Vector Precoding with MMSE for the Fast Fading and Quasi-Static Multi-User Broadcast Channel

Aaron Callard, Amir Khandani
E&CE Department, University of Waterloo
Email: aaron, khandani@shannon2.uwaterloo.ca

Aladdin Saleh
Wireless Technology, Bell Mobility
aomsaleh@Mobility.com

***Abstract*— Vector Precoding is arguably the best form of precoding for the multi-user Multiple Input Multiple Output (MIMO) broadcast channel. However, conventional Vector Precoding schemes are designed to minimize the transmit energy, which is suboptimal in terms of the received signals Mean Square Error (MSE). This paper proposes modifications to Vector Precoding to overcome this shortcoming. Improvements of about 2 dB are realizable in a fast fading environment. Also, conventional Vector Precoding schemes usually result in unbalanced levels of interference, resulting in a poor performance for some users. Noting the above, another improvement is proposed by directly minimizing the bit error rate rather than the MSE. This improvement adds another 1 dB of gain, resulting in an overall gain of 3 dB in a quasi static fading environment. These improvements are also applied to Tomlinson-Harashima Precoding with similar results.**

I. INTRODUCTION

The use of wireless technology is on the rise. As both the transmit power and the available bandwidth are severely limited, transmission schemes with higher spectral efficiency must be used.

One method for achieving this increase in the spectral efficiency is to use multiple antennas at the transmitter and/or receiver. Telatar [1] showed that the capacity of a wireless channel can be increased by a factor of $\min(N_t, N_r)$, where N_t and N_r are the number of transmit and receive antennas, respectively. Therefore, by adding additional antennas at both the transmitter and receiver, the capacity of the underlying wireless channel can be increased dramatically.

However, in most real world applications, the majority of data transmission will be from a central base station to resource-limited mobile units. These mobile units are often quite restricted in both size and complexity, which can make it difficult for multiple receive antennas to be located on such units. Therefore, we only consider the case of single antenna users in this article.

If we restrict ourselves to single antenna users, we can still achieve the capacity gain promised in [1], if we communicate to multiple users at once. Provided that the transmitter knows the channel, the sum capacity increases with the minimum of N_t and N, where N_t is the number of transmit antennas and N is the number of single antenna users [2]. In this paper, we only consider the scenario where the number of transmit antennas is equal to the number of single antenna users (similar to [3]). The methods presented here can be easily extended to the more general case.

Several practical methods are known for the multi-user Multiple Input Multiple Output (MIMO) broadcast channel. They are for the most part generalizations of the works of Tomlinson [4] and Harashima [5] [6]. At the heart of the Tomlinson-Harashima Precoder (THP) is the introduction of a simple modulo operation at the receiver. In effect, this operation extends the constellation periodically. This gives the transmitter an infinite number of possible symbols, all of which represent the same information. The difference between the various precoding schemes is how the transmitter exploits this flexibility to choose the transmitted symbol.

In THP, the symbols are chosen by a method of channel triangularization. The channel matrix is converted, using its LQ decomposition, into an equivalent triangular form. This means that the ith user will experience no interference from the signal sent to the jth user, $j < i$. By choosing the signals successively, the transmitter can decide which symbol to transmit without affecting the previous users received signal. Thus, the transmitter can calculate the interference each user would receive from the previous users, and transmit the symbol closest to that interference, effectively minimizing the transmit energy required for that particular symbol.

Vector Precoding [3], on the other hand, rather than looking at one user at a time, minimizes the transmit power over all the users simultaneously. For this purpose, Peel *et al* [3] employ a 'Sphere Encoder' to compute the transmit symbols, minimizing the energy with only a moderate increase in the complexity.

For many channels, it can be advantageous to allow for some interference among users, in return for a reduction in the transmit energy. To determine how much interference to allow, the most commonly used criterion is the Mean Square Error (MSE). This is the expected squared distance between the desired signal and some scaled version of the received signal. Suboptimal methods which attempt to minimize the MSE exist for both THP and Vector Precoding [3] [7]. These methods show some clear benefits at low to medium power levels in comparison with their zero forcing counterparts [3] [7].

However, the methods discussed in [3] [7] do not consider the impact of the selected symbols on the amount of the interference. This paper proposes modifications to these two methods so that the symbols are chosen to directly minimize the MSE, rather than the transmit power. This simple change

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achieves a 2 dB gain in the Bit Error Rate (BER), with a negligible increase in the complexity.

Conventional Vector Precoding schemes usually result in unbalanced levels of interference among the users. This becomes problematic in the case of quasi-static fading as the level of the interference remains the same for a long period of time. This causes a minority of users to have the majority of the interference. In the fast fading environment, this is not an issue as the different channel instances would favor different users, and on the average, each user would receive the same amount of the interference.

