Diversity-Multiplexing Trade-off in Z-channel

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***Abstract*—We consider a simple structure of Z-Channel and derive the diversity-multiplexing trade-off assuming different scenarios for the signal-to-noise ratio of the links. The diversity-multiplexing trade-off is determined assuming having known/unknown interference channel at the receiver.**

I. Introduction

The fundamental trade-off between the diversity and multiplexing gain has been characterized for MIMO systems in [1] and is extended for MIMO multiple-access channel in [2]. This approach has been applied for other wireless channels and networks [3]–[5]; In [3], the trade-off between the rate and the reliability is studied for different strategies in a wireless relay network. In [5], diversity-multiplexing trade-off upper bounds are obtained for cooperative diversity protocols in a wireless network.

In this paper, we consider a Z-Channel in which the transmitters and receivers are equipped with single antennas. The received signal at the first receiver can be written as

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_0\mathbf{x}_2 + \mathbf{n}_1, \tag{1}$$

and the second link will be an ordinary point to point link as

$$\mathbf{y}_2 = \mathbf{H}_2\mathbf{x}_2 + \mathbf{n}_2, \tag{2}$$

where $\mathbf{x}_i \in \mathbb{C}^{1\times 1}$ is the transmitted signal from the ith transmitter with the power constraint $\mathbb{E}\{\|\mathbf{x}_i\|^2\} \le \rho_i$, and $\mathbf{n}_i \sim \mathcal{CN}(\mathbf{0}, 1)$ is the AWGN at ith receiver for $i = 1, 2$. All the channels in this model are assumed to be quasi-static Rayleigh fading, i.e. the channel coefficients $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$ have complex Gaussian distribution with zero mean and unit variance. It is assumed that $\mathbf{H}_0$ and $\mathbf{H}_1$ are perfectly known at the first receiver and perfectly unknown at the first transmitter. The channel coefficients $\mathbf{H}_2$ is assumed to be perfectly known at the second receiver, while unknown at the second transmitter.

Investigating the diversity-multiplexing trade-off curves for this setup requires the assumption of $\rho_1, \rho_2 \to \infty$. Hence, using the definition of the diversity and multiplexing in [1], for each link, we have

$$\begin{aligned} r_i &= \lim_{\rho_i\to\infty} \frac{\mathcal{R}_i(\rho_i)}{\log \rho_i}, \\ d_i &= \lim_{\rho_i\to\infty} -\frac{\log \mathbb{P}_i(\rho_i)}{\log \rho_i}, \quad i = 1, 2, \end{aligned} \tag{3}$$

[1]This work is funded by Nortel and by matching funds from the federal government of Canada (NSERC) and province of Ontario (OCE).

where $\mathcal{R}_i(\rho_i)$ denotes the transmission rate, and $\mathbb{P}_i(\rho_i)$ represents the average error probability of link i. Assuming large enough block lengths, from [1] it is realized that the average error probability is equal to the outage probability, defined as

$$\mathscr{P}_i(\rho_i) \triangleq \Pr\{C(\rho_i) < \mathcal{R}_i\}, \tag{4}$$

where $C(\rho_i)$ denotes the capacity of link i, almost surely. We focus on characterizing the diversity-multiplexing trade-off curve for the first link, since the second link is an ordinary point-to-point link. For this purpose, we derive the outage probability for this link, for any given multiplexing gain vector (r_1, r_2).

II. Analysis of Diversity-Multiplexing Trade-off

From the first receiver's point of view, the channel is a multiple-access channel (MAC). However, since the first receiver is not interested in the data of the second transmitter, the outage event is different from that of MAC. In fact, the outage event in this case can be written as the intersection of the following events:

$$\begin{aligned} \mathscr{B}_1 &\triangleq \{(\mathcal{R}_1, \mathcal{R}_2) \notin C_{MAC}\}, \\ \mathscr{B}_2 &\triangleq \{\mathcal{R}_1 > I(\mathbf{x}_1; \mathbf{y}_1)\}, \end{aligned} \tag{5}$$

where C_{MAC} denotes the capacity region of the MAC. The first event corresponds to the case that the first receiver can not decode both $\mathbf{x}_1$ and $\mathbf{x}_2$. The second event describes the situation when the first receiver can not decode $\mathbf{x}_1$, considering $\mathbf{x}_2$ as noise. In terms of the channel matrices $\mathbf{H}_0$ and $\mathbf{H}_1$, the above events can be written as follows:

