partitioned into  $M$  clusters such that the number of links in each cluster is the same. Also, the users are randomly divided among the clusters. The bandwidth  $W$  is divided into  $M$  disjoint subchannels, each with bandwidth  $\frac{W}{M}$ . Letting  $\mathcal{C}_j$  denote cluster  $j$ , the links in  $\mathcal{C}_j$  operate in subchannel  $j$ . In this work, we assume that  $M$  is a variable parameter in the range of 1 to  $K$ . We also assume the number of links in each cluster,  $n = \frac{K}{M}$ , is a known information for the users. The link between transmitter

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$k$  and receiver  $i$  in  $\mathcal{C}_j$  is represented by the random variable  $\mathcal{L}_{ki}^{(j)}$ . Under a Rayleigh fading channel model,  $\mathcal{L}_{ki}^{(j)} = h_{ii}^{(j)}$ , for  $k = i$ . Also for  $k \neq i$ , the cross-channel gains are defined based on the shadow-fading model as follows<sup>1</sup>

$$\mathcal{L}_{ki}^{(j)} = \begin{cases} h_{ki}^{(j)}, & \text{with probability } \alpha \\ 0, & \text{with probability } 1 - \alpha. \end{cases} \quad (1)$$

We consider a *quasi-static block fading* channel model, where the channel strength  $h_{ki}^{(j)}$  remains constant while transmitting one block and changes independently from block to block. Under a Rayleigh fading channel,  $h_{ki}^{(j)} = |g_{ki}^{(j)}|^2$ 's are exponentially distributed with unit mean. We also assume that the channel is flat fading. In other words, all the channels are assumed to be constant over the whole bandwidth  $W$ . We also assume that each receiver knows only its direct channel gain. This channel-state information (CSI) is fed back to the corresponding transmitter without any error.

In this work, we assume that all the links utilize the on-off power allocation strategy. Based on this scheme, the average transmit power of user  $i$  is assumed to be  $p_i \in \{0, 1\}$ . The power of additive white Gaussian noise (AWGN) at each receiver is assumed to be  $\frac{N_0 W}{M}$ . Since the maximum transmit power is one,  $\frac{N_0 W}{M}$  is equivalent to  $\frac{1}{SNR}$ , where SNR is the signal to noise ratio. Assuming Gaussian signal transmission, the interference term will be Gaussian with power

$$I_i^{(j)} = \sum_{\substack{k \neq i \\ k=1}}^n \mathcal{L}_{ki}^{(j)} p_k, \quad i, k \in \mathcal{C}_j.$$

Due to the orthogonality of the allocated subchannels, no interference is imposed from links in  $\mathcal{C}_k$  on links in  $\mathcal{C}_j$ ,  $k \neq j$ . Under these assumptions, the achievable data rate of each link is expressed as

$$R_i^{(j)} = \frac{W}{M} \log \left( 1 + \frac{h_{ii}^{(j)} p_i}{I_i^{(j)} + \frac{N_0 W}{M}} \right), \quad i \in \mathcal{N}. \quad (2)$$

We consider a homogeneous network in the sense that all the links have the same configurations and use the same protocols. Thus, the transmission strategy for all the nodes are agreed in advance. Next, we summarize the on-off power allocation strategy in a single-hop wireless network proposed in [1] and [6].

**On-Off Power Allocation Strategy:** In each cluster, all users perform the following steps during each block:

1- Based on the direct channel gain, the transmission policy is

$$p_i = \begin{cases} 1, & \text{if } h_{ii}^{(j)} > \tau_n \\ 0, & \text{Otherwise,} \end{cases}$$

for all  $i \in \mathcal{N}$ , where  $\tau_n$  is a prespecified threshold level.

2- After adjusting the powers, each active user in  $\mathcal{C}_j$  transmits a pilot signal with full power. The receivers measure

<sup>1</sup>It is worth to mention that the superscript  $j$  means that the channel gains belong to cluster  $j$ , and it does not mean that the fading model is frequency-selective fading.

the interference and compute the rate using (2). Then, each receiver feedbacks the rate to its corresponding transmitter.

3- The active user transmits data with the computed rate and with full power.

