Decomposition of the MIMO X Channels

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***Abstract*— In a multiple antenna system with two transmitters and two receivers, a scenario of data communication, known as the X channel, is studied in which each receiver receives data from both transmitters. In this scenario, it is assumed that each transmitter is unaware of the other transmitter's data (non-cooperative scenario). This system can be considered as a combination of two broadcast channels (from the transmitters' point of view) and two multi-access channels (from the receivers' point of view). Taking advantage of both perspectives, two signaling schemes for such a scenario are developed. In these schemes, some linear filters are employed at the transmitters and at the receivers which decompose the system into either two non-interfering multi-antenna broadcast sub-channels or two non-interfering multi-antenna multi-access sub-channels. In addition, these filters are designed such that the maximum multiplexing gain (MG) of the system is attained by exploiting the null spaces of the direct and cross channels. It is shown that the proposed scenario outperforms other known non-cooperative schemes in terms of the achievable MG. In particular, it is shown that for some specific cases, the achieved MG is the same as the MG of the system if the full cooperation is provided either between the transmitters or between the receivers.**

I. Introduction

It is well-known that using multiple antennas at both sides of a wireless channel results in a multiplicative increase in the overall throughput. This multiplicative increase in the rate is measured by a metric known as the *multiplexing gain (MG)*, ρ, defined as the ratio of the sum-rate of the system, R, over the logarithm of the total power P_T in the high power regime, i.e.

$$\rho = \lim_{P_T \to \infty} \frac{R}{\log_2(P_T)}. \tag{1}$$

In a point to point multiple-antenna system, with m transmit and n receive antennas, the achievable MG is $\min(m, n)$. In multi-antenna multi-user systems, when the full cooperation is provided at least at one side of the links (either among the transmitters or among the receivers), the system still enjoys a multiplicative increase in the throughput with the smaller value of the following two quantities: (i) The total number of transmit antennas, and (ii) The total number of receive antennas [1]. However, for the case that cooperation is not available, the performance of the system will be deteriorated due to the interference of the links over each other. For example, in a multiple-antenna interference channel with two transmitters and two receivers, each equipped with n antennas, the MG of the system is n [1].

Extensive research efforts have been devoted to the multiple-antenna interference channels. In [2], the capacity region of the multiple-input single-output (MISO) interference channels with strong interference and the capacity region of the single-input multiple-output (SIMO) interference channels with very strong interference are characterized. In [3], the superposition coding technique is utilized to derive an inner-bound for the capacity of the multiple-input multiple-output (MIMO) interference channels. In [1], the MG of the MIMO interference channels with general configuration for the number of transmit and receive antennas is derived. In [4], the performance of the single-antenna interference channels is evaluated, where the transmitters or receivers rely on the same channel, used for transmission, to provide cooperation. It is shown that the resulting MG is still one, i.e., this type of cooperation is not helpful in terms of MG. In [1], a cooperation scheme in the shared communication medium for the MIMO interference channels is proposed and shown that such scheme does not increase the MG.

In [5], we proposed a new signaling scheme for multiple-antenna systems with two transmitters and two receivers. In this scheme, each receiver receives data from both transmitters. It is assumed that neither the transmitters nor the receivers cooperate in signaling. In other words, each transmitter is unaware of the data of the other transmitter. Similarly, each receiver is unaware of the signal received by the other receiver. This signaling scenario has several applications. For example, (i) in a wireless system where two relay nodes are utilized to extend the coverage area or (ii) in a system where two base stations provide different services to the users. In [5], we proposed a signaling scheme over such channels and showed that such a scheme outperforms the interference channels in terms of the MG of the system. In [6], we extended the scheme proposed in [5] to more general configurations for the number of transmit and receive antennas, and developed two signaling schemes based on: (i) linear operations at the receivers and the dirty paper coding at the transmitters, and (ii) linear operations at the transmitters and the successive decoding at the receivers. In [7], the idea of overlapping the interference terms proposed in [6] has been

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adopted to show that the zero-forcing scheme can achieve the MG of the X channels for some special configurations of the number of transmit and receive antennas.