To prevent this unbalanced distribution of interference, we propose an alternate criterion based on directly minimizing the BER. This equalizes the BER performance between the different users. We assume that the BER is a function of the MSE experienced by the individual users. We further assume that this function is logarithmically very steep at the power levels under consideration. Thus, minimizing the BER is almost equivalent to minimizing the MSE for the worst user. An iterative procedure achieving this objective is proposed. Simulation results show that a gain of about 1 dB can be achieved for the worst user.

The paper is organized as follows; In Section II, we introduce our channel model and develop some notation which allows for a simple distinction of the useful signal from the interference. Section III develops transmission strategies to minimize the MSE for a given set of transmitted signals. In section IV, we examine how to modify Vector Precoding to minimize the MSE. Section V discuses how these modifications can be applied to THP. Section VI discusses transmission strategies for the quasi-static fading environment.

II. The Model

This paper deals with channels which can be represented by the equation

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{s} + \mathbf{n} \qquad (1) \\ s.t. \quad & E\{\|\mathbf{s}\|^2\} \leq \overline{P} \end{aligned}$$

$\mathbf{y}$ is a column vector with elements representing the received signal at each user, $\mathbf{s}$ represents the transmitted signals and has an average power constraint $\overline{P}$, $\mathbf{n}$ represents additive white Gaussian noise (AWGN) with covariance matrix $\sigma^2\mathbf{I}$, and $\mathbf{H}$ represents the channel matrix. We assume $\mathbf{H}$ to be square and invertible, although not necessarily well conditioned. This model can represent several practical systems such as Direct Sequence CDMA, the MIMO Broadcast Channel, or the Digital Subscriber Line (DSL) [8].

While this model is easy to build, it can be cumbersome to work with. By some slight manipulation, the transmitted signal can be divided up into useful information ($\mathbf{u}$) and interference (ϵ). By defining $\mathbf{s}$ to be $\mathbf{H}^{-1}(\mathbf{u}+\epsilon)$ and including the scaling factor γ, the channel can be rewritten without loss of generality as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{H}^{-1}\frac{\sqrt{P}}{\gamma}(\mathbf{u}+\epsilon) + \mathbf{n} \\ &= \frac{\sqrt{P}}{\gamma}(\mathbf{u}+\epsilon) + \mathbf{n} \\ \gamma &= \|\mathbf{H}^{-1}(\mathbf{u}+\epsilon)\| \\ s.t. \quad & E\{P\} \leq \overline{P} \end{aligned}$$

For the users to decode their respective signals, they must have knowledge of the scaling factor $\frac{\sqrt{P}}{\gamma}$. The simplest way for this to occur is for $\frac{\sqrt{P}}{\gamma}$ to remain constant for any value of $\overline{P}$ and $\mathbf{H}$. We have therefore assumed that the value $\frac{\sqrt{P}}{\gamma}$, defined as c henceforth, remains constant. It is shown in [3] that this assumption actually leads to enhanced performance in a fast fading environment.

Furthermore, if we make the reasonable assumption that the scheme performs better when more power is transmitted, we may remove the inequality and define c to be the largest value which satisfies the power constraint. Thus, the received signal can be written as

$$\begin{aligned} \mathbf{y} &= c(\mathbf{u}+\epsilon) + \mathbf{n} \qquad (2) \\ c &= \sqrt{\overline{P}/E\{\gamma^2\}} \\ \gamma &= \|\mathbf{H}^{-1}(\mathbf{u}+\epsilon)\| \end{aligned}$$

This channel model is exceptionally easy to work with, as it allows a clear distinction between the useful and detrimental portions of the signal. It also allows the solutions to the minimum MSE equations to be written simply, and without the need for the assumption of linearity.

For simulation purposes, we have assumed that the elements of $\mathbf{H}$ are independent unit mean complex Gaussian random variables. For the fast fading case, we assume that subsequent transmissions correspond to independent samples of $\mathbf{H}$. In the quasi static case, we assume that $\mathbf{H}$ remains constant throughout a block of transmissions, and is then replaced by a new independent realization.