$$\begin{aligned} \mathscr{B}_1 &= \left\{ \begin{array}{ll} \log(1+\rho_1 h_1) < \mathcal{R}_1 & \bigcup \\ \log(1+\rho_2 h_0) < \mathcal{R}_2 & \bigcup \\ \log(1+\rho_2 h_0 + \rho_1 h_1) < \mathcal{R}_1 + \mathcal{R}_2 & \end{array} \right\}, \\ \mathscr{B}_2 &= \left\{ \log(1 + \frac{\rho_1 h_1}{1+\rho_2 h_0}) < \mathcal{R}_1 \right\}, \end{aligned} \tag{6}$$

where $h_i \triangleq \|\mathbf{H}_i\|^2$, $i = 0, 1$. The intersection of these two events is depicted in Fig. 1. As can be observed, the outage event can be expressed as the union of the events $\mathcal{A}_1$ and $\mathcal{A}_2$. $\mathcal{A}_1$ is the outage event as if the second user does not exist. The effect of interference from the second user is captured in $\mathcal{A}_2$.

Theorem 1 *Assuming $\rho_1 = \rho$ and $\rho_2 = \rho^\beta$, the diversity-multiplexing trade-off for the first user of Z-channel is*

$$d_Z^*(r_1, r_2) = \left\{ \begin{array}{ll} \min(1-r_1, (1+\beta) - 2(r_1 + \beta r_2)) & \mathcal{F}_1 \\ \mu - r_1 - \beta r_2 & \mathcal{F}_2 \end{array} \right.$$

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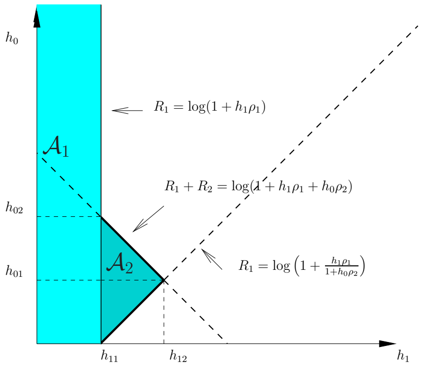

Fig. 1. Outage region

where $\mathcal{F}_1 \equiv r_1 + \beta r_2 < \eta$*,* $\mathcal{F}_2 \equiv \eta < r_1 + \beta r_2 < \mu$*,* $\eta \triangleq \min(1, \beta)$*, and* $\mu \triangleq \max(1, \beta)$*.*

Proof: Defining the total outage event as $\mathscr{B}$, we have

$$\Pr\{\mathscr{B}\} = \Pr\{\mathcal{A}_1\} + \Pr\{\mathcal{A}_2\}. \tag{7}$$

From Fig. 1, the probability of $\mathcal{A}_1$ and $\mathcal{A}_2$ can be written as

$$\Pr\{\mathcal{A}_1\} = \int_0^{h_{11}} f(h_1) dh_1, \tag{8}$$

$$\Pr\{\mathcal{A}_2\} = \int_{h_{11}}^{h_{12}} \int_{m_1(h_1-h_{11})}^{h_{02}+m_2(h_{11}-h_1)} f(h_1) f(h_0) \mathrm{d}h_0 \mathrm{d}h_1, \tag{9}$$

where $h_{11} \triangleq \frac{e^{\mathcal{R}_1}-1}{\rho_1}$, $h_{01} \triangleq \frac{e^{\mathcal{R}_2}-1}{\rho_2}$, $h_{12} \triangleq \frac{e^{\mathcal{R}_2}(e^{\mathcal{R}_1}-1)}{\rho_1}$, $h_{02} \triangleq \frac{e^{\mathcal{R}_1}(e^{\mathcal{R}_2}-1)}{\rho_2}$, $m_1 \triangleq \frac{h_{01}}{h_{12}-h_{11}}$, $m_2 \triangleq \frac{h_{02}-h_{01}}{h_{12}-h_{11}}$, and $f(h_0)$ and $f(h_1)$ are the pdf of h_0 and h_1, respectively. Having the fact that $f(h_0) = e^{-h_0}$ and $f(h_1) = e^{-h_1}$, (8) and (9) can be written as follows:

