An Efficient User Removal Algorithm for Limited-Power Interference Channels

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***Abstract*— In this paper, the problem of maximizing the number of active users satisfying a required quality of service (QoS) in n-user interference channels is investigated. This problem is known as an NP-complete problem. We introduce an efficient suboptimal algorithm, relying on the results for the boundary of the rate region we derived in [1]. The algorithm is developed for different sorts of constraints on the transmit powers, including constraint on the power of the individual transmitters and constraint on the total power of the transmitters. Simulation results show that the performance of the proposed algorithm is very close to the optimum solution, and outperforms alternative algorithms.**

I. Introduction

Sharing the same wireless channel can greatly increase the spectral efficiency of wireless systems. While such a scheme increases the capacity and the coverage area of communication systems, it suffers from the interference of the concurrent links over each other, known as the co-channel interference. Consequently, the signal-to-interference-plus-noise-ratio (SINR) of the links are upper-bounded, even if the transmit powers are unbounded.

There have been some efforts to evaluate the maximum achievable SINR in the interference channels. In [2], the maximum achievable SINR in a satellite network with no power constraint is presented in terms of the Perron-Frobenius (PF) eigenvalue of a non-negative matrix. This result was deployed in many other applications by [3]–[6] afterwards.

Recently, the authors have extended this result to the case that the power of the transmitters are subject to some constraints, including the constraints on power of the individual transmitters, and the constraint on the total transmit power [1].

In practical scenarios, it is desired that the active users satisfy a required quality of service (QoS) . On the other hand, due to the deteriorative effect of the co-channel interference, it is not possible for all users to satisfy such a requirement. Therefore, some of the users should be removed to the advantage of the others. Finding a feasible subset of users (i.e., a subset of users which satisfy the required QoS) with maximum cardinality is claimed to be an NP-complete problem [7]. In the literature, some heuristic algorithms are presented for this problem. In [4], a stepwise removal algorithm (SRA) has been proposed for the case that the transmit power is unbounded. In [8], another algorithm named as stepwise-maximum-interference-removal-algorithm (SMIRA) is proposed, and it is shown that this algorithm outperforms SRA. For the systems with constraint on the power of the individual transmitters, an algorithm known as gradually-removal-distributed-constrained-power-control (GRX-DCPC) is proposed in [7]. This algorithm is presented in the different forms of centralized, distributed, restricted, and non-restricted user selection. The simulation results show that GRN-DCPC (centralized non-restricted algorithm) outperforms other mentioned schemes in [7].

In this paper, we exploit the relationship between the maximum achievable SINR and the PF-eigenvalue of some non-negative matrices, presented in [1], to develop an algorithm for the problem of user removal. The algorithm is proposed for different sorts of power constraints. Simulation results show that the proposed algorithm outperforms the alternative schemes in all cases in terms of the number of active users.

Notation: All boldface letters indicate column vectors (lower case) or matrices (upper case). x_{ij} represents the entry (i, j) of matrix $\mathbf{X}$. $\mathbf{x}_i$ denotes the column i of matrix $\mathbf{X}$, i.e., $\mathbf{X}_{n\times m} = [\mathbf{x}_i]_{n\times m}$. A matrix $\mathbf{X}$ is called *non-negative* if $x_{ij} \geq 0$, for all i and j [9]. $\det(\mathbf{X})$, $\mathrm{Tr}(\mathbf{X})$, and $\mathbf{X}'$ denote the determinant, the trace, and the transpose of the matrix $\mathbf{X}$, respectively. $\psi(\mathbf{X}, \mathbf{y}, \mathcal{S})$ is a matrix defined as a function of three parameters, which are respectively a matrix, a vector and a set of indices. It is defined as

$$\psi(\mathbf{X}, \mathbf{y}, \mathcal{S}) = \mathbf{Z} = [\mathbf{z}_j], \quad \mathbf{z}_j = \begin{cases} \mathbf{x}_j + \mathbf{y} & j \in \mathcal{S} \\ \mathbf{x}_j & \text{otherwise} \end{cases}$$

In addition, $\mathbf{X}^{i^-}$ is the matrix $\mathbf{X}$ whose i^{th} column and row is removed. We use a similar notation for a vector whose i^{th} element is removed.