In order to analyze the performance of the network, we define the network throughput as the *average sum-rate*. Letting  $\bar{R}_{sum}$  denote the average sum-rate of the network, we have

$$\bar{R}_{sum} = \sum_{j=1}^M \bar{R}_{sum}^{(j)}, \quad (3)$$

where  $\bar{R}_{sum}^{(j)}$  is the average sum-rate of cluster  $\mathcal{C}_j$  and is given by

$$\begin{aligned} \bar{R}_{sum}^{(j)} &= \mathbb{E} \left[ \sum_{i=1}^n R_i^{(j)} \right] \\ &= \sum_{i=1}^n \mathbb{E} \left[ \frac{W}{M} \log \left( 1 + \frac{h_{ii}^{(j)} p_i}{I_i^{(j)} + \frac{N_0 W}{M}} \right) \right], \end{aligned}$$

where the expectation is computed with respect to  $h_{ii}^{(j)}$  and  $I_i^{(j)}$ . Also, the probability of the link activation in each cluster is defined as  $q_n \triangleq Pr \left\{ h_{ii}^{(j)} > \tau_n \right\}$ .

Following the same approach as in [1] and [6] with  $M = 1$  and  $\alpha = 1$ , we can easily prove the following lemmas.

**Lemma 1:** Let  $M \sim o(K)$  and  $0 < \alpha \leq 1$  is a fixed parameter. Then, for large values of  $K$ , the optimum threshold level that maximizes the average sum-rate of each cluster is obtained as

$$\tau_n^* = \log \alpha n - 2 \log \log \alpha n + O(1). \quad (4)$$

*Proof:* See [13] for the proof. ■

**Lemma 2:** Under the assumptions in Lemma 1, the probability of the link activation in each cluster is given by

$$q_n = c \frac{(\log \alpha n)^2}{\alpha n}, \quad (5)$$

where  $c$  is a constant.

*Proof:* Under a Rayleigh fading channel condition, we have

$$q_n = Pr \left\{ h_{ii}^{(j)} > \tau_n \right\} = e^{-\tau_n}.$$

Using (4), it is concluded

$$q_n = \frac{(\log \alpha n)^2}{\alpha n} \times e^{-O(1)}.$$

Setting  $c = e^{-O(1)}$ , we obtain (5). ■

**Lemma 3:** Under the assumptions in Lemma 1, the average sum-rate of the network is given by

$$\bar{R}_{sum} \approx \frac{W}{\alpha} (1 - \log q_n) \quad (6)$$

$$= \frac{W}{\alpha} (\log K + o(\log K)). \quad (7)$$

*Proof:* See [13] for the proof. ■

Lemma 3 implies that the average sum-rate of the network for  $M \sim o(K)$  depends on the value of  $0 < \alpha \leq 1$  and scales as  $\frac{W}{\alpha} \log K$ .

$$\begin{aligned}
\bar{R}_{sum} &= \sum_{j=1}^M \sum_{i=1}^m \frac{W}{M} \mathbb{E} \left[ \log \left( 1 + \frac{h_{ii}^{(j)}}{\sum_{k \neq i}^m v_k h_{ki}^{(j)} + \frac{N_0 W}{M}} \right) \right], \quad k \in \mathcal{C}_j \\
&= W \sum_{i=1}^m \sum_{l=0}^m \binom{m}{l} \alpha^l (1-\alpha)^{m-l} \mathbb{E} \left[ \log \left( 1 + \frac{h_{ii}^{(j)}}{\Upsilon_l + \frac{N_0 W}{M}} \right) \right] \\
&= W \sum_{i=1}^m (1-\alpha)^m \mathbb{E} \left[ \log \left( 1 + \frac{h_{ii}^{(j)}}{\frac{N_0 W}{M}} \right) \right] + W \sum_{i=1}^m \sum_{l=1}^m \binom{m}{l} \alpha^l (1-\alpha)^{m-l} \mathbb{E} \left[ \log \left( 1 + \frac{h_{ii}^{(j)}}{\Upsilon_l + \frac{N_0 W}{M}} \right) \right],
\end{aligned} \tag{15}$$

### III. NETWORK ANALYSIS WITH ON-OFF POWER ALLOCATION STRATEGY

In this section, we analyze the average sum-rate in terms of  $M$ ,  $1 \leq M \leq K$ . We prove that the maximum average sum-rate of the network for every value of  $0 < \alpha \leq 1$  is obtained at  $M = 1$ .

*Theorem 1:* Assuming  $0 < \alpha \leq 1$  is fixed, the maximum average sum-rate of the network is achieved at  $M = 1$ .