$$\Pr\{\mathcal{A}_1\} = 1 - e^{-h_{11}}, \tag{10}$$

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &= \int_{h_{11}}^{h_{12}} \int_{m_1(h_1-h_{11})}^{h_{02}+m_2(h_{11}-h_1)} e^{-h_0} e^{-h_1} \mathrm{d}h_0 \mathrm{d}h_1 \\ &= \left\{ \begin{array}{ll} \frac{1}{m_1+1}\left[e^{-h_{11}} - e^{-(h_{12}+h_{01})}\right] & \\ -\frac{1}{1-m_2}\left[e^{-(h_{02}+h_{11})} - e^{-(h_{12}+h_{01})}\right] & m_2 \neq 1 \\ \frac{1}{m_1+1}\left[e^{-h_{11}} - e^{-(h_{12}+h_{01})}\right] & \\ -(h_{12} - h_{11}) e^{-(h_{02}+h_{11})} & m_2 = 1. \end{array} \right.\end{aligned} \tag{11}$$

Setting $\rho_1 = \rho$ and $\rho_2 = \rho^\beta$, where β is an arbitrary constant and using (3), we have

$$\begin{aligned} h_{11} &= \rho^{r_1-1}\left(1 - \rho^{-r_1}\right), \\ h_{12} &= \rho^{r_1+\beta r_2-1}\left(1 - \rho^{-r_1}\right), \\ h_{01} &= \rho^{\beta(r_2-1)}\left(1 - \rho^{-\beta r_2}\right), \\ h_{02} &= \rho^{r_1+\beta r_2-\beta}\left(1 - \rho^{-\beta r_2}\right), \\ m_1 &= \rho^{1-\beta-r_1}\left(1 - \rho^{-r_1}\right)^{-1}, \\ m_2 &= \rho^{1-\beta}. \end{aligned} \tag{12}$$

Noting (10) and (12), we can write

$$\Pr\{\mathcal{A}_1\} \doteq \rho^{r_1-1}, \quad r_1 < 1 \tag{13}$$

where $b \doteq \rho^a$ is equivalent to $\lim_{\rho\to\infty} \frac{\log b}{\log \rho} = a$. We consider 3 scenarios to derive $\Pr\{\mathcal{A}_2\}$:

1) Weak interference ($\beta < 1$)*:* From (12), it follows that $m_2 \to \infty$ as $\rho \to \infty$. Moreover, for the region $r_1 < 1 - \beta$ we have $m_1 \to \infty$. Hence, (11) can be written as

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &= \frac{1}{m_1+1} e^{-h_{11}}\left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) - \\ &\quad \frac{1}{m_2-1} e^{-(h_{12}+h_{01})}\left(1 - e^{-(m_2-1)(h_{12}-h_{11})}\right) \\ &\overset{(a)}{\backsimeq} e^{-h_{11}}(h_{12} - h_{11}) - \\ &\quad \frac{1}{m_2-1} e^{-(h_{12}+h_{01})}\left(1 - e^{-(m_2-1)(h_{12}-h_{11})}\right),\end{aligned} \tag{14}$$

where (a) comes from applying the approximation $1 - e^{-(1+m_1)(h_{12}-h_{11})} \backsimeq (1 + m_1)(h_{12} - h_{11})$, since $(1 + m_1)(h_{12} - h_{11}) \doteq \rho^{1-\beta}\rho^{r_1+\beta r_2-1} = \rho^{\beta(r_2-1)}$, and as a result $(1 + m_1)(h_{12} - h_{11}) \to 0$ for $r_2 < 1$. For the case that $r_1 + \beta r_2 > \beta$, noting that $(m_2 - 1)(h_{12} - h_{11}) \to \infty$, the second term in the above equation can be approximated with $\frac{1}{m_2-1} \backsimeq \rho^{\beta-1}$. Consequently, we have

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &\backsimeq \rho^{r_1+\beta r_2-1} - \rho^{\beta-1} \\ &\doteq \rho^{r_1+\beta r_2-1}.\end{aligned} \tag{15}$$