II. System Model and Previous Results

The Gaussian interference channel with n links (users), is represented by the gain matrix $\mathbf{G} = [g_{ij}]_{n\times n}$ where g_{ij} is the power coefficient from transmitter j to receiver i. This coefficient can be the result of fading, shadowing, or the processing gain of the CDMA system. A white Gaussian noise with zero mean and variance σ_i^2 is added to the received signal at the receiver i terminal. The SINR of each user, denoted by

[1]This work is financially supported by Nortel and by matching funds from the federal government of Canada (NSERC) and province of Ontario (OCE).

γ_i, is obtained by

$$\gamma_i = \frac{g_{ii} p_i}{\sigma_i^2 + \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{ij} p_j}, \quad \forall i \in \{1, \ldots, n\},$$

where p_i is the power of transmitter i. The users are required to attain a minimum SINR denoted by γ i.e., $\gamma_i \geq \gamma$. In the sequel, we obtain the maximum possible γ for which this condition is satisfied for all users. We define the normalized gain matrix, $\mathbf{A}$, as

$$\mathbf{A} = [a_{ij}]_{n\times n}, \; a_{ij} = \begin{cases} \frac{g_{ij}}{g_{ii}} & i \neq j \\ 0 & i = j \end{cases} \tag{1}$$

Based on this definition, the QoS constraint is presented as,

$$\frac{p_i}{\eta_i + \sum_{j=1}^{n} a_{ij} p_j} \geq \gamma, \quad \forall i \in \{1, \ldots, n\}, \tag{2}$$

where $\eta_i = \frac{\sigma_i^2}{g_{ii}}$. By defining $\boldsymbol{\eta} = [\eta_i]_{n\times 1}$, the inequalities in (2) can be reformulated in a matrix form as

$$(\frac{1}{\gamma}\mathbf{I} - \mathbf{A})\mathbf{p} \geq \boldsymbol{\eta}. \tag{3}$$

When there is no constraint on the power vector (rather than the trivial constraint of $\mathbf{p} \geq \mathbf{0}$), the maximum achievable γ in (3), denoted by γ^*, is characterized as

$$\gamma^* = \frac{1}{\lambda^*(\mathbf{A})}, \tag{4}$$

where $\lambda^*(\mathbf{A})$ is the PF-eigenvalue of $\mathbf{A}$. Based on the Perron-Frobenius theorem, any irreducible matrix has an eigenvalue which is real and positive and takes the largest norm among all the eigenvalues. This eigenvalue is called Perron-Frobenius eigenvalue (PF-eigenvalue) of the matrix. For more details see [9, Chapters 1 and 2].

For the case that the total power of a subset of users is constrained, the authors showed that the maximum achievable SINR for a system is obtained according to the following theorem [1].

Theorem 1 *The maximum achievable γ in an interference channel with n links and normalized gain matrix $\mathbf{A}$, with power constraints $\mathbf{p} \geq \mathbf{0}$ and $\sum_{i\in\Omega} p_i \leq \overline{p}_\Omega$ is equal to*

$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_\Omega}, \Omega)\big)},$$

where $\Omega \subseteq \{1, 2, \ldots, n\}$ is an arbitrary subset of the users.

As a result of this theorem, when the total power of all users is constrained as $\sum_{i=1}^{n} p_i \leq \overline{p}_t$, the maximum achievable SINR is

$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, \ldots, n\})\big)}. \tag{5}$$

According to [1], Theorem 1 can be used to show that if $p_i \leq \overline{p}_i, \quad \forall i \in \{1, \ldots, n\}$, the maximum achievable SINR is

$$\gamma^* = \min_i \{\frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_i}, \{i\})\big)}\}. \tag{6}$$

In a congested system, all the users can not satisfy the QoS requirement. Therefore, some of the users should be removed in order to reduce effective interference on the active users and consequently ameliorate the achievable SINR. In what follows, we use (4), (5), and (6) to develop a suboptimum algorithm for obtaining a subset of the users with maximum cardinality satisfying the rate requirement (2).

III. Removal Algorithm

To find the optimum set of active users, satisfying the QoS requirement, we have to examine all the combinations of the users and select a feasible subset with the maximum cardinality. Clearly, this scheme is computationally exponential. As a suboptimum alternative scheme, we propose a greedy removal algorithm. The main idea behind the presented algorithm is as follows. At each step, if the active users do not satisfy the required SINR, one user is removed. This user is the one which provides the highest increase in the maximum achievable SINR if it is removed. We call this user the *worst user*. The proposed algorithm is presented for different sorts of power constraints.