In this paper, it is shown that by using some linear filters at the transmitters and the receivers, the system is decomposed to either two non-interfering multi-antenna broadcast sub-channels or two non-interfering multi-antenna multi-access sub-channels. In this scheme, the null spaces of the direct and cross links are exploited to attain the maximum MG. The scheme is general and is applicable for all the configurations of the number of transmit and receive antennas.

Notation: All boldface letters indicate vectors (lower case) or matrices (upper case). $(.)^\dagger$ denotes transpose-conjugate operation, and $\mathcal{C}$ represents the set of complex numbers. $\mathcal{OC}^{m\times n}$ represents the set of all $m \times n$ complex matrices with mutually orthogonal and normal columns. $\mathbf{A}\perp\mathbf{B}$ means that every column of the matrix $\mathbf{A}$ is orthogonal to all columns of the matrix $\mathbf{B}$. The sub-space spanned by columns of $\mathbf{A}$ is represented by $\Omega(\mathbf{A})$. The null space of the matrix $\mathbf{A}$ is denoted by $\mathrm{N}(\mathbf{A})$. The identity matrix is represented by $\mathbf{I}$.

II. Channel Model

We consider a MIMO system with two transmitters and two receivers. Transmitter t, $t = 1, 2$, is equipped with m_t antennas and receiver r, $r = 1, 2$, is equipped with n_r antennas. This configuration of antennas is shown by (m_1, m_2, n_1, n_2). For simplicity and without loss of generality, it is assumed that $m_1 \geq m_2$ and $n_1 \geq n_2$.

Assuming flat fading environment, the channel between transmitter t and receiver r is represented by the channel matrix $\mathbf{H}_{rt}$, where $\mathbf{H}_{rt} \in \mathcal{C}^{n_r\times m_t}$. The received vector $\mathbf{y}_r \in \mathcal{C}^{n_r\times 1}$ by receiver r, $r = 1, 2$, is given by,

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{H}_{11}\mathbf{s}_1 + \mathbf{H}_{12}\mathbf{s}_2 + \mathbf{w}_1, \\ \mathbf{y}_2 &= \mathbf{H}_{21}\mathbf{s}_1 + \mathbf{H}_{22}\mathbf{s}_2 + \mathbf{w}_2, \end{aligned} \tag{2}$$

where $\mathbf{s}_t \in \mathcal{C}^{m_t\times 1}$ represents the transmitted vector by transmitter t. The vector $\mathbf{w}_r \in \mathcal{C}^{n_r\times 1}$ is a white Gaussian noise with zero mean and identity covariance matrix. The power of $\mathbf{s}_t$ is subject to the constraint $\mathrm{Tr}(E[\mathbf{s}_t\mathbf{s}_t^\dagger]) \leq P_t$, $t = 1, 2$. P_T denotes the total transmit power, i.e. $P_T = P_1 + P_2$.

In the proposed scenario, each transmitter sends two sets of data streams. The transmitter t sends μ_{1t} data streams to receiver 1 and μ_{2t} data streams to receiver 2.

III. Decomposition Schemes

In what follows, we propose two signaling schemes. In the first scheme, by using linear transformations at the transmitters and the receivers, the system is decomposed into two non-interfering broadcast sub-channels. Then, we can use the known signaling schemes over the resulting broadcast sub-channels.

As a dual of the first scheme, in the second scheme, linear transformations are utilized to decompose the system into two non-interfering multi-access sub-channels.

It is assumed that $m_1 < n_1 + n_2$ and $n_1 < m_1 + m_2$. Otherwise, if $m_1 \geq n_1 + n_2$, the maximum MG of $n_1 + n_2$ is achievable by a simple broadcast channel including the first transmitter and the two receivers. Similarly, if $n_1 \geq m_1 + m_2$, then the maximum MG of $m_1 + m_2$ is achievable by a simple multi-access channel including the two transmitters and the first receiver.