*Proof:* We prove the theorem in the following two cases:

Case 1:  $M \sim o(K)$

From (6), the average sum-rate of the network for  $M \sim o(K)$  is obtained as

$$\bar{R}_{sum} \approx \frac{W}{\alpha} (1 - \log q_n). \tag{8}$$

Taking the first-order derivative of (8) with respect to  $M$  yields,

$$\frac{\partial \bar{R}_{sum}}{\partial M} = -\frac{W}{\alpha} \frac{\partial q_n}{\partial M} \frac{1}{q_n}.$$

Using (5) and noting that  $n = \frac{K}{M}$ , we have

$$q_n = \frac{c}{\alpha K} M \left( \log \frac{\alpha K}{M} \right)^2. \tag{9}$$

Since,

$$\frac{\partial q_n}{\partial M} = \frac{c}{\alpha K} \log \frac{\alpha K}{M} \times \left( \log \frac{\alpha K}{M} - 2 \right) > 0,$$

it is concluded that (8) is a monotonically decreasing function of  $M$ . Thus for  $M \sim o(K)$ , the maximum average sum-rate of the network is obtained at  $M = 1$ .

Case 2:  $M \sim \Theta(K)$

Letting  $m$  denote the number of active links in each cluster, it is concluded that  $m \in [1, n]$  does not grow with  $K$ . The average sum-rate of the network is given by

$$\bar{R}_{sum} = \sum_{j=1}^M \sum_{i=1}^m \frac{W}{M} \mathbb{E} \left[ \log \left( 1 + \frac{h_{ii}^{(j)}}{I_i^{(j)} + \frac{N_0 W}{M}} \right) \right]. \tag{10}$$

For simplicity of derivations, we assume that the number of active links in all the clusters is the same denoted by  $m$ . It can be shown that this assumption does not affect the validity of the results [13]. We first evaluate the average sum-rate for

$m = 1$ . In this case, we have  $I_i^{(j)} = 0$ . Thus, the maximum achievable throughput is obtained as

$$\bar{R}_{sum} = W \mathbb{E} \left[ \log \left( 1 + \frac{M}{N_0 W} h_{max}^{(j)} \right) \right], \tag{11}$$

where  $h_{max}^{(j)} = \max_{i=1, \dots, n} h_{ii}^{(j)}$  is a random variable. Since  $\log x$  is a concave function of  $x$ , an upper bound of (11) is obtained through *Jensen's inequality*,  $\mathbb{E}[\log x] \leq \log(\mathbb{E}[x])$ ,  $x > 0$ . Thus,

$$\bar{R}_{sum} \leq W \log \left( 1 + \frac{M}{N_0 W} \mathbb{E}[Y] \right), \tag{12}$$

where  $Y \triangleq h_{max}^{(j)}$ . Noting that  $h_{ii}^{(j)}$ 's are i.i.d. over  $i \in \mathcal{N}$ , we have

$$\begin{aligned}
F_Y(y) &= Pr\{Y \leq y\}, \quad y > 0 \\
&= \prod_{i=1}^n Pr\{h_{ii}^{(j)} \leq y\} \\
&= (1 - e^{-y})^n,
\end{aligned}$$

where  $F_Y(\cdot)$  is the cumulative distribution function (cdf) of  $Y$ . Hence,

$$\mathbb{E}[Y] = \int_0^\infty n y e^{-y} (1 - e^{-y})^{n-1} dy.$$

Since  $(1 - e^{-y})^{n-1} \leq 1$ , we have

$$\mathbb{E}[Y] \leq \int_0^\infty n y e^{-y} dy = n. \tag{13}$$

Consequently, the upper bound of  $\bar{R}_{sum}$  obtained in (12) can be simplified as

$$\bar{R}_{sum} \leq W \log \left( 1 + \frac{K}{N_0 W} \right). \tag{14}$$

Hence, the throughput of the network for  $m = 1$  scales at most as  $W \log K$  which is less than or equal to  $\frac{W}{\alpha} \log K$  for  $M = 1$ .

For  $m > 1$ , the average sum-rate can be written as (15), where  $v_k$ 's are binomial random variables with parameters  $(m, \alpha)$ , and  $\Upsilon_l$  is the sum of  $l$  i.i.d random variables with  $\chi^2(2)$  distribution. Noting that  $M \sim \Theta(K)$ , we ignore the term  $\frac{N_0 W}{M}$ . Also for  $m > 1$ ,  $\Upsilon_l$  is greater than the interference term caused by one interfering link. Thus, an upper bound for the throughput of the network is given by (16), where

$$\bar{R}_{sum} \leq Wm(1-\alpha)^m \mathbb{E} \left[ \log \left( 1 + \frac{Y}{\frac{N_0 W}{M}} \right) \right] + W \sum_{i=1}^m \sum_{l=1}^m \binom{m}{l} \alpha^l (1-\alpha)^{m-l} \mathbb{E} \left[ \log \left( 1 + \frac{Y}{Z} \right) \right] \quad (16)$$

$$= Wm(1-\alpha)^m \mathbb{E} \left[ \log \left( 1 + \frac{Y}{\frac{N_0 W}{M}} \right) \right] + Wm(1-(1-\alpha)^m) \mathbb{E} \left[ \log \left( 1 + \frac{Y}{Z} \right) \right]. \quad (17)$$