According to (4) and Theorem 1, in general, the maximum γ is equal to the inverse of the PF-eigenvalue of a matrix $\mathbf{X}$, i.e., $\gamma^* = \frac{1}{\lambda^*(\mathbf{X})}$. In a system with a large number of users, computing the PF-eigenvalue is computationally extensive. In this case, it is beneficial to use an approximation of the PF-eigenvalue as follows. When a matrix is raised to a power, its eigenvalues are raised to the same power as well [10], i.e., $\lambda(\mathbf{X}^q) = \lambda^q(\mathbf{X})$. On the other hand, the trace of a matrix is equal to the summation of the eigenvalues of that matrix [10]; therefore, $Tr(\mathbf{X}^q) = \sum_i \lambda_i^q$. Since the PF-eigenvalue of an irreducible matrix has the largest norm among all the eigenvalues of that matrix [10], we can approximate $\lambda^{*q}(\mathbf{X})$ with the $Tr(\mathbf{X}^q)$, i.e., $\lambda^{*q}(\mathbf{X}) \approx Tr(\mathbf{X}^q)$. This approximation is stronger if the power q is larger. However, the simulation results show that $q = 2$ yields a very good approximation of the exact value in our problem. Therefore, we use

$$\gamma^* \approx \big(Tr(\mathbf{X}^2)\big)^{-\frac{1}{2}} \tag{7}$$

as an approximate value for γ^*. In what follows, we investigate the problem of user removal for various power constraint scenarios and give an efficient algorithm for each case.

Case One: No Power Constraint

Based on the previous discussions on the worst link determination and according to (4), when there is no power constraint, the index of the user to be removed, $\hat{i}$, is obtained as $\hat{i} = \arg\max_i \{\frac{1}{\lambda^*(\mathbf{A}^{i^-})}\}$. If this link is removed and still the maximum achievable SINR computed through (4) does not meet the required SINR, additional links are removed in a recursive manner till the remaining users become feasible. This algorithm is called the *Removal Algorithm I-A* throughout this paper.

To avoid the complexity of computing PF-eigenvalues in each iteration, we present the following algorithm which is an approximate version of algorithm I-A. According to (4) and (7), for the unconstrained power scenario, we have

$$\gamma^* = \frac{1}{\lambda^*(\mathbf{A})} \approx \frac{1}{\sqrt{Tr(\mathbf{A}^2)}} = \Big(\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}a_{ji}\Big)^{-\frac{1}{2}}. \qquad (8)$$

We define the vector $\mathbf{w}$ as $\mathbf{w} = [w_i]_{n\times 1}$, $w_i = \sum_{j=1}^{n} a_{ij}a_{ji}$. Then we have $\gamma^* \approx (\sum_{i=1}^{n} w_i)^{-\frac{1}{2}}$. It is easy to show that by removing user i, $2w_i$ is subtracted from the trace of $\mathbf{A}^2$. An immediate conclusion is that if we want to remove one link to obtain the largest increase in the maximum achievable SINR, the best choice (worst link) is to remove the one with the largest w_i. Therefore, $\hat{i} = \arg\max_i w_i$. Based on this result, an efficient algorithm for gradually removing the users is presented as follows.

Removal Algorithm I-B

1) Set $\mathbf{A}$ as in (1), $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable SINR as $\gamma^* = \dfrac{1}{\lambda^*(\mathbf{A})}$.
3) If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.
4) Update the vector $\mathbf{w}_{m\times 1}$ as $w_i = \sum_{j=1}^{m} a_{ij}a_{ji}$.
5) Determine the worst link as $\hat{i} = \arg\max_i w_i$.
6) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}^-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}^-}$, $m \leftarrow m-1$, and go to step 2.

Case Two: Constraints on the Power of Individual Transmitters

Assume there is a constraint on the power of individual transmitters. Based on (6), we design an efficient suboptimal algorithm to find the maximum cardinality subset of the users satisfying a minimum SINR requirement. We define the matrix $\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})$ as the matrix $\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})$ whose i^{th} column and row are removed. Therefore, the worst link is

$$\hat{i} = \arg\max_i \min_{\substack{j\\ j\neq i}} \frac{1}{\lambda^*\big(\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)}. \qquad (9)$$

The users are removed one by one based on (9) until all of the active users satisfy the rate requirement. We call this algorithm the *Removal Algorithm II-A*.