A. Decomposition of the System into Two Broadcast Sub-Channels

As depicted in Fig. 1, in this scheme, the transmit filter $\mathbf{Q}_t \in \mathcal{OC}^{M_t\times(\mu_{1t}+\mu_{2t})}$ is employed at transmitter t, $t = 1, 2$. Therefore, the transmitted vectors $\mathbf{s}_t$, $t = 1, 2$, are equal to

$$\mathbf{s}_t = \mathbf{Q}_t\tilde{\mathbf{s}}_t, \tag{3}$$

where $\tilde{\mathbf{s}}_t \in \mathcal{C}^{(\mu_{1t}+\mu_{2t})\times 1}$ contains μ_{1t} data streams for receiver one and μ_{2t} data streams for receiver two. The transmit filters $\mathbf{Q}_t$, $t = 1, 2$, have two functionalities: (i) Confining the transmit signal from transmitter t to a $(\mu_{1t} + \mu_{2t})$-dimensional sub-space, which provides the possibility of decomposing the system into two broadcast sub-channels by using linear filters at the receivers, (ii) Exploiting the null spaces of the channel matrices to achieve the highest MG.

At each receiver, two parallel receive filters are employed. The received vector $\mathbf{y}_1$ is passed through the filter $\mathbf{\Psi}_{11}^\dagger$, which is used to null out the signal coming from the second transmitter. The μ_{11} data streams, sent by transmitter one intended to receiver one, can be decoded from $\mathbf{y}_{11}$, the output of $\mathbf{\Psi}_{11}^\dagger$. Similarly, to decode μ_{12} data streams, sent by transmitter two to receiver one, the received vector $\mathbf{y}_1$ is passed through the receive filter $\mathbf{\Psi}_{12}^\dagger$, which is used to null out the signal coming from transmitter one. Receiver two has a similar structure with parallel receive filters $\mathbf{\Psi}_{21}^\dagger$ and $\mathbf{\Psi}_{22}^\dagger$. Later, it is shown that if the numbers of data streams μ_{rt}, $r, t = 1, 2$, satisfy a set of inequalities, then it is possible to deign $\mathbf{Q}_t$ and $\mathbf{\Psi}_{rt}$ to meet the desired features explained earlier. It means that the system is decomposed into two non-interfering MIMO broadcast sub-channels (see Fig. 2).

Next, we explain how to select the design parameters including the number of data streams μ_{rt}, $r, t = 1, 2$ and the transmit/receive filters. The primary objective is to prevent the saturation of the rate of each stream in the high SNR regime. In other words, the MG of the system is $\mu_{11} + \mu_{12} + \mu_{21} + \mu_{22}$.

The integer variables ζ_{11}, ζ_{21}, ζ_{12}, and ζ_{22} are defined as the dimensions of $\Omega(\mathbf{H}_{12}\mathbf{Q}_2)$, $\Omega(\mathbf{H}_{22}\mathbf{Q}_2)$, $\Omega(\mathbf{H}_{11}\mathbf{Q}_1)$, and $\Omega(\mathbf{H}_{21}\mathbf{Q}_1)$, respectively.

The design scheme varies depending on (m_1, m_2, n_1, n_2). In the sequel, we categorize the design scheme into the four general cases depending on (m_1, m_2, n_1, n_2). To facilitate the derivations, we use the auxiliary variables m'_t, n'_r, and μ'_{rt}, for $r,t=1,2$. As will be explained later, for each case, m'_t and n'_r are computed directly as a function of m_t and n_r for $r,t=1,2$. Then, μ'_{rt}, $r,t=1,2$, are selected such that the following constraints are satisfied,

$$\begin{aligned}
\mu'_{11}: &\quad \mu'_{11}+\mu'_{12}+\mu'_{22}\le n'_1, &(4)\\
\mu'_{12}: &\quad \mu'_{12}+\mu'_{11}+\mu'_{21}\le n'_1, &(5)\\
\mu'_{22}: &\quad \mu'_{22}+\mu'_{21}+\mu'_{11}\le n'_2, &(6)\\
\mu'_{21}: &\quad \mu'_{21}+\mu'_{22}+\mu'_{12}\le n'_2, &(7)\\
&\quad \mu'_{11}+\mu'_{21}\le m'_1, &(8)\\
&\quad \mu'_{22}+\mu'_{12}\le m'_2. &(9)
\end{aligned}$$

Each of the first four inequalities corresponds to one of the parameters μ'_{rt}, $r,t=1,2$, in the sense that if μ'_{rt}, $r,t=1,2$, is zero, the corresponding inequality is removed from the set of constraints. After choosing μ'_{rt}, $r,t=1,2$, for each case, μ_{rt}, $r,t=1,2$, are computed as function of μ'_{rt}, $r,t=1,2$, as will be explained later.