Indeed, for the case $r_1 + \beta r_2 < \beta$, rewriting (14) as

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &\backsimeq e^{-h_{11}}(h_{12} - h_{11}) - \\ &\quad \frac{1}{m_2-1} e^{-(h_{11}+h_{02})}\left(e^{(m_2-1)(h_{12}-h_{11})} - 1\right),\end{aligned} \tag{16}$$

and noting that $(m_2 - 1)(h_{12} - h_{11}) \to 0$, we have $\frac{e^{(m_2-1)(h_{12}-h_{11})}-1}{m_2-1} \backsimeq (h_{12} - h_{11})$. Substituting in the above equation yields,

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &\backsimeq e^{-h_{11}}\left(1 - e^{-h_{02}}\right)(h_{12} - h_{11}) \\ &\overset{(a)}{\backsimeq} h_{02}(h_{12} - h_{11}) \\ &\doteq \rho^{2(r_1+\beta r_2)-(1+\beta)},\end{aligned} \tag{17}$$

where (a) comes from the fact that since $h_{11}, h_{02} \to 0$, $e^{-h_{11}} \backsimeq 1$, $e^{-h_{02}} \backsimeq 1 - h_{02}$. With a similar argument, we can obtain $\Pr\{\mathcal{A}_2\}$ for the case $\beta < r_1 + \beta r_2$ and in summary, we have

$$\Pr\{\mathcal{A}_2\} \doteq \left\{ \begin{array}{ll} \rho^{2(r_1+\beta r_2)-(1+\beta)} & r_1 + \beta r_2 < \beta \\ \rho^{r_1+\beta r_2-1} & \beta < r_1 + \beta r_2 < 1 \end{array} \right. \tag{18}$$

2) Moderate interference ($\beta = 1$)*:* Noting that $m_2 = 1$ in this scenario, from (11), we can write

$$\begin{aligned}\Pr\{\mathcal{A}_2\} &= \frac{1}{m_1+1} e^{-h_{11}}\left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) \\ &\quad - (h_{12} - h_{11}) e^{-(h_{11}+h_{02})}.\end{aligned} \tag{19}$$

Having the fact that $m_1 \to 0$, the necessary condition to have $\Pr\{\mathcal{A}_2\} \to 0$ is $h_{12} \to 0$, which incurs $r_1 + r_2 < 1$. Using (19) and the approximation $\frac{1}{m_1+1}\left(1 - e^{-(1+m_{11})(h_{12}-h_{11})}\right) \simeq (h_{12} - h_{11})$, we have

$$
\begin{aligned}
\Pr\{\mathcal{A}_2\} &\simeq e^{-h_{11}}\left(1 - e^{-h_{02}}\right)(h_{12} - h_{11}) \\
&\simeq h_{02}(h_{12} - h_{11}) \\
&\doteq \rho^{2(r_1+r_2-1)}. \qquad (20)
\end{aligned}
$$

3) Strong interference ($\beta > 1$): From (12), it follows that $m_1, m_2 \to 0$, and as a result, (11) can be written as

$$
\begin{aligned}
\Pr\{\mathcal{A}_2\} = &\frac{1}{m_1+1} e^{-h_{11}}\left(1 - e^{-(1+m_1)(h_{12}-h_{11})}\right) - \\
&\frac{1}{1-m_2} e^{-(h_{11}+h_{02})}\left(1 - e^{-(1-m_2)(h_{12}-h_{11})}\right) \qquad (21)
\end{aligned}
$$

Here, we consider two cases: i) $r_1 + \beta r_2 < 1$: In this case, it is easy to show that the first term behaves as $(h_{12} - h_{11})$, and the second term as $e^{-h_{02}}(h_{12} - h_{11})$. Since $\beta > 1$, this condition also incurs that $h_{02} \sim \rho^{r_1+\beta r_2-\beta} \to 0$, and as a result,

$$
\begin{aligned}
\Pr\{\mathcal{A}_2\} &\simeq \left(1 - e^{-h_{02}}\right)(h_{12} - h_{11}) \\
&\simeq h_{02}(h_{12} - h_{11}) \\
&\doteq \rho^{2(r_1+\beta r_2)-(1+\beta)}. \qquad (22)
\end{aligned}
$$