To reduce the complexity of this algorithm, we use the following approximation scheme. According to (6) and (7), we have $\gamma^* \approx \min_j \Big(Tr\big(\psi^2(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)\Big)^{-\frac{1}{2}}$, which can be rewritten as $\gamma^* \approx \min_j \Big(\big(\frac{\eta_j}{\overline{p}_j}\big)^2 + \sum_{k=1}^{n}\sum_{l=1}^{n} a_{kl}a_{lk} + 2\sum_{k=1}^{n} \frac{\eta_k}{\overline{p}_k} a_{jk}\Big)^{-\frac{1}{2}}$. We define the matrix $\mathbf{W}$ as $\mathbf{W} = [w_{ij}]_{n\times n}$,

$$w_{ij} = \begin{cases} \big(\frac{\eta_j}{\overline{p}_j}\big)^2 + \sum\limits_{\substack{k=1\\k\neq i}}^{n}\sum\limits_{\substack{l=1\\l\neq i}}^{n} a_{kl}a_{lk} + 2\sum\limits_{\substack{k=1\\k\neq i}}^{n} \frac{\eta_k}{\overline{p}_k} a_{jk} & j \neq i \\ 0 & j = i \end{cases}$$

According to [11], equation (9) can be simplified to $\hat{i} = \arg\min_i \max_j w_{ij}$. Based on this result, the following algorithm is developed.

Removal Algorithm II-B

1) Set $\mathbf{A}$ as in (1), $\overline{\mathbf{p}} = [\overline{p}_i]$, $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable SINR as
$$\gamma^* = \min_j \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_j}, \{j\})\big)}.$$
3) If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.
4) Update $\mathbf{W}_{m\times m}$ as
$$w_{ij} = \begin{cases} \big(\frac{\eta_j}{\overline{p}_j}\big)^2 + \sum\limits_{\substack{k=1\\k\neq i}}^{m}\sum\limits_{\substack{l=1\\l\neq i}}^{m} a_{kl}a_{lk} + 2\sum\limits_{\substack{k=1\\k\neq i}}^{m} \frac{\eta_k}{\overline{p}_k} a_{jk} & j \neq i \\ 0 & j = i \end{cases}.$$
5) Determine the worst link as $\hat{i} = \arg\min_i \max_j w_{ij}$.
6) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}^-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}^-}$, $\overline{\mathbf{p}} \leftarrow \overline{\mathbf{p}}^{\hat{i}^-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}^-}$, and $m \leftarrow m-1$, and go to step 2.

Case Three: Total Transmit Power Constraint

When the total power is constrained by $\overline{p}_t$, the maximum achievable SINR is computed through (5). In this case, the worst user is determined as

$$\hat{i} = \arg\max_i \Big\{\frac{1}{\lambda^*\big(\psi^{i^-}(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, 2, \ldots, n\})\big)}\Big\}. \qquad (10)$$

We call this algorithm the *Removal Algorithm III-A*. Similar to the previous discussions, we propose the following low-complexity algorithm for the user removal with total power constraint (see [11] for details) .

Removal Algorithm III-B

1) Set $\mathbf{A}$ as in (1), $m = n$, $\mathcal{R} = \varnothing$, and $\mathbf{v} = [1, 2, \ldots, n]'$.
2) Find the maximum achievable SINR as
$$\gamma^* = \frac{1}{\lambda^*\big(\psi(\mathbf{A}, \frac{\boldsymbol{\eta}}{\overline{p}_t}, \{1, \ldots, m\})\big)}.$$
3) If $\gamma^* \geq \gamma_{th}$, $\mathbf{v}$ is the set of active users, stop.
4) Update the vector $\mathbf{w}_{m\times 1}$ as $w_i = (\frac{\eta_i}{\overline{p}_t})^2 + 2\sum_{j=1}^{m} a_{ij}a_{ji} +$
$$2\frac{\eta_i}{\overline{p}_t}\sum_{j=1}^{m} a_{ji} + 2\sum_{j=1}^{m} \frac{\eta_j}{\overline{p}_t} a_{ij} + 2\frac{\eta_i}{\overline{p}_t}\sum_{\substack{j=1\\j\neq i}}^{m} \frac{\eta_j}{\overline{p}_t}.$$