Note that we have many options to choose μ'_{rt}, $r,t=1,2$. It is shown that as long as the integers μ'_{rt}, $r,t=1,2$, satisfy (4) to (9), the system achieves the MG of $\mu_{11}+\mu_{12}+\mu_{21}+\mu_{22}$. However, it turns out that to achieve the highest MG, μ'_{rt}, $r,t=1,2$, should be selected such that $\mu'_{11}+\mu'_{12}+\mu'_{21}+\mu'_{22}$ is maximum.

In what follows, for each of the four cases, we explain: (i) How to compute the auxiliary variables m'_t and n'_r as a function of m_t and n_r, $r,t=1,2$, (ii) After choosing the auxiliary variables μ'_{rt}, $r,t=1,2$, satisfying (4) to (9), how to compute μ_{rt}, $r,t=1,2$, (iii) How to choose the transmit filters $\mathbf{Q}_t$, $t=1,2$, and finally, (iv) How to compute ζ_{rt}, $r,t=1,2$.

Having completed these steps, the procedure of computing the receive filters $\mathbf{\Psi}^{\dagger}_{rt}$, $r,t=1,2$, is similar for all cases. Later, we will show that this scheme decomposes the system into two non-interfering broadcast sub-channels.

Scheme I – Case I: $n_1\ge n_2\ge m_1\ge m_2$

In this case, $n'_r=n_r$, $r=1,2$, and $m'_t=m_t$, $t=1,2$. Using the above parameters, we choose μ'_{rt}, $r,t=1,2$, subject to (4)-(9) constraints. In this case, μ_{rt}, the number of data streams sent from transmitter t to receiver r, is obtained by $\mu_{rt}=\mu'_{rt}$, $r,t=1,2$. In addition, $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are randomly chosen from $\mathcal{OC}^{m_1\times(\mu_{11}+\mu_{21})}$ and $\mathcal{OC}^{m_2\times(\mu_{12}+\mu_{22})}$, respectively. Regarding the definition of ζ_{rt}, $r,t=1,2$, it is easy to see that $\zeta_{11}=\mu_{12}+\mu_{22}$, $\zeta_{12}=\mu_{11}+\mu_{21}$, $\zeta_{21}=\mu_{12}+\mu_{22}$, and $\zeta_{22}=\mu_{11}+\mu_{21}$.

Scheme I – Case II: $n_1\ge m_1>n_2\ge m_2$

In this case, we have, $n'_1=n_1+n_2-m_1$, $n'_2=n_2$, $m'_1=n_2$, and $m'_2=m_2$. In addition, $\mu_{11}=\mu'_{11}+m_1-n_2$, $\mu_{12}=\mu'_{12}$, $\mu_{21}=\mu'_{21}$, and $\mu_{22}=\mu'_{22}$. Furthermore, $\zeta_{11}=\mu_{12}+\mu_{22}$, $\zeta_{12}=\mu_{11}+\mu_{21}$, $\zeta_{21}=\mu_{12}+\mu_{22}$, and $\zeta_{22}=\mu'_{11}+\mu_{21}$. $\mathbf{Q}_1$ is chosen as $\mathbf{Q}_1=[\mathbf{\Sigma}_1,\mathbf{\Sigma}_2]$ where,

$$\mathbf{\Sigma}_1\in\mathcal{OC}^{m_1\times(n_1-m_2)},\quad \mathbf{\Sigma}_1\in\mathrm{N}(\mathbf{H}_{21}),$$
$$\mathbf{\Sigma}_2=\mathcal{OC}^{m_1\times(\mu'_{11}+\mu_{21})},\quad \mathbf{\Sigma}_2\perp\mathbf{\Sigma}_1.$$

$\mathbf{Q}_2$ is randomly chosen from $\mathcal{OC}^{m_2\times(\mu_{12}+\mu_{22})}$.