ii) $1 < r_1 + \beta r_2 < \beta$: In this case, it is easy to see that $h_{12} - h_{11} \to \infty$, and hence,

$$
\begin{aligned}
\Pr\{\mathcal{A}_2\} &\simeq \frac{1}{1+m_1} - \frac{1}{1-m_2} e^{-h_{02}} \\
&\overset{(a)}{\simeq} (1 - m_1) - (1 + m_2)e^{-h_{02}} \\
&\overset{(b)}{\simeq} h_{02} - (m_1 + m_2 e^{-h_{02}}) \\
&\overset{(c)}{\doteq} \rho^{r_1+\beta r_2-\beta}, \qquad (23)
\end{aligned}
$$

where (a) comes from using the approximation $\frac{1}{1-x} \simeq 1 + x$ for $x \ll 1$, (b) results from the assumption of $r_1 + \beta r_2 < \beta$ which incurs $h_{02} \to 0$, and finally, (c) results from the fact that since $r_1 + \beta r_2 > 1$, $h_{02} \sim \rho^{r_1+\beta r_2-\beta}$ dominates m_1 and m_2. Unifying the expression of $\Pr\{\mathcal{A}_2\}$ from (18), (20), (22), and (23), we obtain

$$
\Pr\{\mathcal{A}_2\} \doteq \begin{cases} \rho^{2(r_1+\beta r_2)-(1+\beta)} & r_1 + \beta r_2 < \eta \\ \rho^{r_1+\beta r_2-\mu} & \eta < r_1 + \beta r_2 < \mu \end{cases} \qquad (24)
$$

where $\eta \triangleq \min(1, \beta)$, $\mu \triangleq \max(1, \beta)$. Using (7), (13) and (24), the result of the theorem is obtained. ■

Fig. 2 depicts the optimal diversity-multiplexing trade-off curve for $\beta = 0.5$, $\beta = 1$ and $\beta = 1.3$. As can be observed, the curve corresponding to $\beta = 1.3$, outperforms the other curves. Moreover, comparing the two curves corresponding to $\beta = 0.5$ and $\beta = 1$, it is realized that for moderate values of multiplexing gain, the curve corresponding to $\beta = 1$ yields the higher diversity gain, while for the high multiplexing gain values $\beta = 0.5$ is preferable. Fig. 3 shows the maximum diversity gain versus β, for the fixed multiplexing gain values

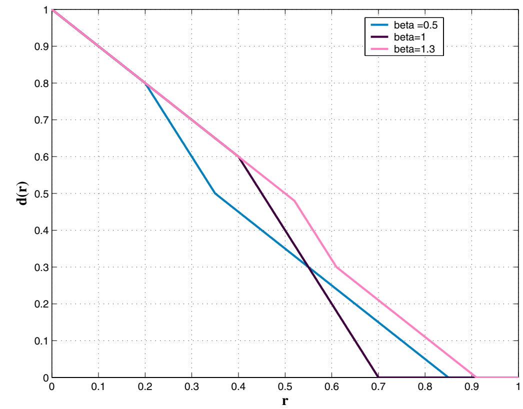

Fig. 2. Diversity-Multiplexing trade-off for various values of β, $r_2 = 0.3$.

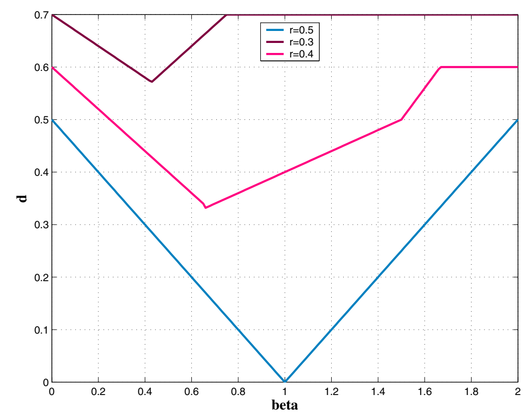

Fig. 3. Diversity vs. β for various values of multiplexing gains, $r_1 = r_2 = r$.

of 0.3, 0.4, and 0.5, assuming $r_1 = r_2$. As can be observed, all the curves have a global minimum, depending on the value of the multiplexing gain.