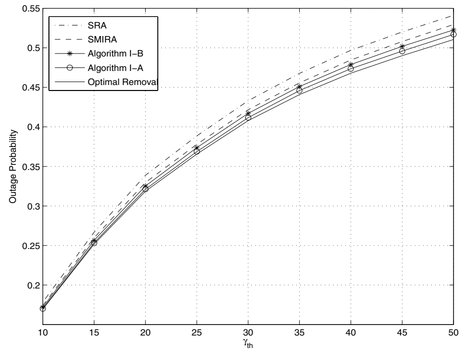

Fig. 1. No Power Constraints, ${\sigma_i}^2 = 10^{-16}\ \forall i$

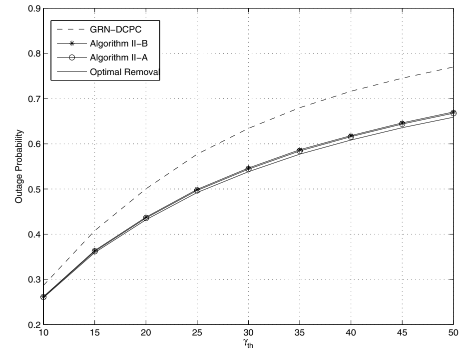

Fig. 2. Constraints on the Power of Individual Transmitters, ${\sigma_i}^2 = 10^{-2}$, $\overline{p}_i = 1\text{w}\ \forall i$

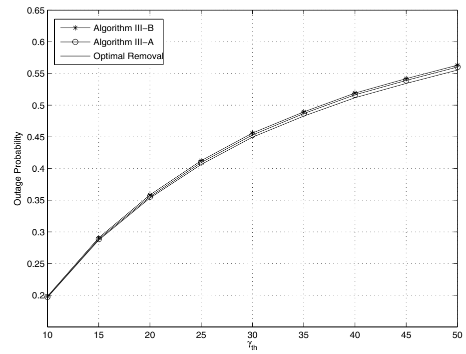

Fig. 3. Constraint on the Total Power, ${\sigma_i}^2 = 10^{-3}\ \forall i$, $\overline{p}_t = 1\text{w}$

5) Determine the worst link as $\hat{i} = \arg\max_i w_i$.
6) Set $\mathcal{R} \leftarrow \mathcal{R} \cup \{v_{\hat{i}}\}$, $\mathbf{A} \leftarrow \mathbf{A}^{\hat{i}^-}$, $\mathbf{v} \leftarrow \mathbf{v}^{\hat{i}^-}$, $\boldsymbol{\eta} \leftarrow \boldsymbol{\eta}^{\hat{i}^-}$, $m \leftarrow m-1$, and go to step 2.

IV. Numerical Results

The simulation results are presented for a Rayleigh fading channel with $n = 8$. For the results in a cellular network see [11]. The parameters g_{ij} follow an exponential distribution with mean and variance one for the forward gains, and mean 10^{-2} and variance 10^{-4} for the cross gains.

We define *Outage Probability* as the ratio between the number of the inactive users to the total number of the users. This probability shows the percentage of the users that fail to attain the required QoS. We use this function as a metric to compare different algorithms, as it is used in [4], [5].

For the case that there is no constraint on the users' power, the curves of the outage probability for different user removal algorithms are depicted in Fig. 1. Since in SMIRA and SRA algorithms the noise power is considered zero, we assigned a very small value to the noise power to be able to compare all algorithms. As shown in Fig. 1, algorithms I-A and I-B outperform SMIRA and SRA algorithm. Another observation is that the performance of algorithm I-B is very close to that of algorithm I-A, while it enjoys much less computational complexity. In [7], a number of removal algorithms when the power of transmitters are individually constrained are proposed. We selected centralized GRN-DCPC to compare it with our results since according to [7], it outperforms the other presented algorithms in that work. The simulation results in Fig. 2 show a significant improvement in the outage probability of the algorithms II-A and II-B compared to GRN-DCPC. As depicted in Fig. 3, when the total power is bounded, the performance of algorithms III-A and III-B is very close to the optimal result. To the best of our knowledge, there is no alternative algorithms for the case that the total power is upper-bounded.

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