In this case, we take advantage of m_1-n_2 dimensions of $\mathrm{N}(\mathbf{H}_{21})$, to exclusively send data from transmitter one to receiver one, without imposing any interference on receiver two. Therefore, transmitter one and receiver one effectively lose m_1-n_2 of the available space dimensions. Consequently, the resulting system is equivalent to a system with effective number of antennas as $(m'_1,m'_2,n'_1,n'_2)=\Big(m_1-\{m_1-n_2\},m_2,n_1-\{m_1-n_2\},n_2\Big)$. The equivalent system with (m'_1,m'_2,n'_1,n'_2) antennas satisfies the condition of the first case, i.e. $m'_1\ge m'_2\ge n'_1\ge n'_2$. Therefore, it is categorized in the same category as the previous case.

Scheme I – Case III: $n_1\ge m_1>m_2\ge n_2$ and $n_1+n_2\ge m_1+m_2$

In this case, we have $n'_1=n_1+2n_2-m_1-m_2$, $n'_2=n_2$, $m'_1=n_2$, and $m'_2=n_2$. In addition, $\mu_{11}=\mu'_{11}+m_1-n_2$, $\mu_{12}=\mu'_{12}+m_2-n_2$, $\mu_{21}=\mu'_{21}$, and $\mu_{22}=\mu'_{22}$. It is easy to see that $\zeta_{11}=\mu_{12}+\mu_{22}$, $\zeta_{12}=\mu_{11}+\mu_{21}$, $\zeta_{21}=\mu'_{12}+\mu_{22}$, and $\zeta_{22}=\mu'_{11}+\mu_{21}$. $\mathbf{Q}_1$ is chosen as $\mathbf{Q}_1=[\mathbf{\Sigma}_1,\mathbf{\Sigma}_2]$, where

$$\mathbf{\Sigma}_1\in\mathcal{OC}^{m_1\times(m_1-n_2)},\quad \mathbf{\Sigma}_1\in\mathrm{N}(\mathbf{H}_{21}),$$
$$\mathbf{\Sigma}_2=\mathcal{OC}^{m_1\times(\mu'_{11}+\mu_{21})},\quad \mathbf{\Sigma}_2\perp\mathbf{\Sigma}_1.$$

$\mathbf{Q}_2$ is chosen as $\mathbf{Q}_2=[\mathbf{\Sigma}_3,\mathbf{\Sigma}_4]$, where

$$\mathbf{\Sigma}_3\in\mathcal{OC}^{m_2\times(m_2-n_2)},\quad \mathbf{\Sigma}_3\in\mathrm{N}(\mathbf{H}_{22}),$$
$$\mathbf{\Sigma}_4=\mathcal{OC}^{m_2\times(\mu'_{12}+\mu_{22})},\quad \mathbf{\Sigma}_4\perp\mathbf{\Sigma}_3.$$

Scheme I – Case IV: $m_1\ge n_1>n_2\ge m_2$ and $n_1+n_2\ge m_1+m_2$

In this case, we have $n'_1=n_1+n_2-m_1$, $n'_2=n_1+n_2-m_1$, $m'_1=n_1+n_2-m_1$, and $m'_2=m_2$. In addition, $\mu_{11}=\mu'_{11}+m_1-n_2$, $\mu_{12}=\mu'_{12}$, $\mu_{21}=\mu'_{21}+m_1-n_1$, and $\mu_{22}=\mu'_{22}$. Furthermore, $\zeta_{11}=\mu_{12}+\mu_{22}$, $\zeta_{12}=\mu_{11}+\mu'_{21}$, $\zeta_{21}=\mu_{12}+\mu_{22}$, and $\zeta_{22}=\mu'_{11}+\mu_{21}$. In addition, $\mathbf{Q}_1$ is chosen as $\mathbf{Q}_1=[\mathbf{\Sigma}_1,\mathbf{\Sigma}_2]$ where,