$Y \triangleq h_{max}^{(j)} = \max_{i=1, \dots, n} h_{ii}^{(j)}$  and  $Z \triangleq h_{ki}^{(j)}$ . Letting  $X = \frac{Y}{Z}$ , the cdf of  $X$  is obtained as

$$\begin{aligned} F_X(x) &= \Pr\{X \leq x\}, \quad x > 0 \\ &= \Pr\{Y \leq Zx\} \\ &= \int_0^\infty \Pr\{Y \leq Zx|Z\} f_Z(z) dz \\ &= \int_0^\infty (1 - e^{-zx})^n e^{-z} dz. \end{aligned}$$

Thus, the probability distribution function (pdf) of  $X$  can be written as

$$\begin{aligned} f_X(x) &= \frac{dF_X(x)}{dx} \\ &= \int_0^\infty nze^{-z(1+x)} (1 - e^{-zx})^{n-1} dz \\ &\leq \int_0^\infty nze^{-z(1+x)} dz = \frac{n}{(1+x)^2}. \end{aligned} \quad (18)$$

Using (11)-(14) and (18), the inequality (17) is simplified as

$$\begin{aligned} \bar{R}_{sum} &\leq Wm(1-\alpha)^m \log \left( 1 + \frac{K}{N_0 W} \right) \\ &+ Wm(1-(1-\alpha)^m) \int_0^\infty \log(1+x) f_X(x) dx \\ &\leq Wm(1-\alpha)^m \log \left( 1 + \frac{K}{N_0 W} \right) \\ &+ Wnm(1-(1-\alpha)^m) \int_0^\infty \frac{\log(1+x)}{(1+x)^2} dx \\ &= Wm(1-\alpha)^m \log \left( 1 + \frac{K}{N_0 W} \right) + \Lambda_n, \end{aligned} \quad (19)$$

where  $\Lambda_n = Wnm(1-(1-\alpha)^m)$ . In order to show that the achievable average sum-rate obtained in (19) is less than that of  $M=1$ , it is sufficient to prove  $m(1-\alpha)^m < \frac{1}{\alpha}$ . Letting  $f(\alpha) = \alpha m(1-\alpha)^m$ , we have

$$\frac{\partial f(\alpha)}{\partial \alpha} = m(1-\alpha)^{m-1}(1-\alpha-\alpha m) = 0.$$

Thus, the extremum points of  $f(\alpha)$  are located at  $\alpha = 1$  and  $\alpha = \frac{1}{m+1}$ . It is seen that

$$f(1) = 0 < 1,$$

and

$$f\left(\frac{1}{m+1}\right) = \left(\frac{m}{m+1}\right)^{m+1} < 1.$$

Therefore, since  $\Lambda_n$  does not grows with  $K$  and also  $f(\alpha) < 1$ , we can conclude that the throughput of the network is less than that of  $M=1$ . ■

*Corollary 1:* Note that although in the proof of Theorem 1, it is assumed that  $\alpha$  is fixed, in the case of  $M \sim \Theta(K)$ , following the same proof steps, it can be shown that for  $\alpha \rightarrow 0$  the throughput of the network is less than that of  $M=1$ .

*Proof:* See [13] for the proof. ■

In the case of  $M \sim \Theta(K)$ , which includes  $M=K$ , we obtained an upper bound for  $\bar{R}_{sum}$ . In the next corollary, we derive the exact achievable throughput of the network for  $M=K$ .

*Corollary 2:* Assuming  $M=K$ , the average sum-rate of the network is obtained by

$$\bar{R}_{sum} \approx W(\log K - \log N_0 W - \gamma),$$

where  $\gamma$  is Euler's constant.