III. Comparison with Multiple-Access channel and No-CSIR

In this section, we compare the diversity-multiplexing trade-off curve, derived in the previous section, with two other scenarios; i) Multiple-Access channel (MAC), where the first receiver decodes the transmitted data from both senders, and ii) No-CSIR scenario, where the receiver does not have any information about the interference channel $\mathbf{H}_0$. The former is studied in [2], for the case that the signal-to-noise ratio (SNR) of all links are the same. In the following, we will briefly go over this scenario, assuming that $\rho_1 = \rho$ and $\rho_2 = \rho^\beta$.

A. Multiple-Access channel

The probability of the outage event, denoted as $\mathscr{B}_{MAC}$ can be written as

$$\begin{aligned}\Pr\{\mathscr{B}_{MAC}\} =& \int_0^{h_{11}} e^{-h_1}\mathrm{d}h_1 \\ &+ \int_{h_{11}}^{h_{12}}\int_0^{h_{02}+m_2(h_{11}-h_1)} e^{-h_0}e^{-h_1}\mathrm{d}h_0\mathrm{d}h_1 \\ &+ \int_{h_{12}}^{\infty}\int_0^{h_{01}} e^{-h_0}e^{-h_1}\mathrm{d}h_0\mathrm{d}h_1 \\ =& 1-e^{-h_{11}}+\chi+(1-e^{-h_{01}})e^{-h_{12}}, \end{aligned} \tag{25}$$

where

$$\chi = \begin{cases} \left(e^{-h_{11}}-e^{-h_{12}}\right) \\ -\frac{1}{m_2-1}\left[e^{-(h_{01}+h_{12})}-e^{-(h_{02}+h_{11})}\right] & m\neq 1 \\ \left(e^{-h_{11}}-e^{-h_{12}}\right) \\ -e^{-(h_{01}+h_{12})}\left(h_{12}-h_{11}\right) & m=1 \end{cases} \tag{26}$$

Following similar arguments in the proof of Theorem 1, we can easily show that

$$\chi \doteq \begin{cases} \rho^{2(r_1+\beta r_2)-(1+\beta)} & r_1+\beta r_2<\eta \\ \rho^{r_1+\beta r_2-\mu} & \eta<r_1+\beta r_2<\mu \end{cases}, \tag{27}$$

where $\eta = \min(1,\beta)$ and $\mu = \max(1,\beta)$. We interpret diversity-multiplexing gain of MAC in terms of that of Z-channel as follows:

$$d^*_{MAC}(r_1,r_2) = \begin{cases} d^*_Z(r_1,r_2) & 1<r_1+\beta r_2<\beta \\ \min(d^*_Z(r_1,r_2),\beta(1-r_2)) & \text{Otherwise} \end{cases}$$

B. No-CSIR

In this scenario, the first receiver only knows its own channel and does not have any information about the interference channel ($\mathbf{H}_0$). Denoting the outage event as $\mathscr{B}_{NCSIR}$, we have

$$\Pr\{\mathscr{B}^L_{NCSIR}\} \leq \Pr\{\mathscr{B}_{NCSIR}\} \leq \Pr\{\mathscr{B}^U_{NCSIR}\}, \tag{28}$$

where

$$\mathscr{B}^U_{NCSIR} \triangleq \{\mathcal{R}_1 > I(\mathbf{x}_1;\mathbf{y}_1|\mathbf{H}_1)\}, \tag{29}$$

and

$$\mathscr{B}^L_{NCSIR} \triangleq \{\mathcal{R}_1 > I(\mathbf{x}_1;\mathbf{y}_1|\mathbf{x}_2,\mathbf{H}_1)\}. \tag{30}$$

$\mathscr{B}^L_{NCSIR}$ denotes the outage event when the first receiver considers $\mathbf{x}_2$ as noise and $\mathscr{B}^U_{NCSIR}$ is the multiple-access upper-bound when the second users' data is decoded correctly at the first receiver. We can write