$$\mathbf{\Sigma}_1\in\mathcal{OC}^{m_1\times(m_1-n_2+m_1-n_2)},\quad \mathbf{\Sigma}_1\in\mathrm{N}(\mathbf{H}_{21})\cup\mathrm{N}(\mathbf{H}_{11}),$$
$$\mathbf{\Sigma}_2=\mathcal{OC}^{m_1\times(\mu'_{11}+\mu'_{21})},\quad \mathbf{\Sigma}_2\perp\mathbf{\Sigma}_1.$$

$\mathbf{Q}_2$ is randomly chosen from $\mathcal{OC}^{m_2\times(\mu_{12}+\mu_{22})}$.

The next steps of the algorithm are the same for all of the aforementioned cases. We define

$$\widetilde{\mathbf{H}}_{rt}=\mathbf{H}_{rt}\mathbf{Q}_t,\ \ r,t=1,2. \tag{10}$$

$\mathbf{\Psi}_{rt}\in\mathcal{OC}^{N_t\times(N_t-\zeta_{rt})}$, $r,t=1,2$, are chosen such that $\mathbf{\Psi}_{11}\perp\widetilde{\mathbf{H}}_{12}$, $\mathbf{\Psi}_{12}\perp\widetilde{\mathbf{H}}_{11}$, $\mathbf{\Psi}_{21}\perp\widetilde{\mathbf{H}}_{22}$, and $\mathbf{\Psi}_{22}\perp\widetilde{\mathbf{H}}_{21}$. According to the definition of ζ_{rt}, one can always choose such matrices. Clearly, any signal sent by transmitter one does not pass through the filters $\mathbf{\Psi}^{\dagger}_{12}$ and $\mathbf{\Psi}^{\dagger}_{22}$. Similarly, any signal sent by transmitter two does not pass through the filters $\mathbf{\Psi}^{\dagger}_{21}$ and $\mathbf{\Psi}^{\dagger}_{11}$.

We define $\overline{\mathbf{H}}_{rt}=\mathbf{\Psi}^{\dagger}_{rt}\widetilde{\mathbf{H}}_{rt}$, $\mathbf{w}_{rt}=\mathbf{\Psi}^{\dagger}_{rt}\mathbf{w}_r$, and $\mathbf{y}_{rt}=\mathbf{\Psi}^{\dagger}_{rt}\mathbf{y}_r$, for $r,t=1,2$. Therefore, the system is decomposed into two non-interfering broadcast sub-channels. The MIMO broadcast sub-channel viewed from transmitter 1 is modeled by $\mathbf{y}_{11}=\overline{\mathbf{H}}_{11}\tilde{\mathbf{s}}_1+\mathbf{w}_{11}$ and $\mathbf{y}_{21}=$

$\overline{\mathbf{H}}_{21}\tilde{\mathbf{s}}_1+\mathbf{w}_{21}$, and the MIMO broadcast sub-channel viewed from transmitter two is modeled by $\mathbf{y}_{12} = \overline{\mathbf{H}}_{12}\tilde{\mathbf{s}}_2+\mathbf{w}_{12}$ and $\mathbf{y}_{22} = \overline{\mathbf{H}}_{22}\tilde{\mathbf{s}}_2 + \mathbf{w}_{22}$ (see Fig. 2).