*Proof:* Since for  $M=K$ , the users transmit with full power over the orthogonal subchannels, it is concluded that  $I_i^{(j)} = 0$  for  $i=1, \dots, K$ . Thus, the average sum-rate of the network is given by

$$\begin{aligned} \bar{R}_{sum} &= \mathbb{E} \left[ \sum_{i=1}^K R_i^{(j)} \right] \\ &= \frac{W}{K} \sum_{i=1}^K \mathbb{E} \left[ \log \left( 1 + \frac{h_{ii}^{(j)}}{\frac{N_0 W}{K}} \right) \right], \end{aligned}$$

where the expectation is computed with respect to  $h_{ii}^{(j)}$ . Under a Rayleigh fading channel model, we have

$$\bar{R}_{sum} = \frac{W}{K} \sum_{i=1}^K \int_0^\infty e^{-x} \log(1+\lambda x) dx,$$

where  $\lambda \triangleq \frac{K}{N_0 W}$ . Thus,

$$\begin{aligned} \bar{R}_{sum} &= W \int_0^\infty e^{-x} \log(1+\lambda x) dx \\ &= We^{\frac{1}{\lambda}} \int_{1/\lambda}^\infty \frac{e^{-x}}{x} dx \\ &= We^{\frac{1}{\lambda}} E_1 \left( \frac{1}{\lambda} \right), \end{aligned} \quad (20)$$

where  $E_1(x)$  is obtained by the exponential-integral function defined as [14]

$$E_n(x) \triangleq \int_1^\infty \frac{e^{-tx}}{t^n} dt.$$

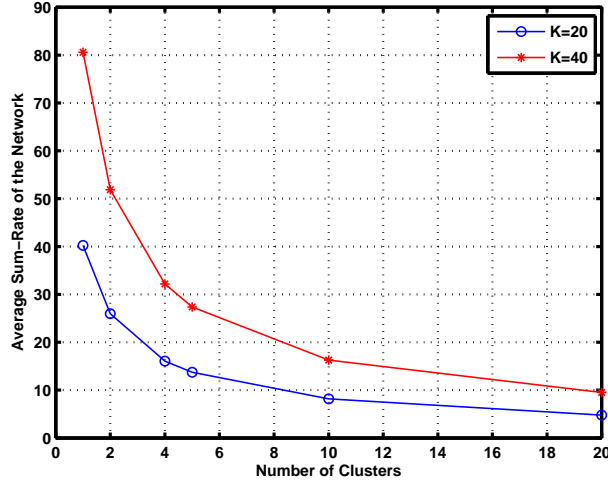


Fig. 1. Average sum-rate of the network vs.  $M$  for  $K = 20$  and  $K = 40$ .

To simplify (20), we use the following series representation for  $E_1(x)$ ,

$$E_1(x) = -\gamma - \log x + \sum_{s=1}^{\infty} \frac{(-1)^{s+1} x^s}{s \cdot s!}, \quad (21)$$

where  $\gamma$  is Euler's constant and is defined by the limit [14]

$$\gamma = \lim_{s \rightarrow \infty} \left( \sum_{k=1}^s \frac{1}{k} - \log s \right) = 0.577215665\dots$$

Thus, the average sum-rate of the network is obtained as

$$\bar{R}_{sum} = W e^{\frac{1}{\lambda}} \left( -\gamma + \log \lambda + \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{s \cdot s!} \left( \frac{1}{\lambda} \right)^s \right).$$

For sufficiently large values of  $K$ , we have  $\lambda = \frac{K}{N_0 W} \gg 1$ , which results in  $e^{\frac{1}{\lambda}} \approx 1$  and

$$\sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{s \cdot s!} \left( \frac{1}{\lambda} \right)^s \approx 0.$$

Consequently for  $M = K$ , the average sum-rate of the network is asymptotically obtained by

$$\bar{R}_{sum} \approx W (\log K - \log N_0 W - \gamma). \quad (22)$$

**Corollary 3:** For  $M = K$ , the average sum-rate of the network is of order  $\log K$ . Through comparing (22) with (7), it is concluded that the throughput of the network with  $M = K$  is less than or equal to that of  $M = 1$ .

We finally evaluate the throughput of the network versus the number of clusters for finite values of  $K$  through simulation results. Fig. 1 illustrates the maximum average sum-rate of the network versus  $M$  for  $K = 20$ ,  $K = 40$  and  $\alpha = 0.1$ . It is seen that the average sum-rate is a decreasing function of  $M$ . Also, the maximum value of  $\bar{R}_{sum}$  is achieved at  $M = 1$ .

#### IV. CONCLUSION

We have analyzed the average sum-rate of a single-hop wireless network in terms of  $M$  and under the shadowing effect with probability  $\alpha$ . It has been demonstrated that for  $M \sim o(K)$  and  $0 < \alpha \leq 1$ , the average sum-rate of the network is of order  $\frac{W}{\alpha} \log K$ . Also, it has been proved that the maximum average sum-rate of the network for every value of  $0 < \alpha \leq 1$  and  $1 \leq M \leq K$  is achieved at  $M = 1$ . In fact, in the proposed model, partitioning the bandwidth  $W$  into  $M$  subchannels has no gain in terms of enhancing the throughput.

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