$$\begin{aligned} I(\mathbf{x}_1;\mathbf{y}_1|\mathbf{H}_1) &= h(\mathbf{y}_1|\mathbf{H}_1) - h(\mathbf{y}_1|\mathbf{H}_1,\mathbf{x}_1) \\ &= h(\mathbf{H}_1\mathbf{x}_1+\mathbf{H}_0\mathbf{x}_2+\mathbf{n}_1|\mathbf{H}_1) \\ &- h(\mathbf{H}_0\mathbf{x}_2+\mathbf{n}_1) \\ &\overset{(a)}{\geq} h(\mathbf{H}_1\mathbf{x}_1+\mathbf{n}_1|\mathbf{H}_1) \\ &- \log\left(2\pi e\mathrm{Var}(\mathbf{H}_0\mathbf{x}_2+\mathbf{n}_1)\right) \\ &= \log(2\pi e(\rho_1 h_1+1)) - \log(2\pi e(\rho_2+1)) \\ &\backsimeq \log(\rho^{1-\beta}h_1), \end{aligned} \tag{31}$$

where (a) comes from the fact that $h(X+Y) > h(X)$, for independent X and Y, and $h(X) \leq \log(2\pi e\mathrm{Var(X)})$. Substituting (31) in (29) and assuming $r_1 < 1-\beta$, we obtain

$$\Pr\{\mathscr{B}^U_{NCSIR}\} \lesssim \rho^{1-\beta-r_1}. \tag{32}$$

For calculating the lower-bound, we first compute $I(\mathbf{x}_1;\mathbf{y}_1|\mathbf{x}_2,\mathbf{H}_1)$ as follows:

$$\begin{aligned} I(\mathbf{x}_1;\mathbf{y}_1|\mathbf{x}_2,\mathbf{H}_1)\} &= h(\mathbf{y}_1|\mathbf{H}_1,\mathbf{x}_2) - h(\mathbf{y}_1|\mathbf{H}_1,\mathbf{x}_1,\mathbf{x}_2) \\ &= h(\mathbf{H}_1\mathbf{x}_1+\mathbf{H}_0\mathbf{x}_2+\mathbf{n}_1|\mathbf{H}_1,\mathbf{x}_2) \\ &- h(\mathbf{H}_0\mathbf{x}_2+\mathbf{n}_1|\mathbf{x}_2) \\ &\overset{(a)}{=} \mathbb{E}_{\mathbf{x}_2}\left\{\log(2\pi e(h_1\rho_1+|\mathbf{x}_2|^2+1))\right\} \\ &- \mathbb{E}_{\mathbf{x}_2}\left\{\log(2\pi e(|\mathbf{x}_2|^2+1))\right\} \\ &\overset{(b)}{\leq} \log\left(1+\mathbb{E}_{\mathbf{x}_2}\left\{\frac{\rho_1 h_1}{1+|\mathbf{x}_2|^2}\right\}\right) \\ &\backsimeq \log(1+h_1\rho^{1-\beta}), \quad r_1<1-\beta \end{aligned} \tag{33}$$

where (a) results from the fact that conditioned on $\mathbf{H}_1$ and $\mathbf{x}_2$, $\mathbf{H}_1\mathbf{x}_1$ and $\mathbf{H}_0\mathbf{x}_2$ are independent Gaussian variables with variances $h_1\rho_1 = |\mathbf{H}_1|^2\rho_1$ and $|\mathbf{x}_2|^2$, respectively and (b) results from the concavity of $\log$ function and Jensen inequality. As a result, the lower-bound on the outage probability can be expressed as

$$\mathscr{B}^L_{NCSIR} \gtrsim \rho^{1-\beta-r_1}. \tag{34}$$

From (32) and (34) it is concluded that

$$\Pr\{\mathscr{B}_{NCSIR}\} \doteq \rho^{1-\beta-r_1}, \quad r_1<1-\beta, \tag{35}$$

or equivalently,

$$d^*_{NCSIR}(r_1) = \max(1-\beta-r_1,0), \quad r_1\geq 0. \tag{36}$$

Hence, No-CSIR scenario can be considered as perfect CSIR scenario with $r_2 = 1$. Note that in No-CSIR scenario, β is limited to be strictly less than one (otherwise the diversity is always zero).

IV. Conclusion

In this paper, the diversity-multiplexing trade-off curve is characterized for Z-channel. The diversity-multiplexing gain of MAC is obtained assuming different scenarios for the SNR of the links and is interpreted in terms of the diversity-multiplexing gain of Z-channel. Moreover, the diversity-multiplexing trade-off curve for No-CSIR scenario is obtained.

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