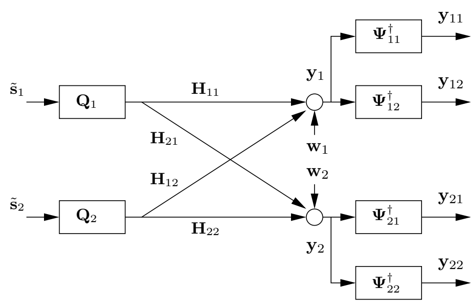

Fig. 1. Scheme One: Decomposition of the System into Two Broadcast Sub-Channels

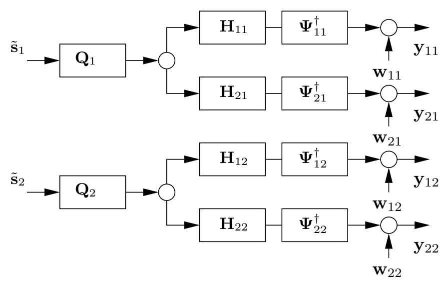

Fig. 2. Scheme One: The Resulting Non-Interfering MIMO Broadcast Sub-Channels

B. Scheme 2 - Decomposition of the System into Two Multi-access Sub-Channels

This scheme is indeed the dual of the scheme one, detailed in subsection III-A. In this scheme, the system is decomposed into two non-interfering MIMO multi-access sub-channels. ,Similar to the scheme one, this scheme is designed for four cases. Regarding lack of space, we do not explain the scheme two here. The complete detail of the scheme two is provided in [8]. The eight cases, listed for scheme one and two, cover all the possible configurations of the number of transmit and receive antennas.

IV. Performance Evaluation

In what follows, the MG of the X channel is studied. In addition, for some special cases, a metric known as the *power offset*, is evaluated. For the proof of the results please refer to [8].

Theorem 1 *The MIMO X channel with* (m_1, m_2, n_1, n_2) *antennas achieves the MG of* $\mu_{11} + \mu_{21} + \mu_{12} + \mu_{22}$*, if* μ_{rt}*,* $r, t = 1, 2$*, are selected according to the schemes presented in Section III.*

For example, the MGs of a X channels with $(3, 3, 3, 3)$, $(4, 3, 4, 3)$, $(9, 5, 8, 7)$ antennas are 4, 5, and 11 respectively, while the MGs of the interference channels with the same number of antennas are respectively 3, 4, and 9. In addition, in the X channel with $(\lceil \frac{1}{2} \lfloor \frac{4n}{3} \rfloor \rceil, \lfloor \frac{1}{2} \lfloor \frac{4n}{3} \rfloor \rfloor, n, n)$ or $(n, n, \lceil \frac{1}{2} \lfloor \frac{4n}{3} \rfloor \rceil, \lfloor \frac{1}{2} \lfloor \frac{4n}{3} \rfloor \rfloor)$ antennas, the MG of $\lfloor \frac{4n}{3} \rfloor$ is achievable, which is the MG of the system where full-cooperation between transmitters or between receivers is provided. However, it does not mean that the system does not gain any improvement by cooperation. The gain of the cooperation is reflected in a metric known as the *power offset*. The power offset is the negative of the zero-order term in the expansion of the sum-rate, normalized with MG, with respect to the total power [9], i.e., $R = \rho(\log_2(P_T) - L_\infty) + o(1)$, where P_T denotes the total power, and L_∞ denotes the power offset in 3dB unit. In this definition, it is assumed that the noise is normalized as in the system model (2).

Theorem 2 *In an X channel with* $(m_1, m_2, n_1, n_2) = (2k, 2k, 3k, 3k)$ *antennas, where the entries of channel matrices have Rayleigh distribution, if the decomposition scheme is employed, the power offset is equal to,*

$$L_\infty(m_1, m_2, n_1, n_2) = L_\infty(2k, 2k) - \frac{1}{2}\log_2\Big(\alpha(1-\alpha)\Big),$$

in 3dB units, where $P_1 = \alpha P_T$*,* $P_2 = (1-\alpha)P_T$*,* $0 \leq \alpha \leq 1$*, and* $L_\infty(m, m) = \log_2 m + \frac{1}{\ln(2)}\left(0.5772 + 1 - \sum_{i=1}^{m} i^{-1}\right)$.

According to this theorem, the power offset of an X channel with $(2, 2, 3, 3)$ and $\alpha = .5$ is 9.3341 dB, while the power offset of a MIMO system (see [9]) with 4 transmit and 6 receive antennas is 3.1666 dB. Therefore, although cooperation does not increase the MG of the system (both systems have MG of 4), but it improves the performance of the system about 6.2 dB in high SNR.

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