III. Minimum Mean Square Error for the MIMO BC Channel

The MMSE approach aims to minimize the mean square distance between the received signal and some desired signal $\mathbf{u}$, subject to a given power constraint. This problem can be formulated as

$$\text{MSE} = E\left\{\left\|\frac{\mathbf{y}}{c} - \mathbf{u}\right\|^2\right\} \qquad (3)$$

$$\mathbf{y} = c(\mathbf{u}+\epsilon) + \mathbf{n} \qquad (4)$$

The only free variable in the above equation is ϵ, namely the vector representing the interference among the users, which should be selected to minimize the MSE. To do this, we first simplify (3) so that we can consider the MSE on a transmission by transmission basis. By substituting (2) into (3), we obtain

$$MSE = E\left\{\left\|\epsilon + \frac{\mathbf{n}}{c}\right\|^2\right\} \quad (5)$$

$$= E\{\|\epsilon\|^2\} + \frac{2N\sigma^2}{c^2} \quad (6)$$

$$= E\{\|\epsilon\|^2\} + \frac{2N\sigma^2}{P}E\{\gamma^2\} \quad (7)$$

$$= E\left\{\|\epsilon\|^2 + \alpha\|\mathbf{H}^{-1}(\mathbf{u}+\epsilon)\|^2\right\} \quad (8)$$

For ease of notation, we define α as $\frac{2N\sigma^2}{P}$. To optimize this function, we simply minimize the value inside the expectation, termed square error (SE). This can be done by simple vector calculus as shown below:

$$\frac{\partial SE}{\partial \epsilon} = 2\epsilon + 2\alpha(\mathbf{H}\mathbf{H}^H)^{-1}(\mathbf{u}+\epsilon) \quad (9)$$

$$\mathbf{0} = 2(\mathbf{I} + \alpha(\mathbf{H}\mathbf{H}^H)^{-1})\epsilon + 2\alpha(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{u} \quad (10)$$

$$\mathbf{0} = (\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I})\epsilon + \alpha\mathbf{u} \quad (11)$$

$$\epsilon = -\alpha(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I})^{-1}\mathbf{u} \quad (12)$$

Note that this minimization applies to both the fast and the quasi-static fading environments. Thus, given a set of symbols $\mathbf{u}$, the optimal choice (in the MSE sense) for a transmitted signal is

$$\mathbf{s} = c\mathbf{H}^{-1}(\mathbf{I} - \alpha(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I})^{-1})\mathbf{u} \quad (13)$$

By using the matrix inversion lemma [9], we can transform (13) into the following more recognizable form

$$\mathbf{s} = c\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}\right)^{-1}\mathbf{u} \quad (14)$$

The above matrix appears in many places under names such as the transmit Wiener Filter [10], or the MMSE detector [11].

IV. Vector Precoding with MMSE

The receiver for Vector Precoding is equipped with a modulo operator identical to the one used for THP. This means that any integer multiple of some constant τ can be added to the signal of any user without affecting the data. Thus, given a vector of original data symbols $\mathbf{u}_0$, any signal of the form

$$\mathbf{H}\mathbf{s} = c\mathbf{H}\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}\right)^{-1}(\mathbf{u}_0 + \tau\mathbf{p})$$

$$\mathbf{p} \in \left\{\mathbf{a} + i\mathbf{b} \,|\, \mathbf{a} \in \mathbf{Z}^N, \mathbf{b} \in \mathbf{Z}^N\right\}$$

represents the same data.

Vector Precoding [3] exploits this freedom by choosing the vector of integers $\mathbf{p}$ to minimize the transmit power γ^2. In other words,

$$\mathbf{p} = \underset{\mathbf{p}}{\operatorname{argmin}}\left\|\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}\right)^{-1}(\mathbf{u}_0 + \tau\mathbf{p})\right\|^2 \quad (15)$$

This results in a significant reduction in the transmit power required; however, it also results in a large increase in the amount of interference among the users. This is because the minimization of the transmit power does not consider the effect of vector $\mathbf{p}$ on the magnitude of ϵ. To compensate for the increase in interference, Peel et al. [3] reduce α from $\frac{2N\sigma^2}{P}$ to $\frac{\sigma^2}{5P}$, relying on computer simulations to justify this decision.

A better solution is to consider both the transmit power and the effect of the interference simultaneously. This is done by selecting $\mathbf{p}$ to minimize the MSE directly. This can be done without any increase in the complexity by a simplification of the expression for the MSE. If we substitute (12) into our original equation for MSE in (8), we can simplify the resulting expression by defining $\mathbf{A}$ as $\left(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}\right)^{-1}$ and $\mathbf{u}$ as $\mathbf{u}_0 + \tau\mathbf{p}$, resulting in

$$MSE = E\{\|\epsilon\|^2 + \alpha\gamma^2\} \quad (16)$$

$$= E\{\|\alpha\mathbf{A}\mathbf{u}\|^2 + \alpha\|\mathbf{H}^H\mathbf{A}\mathbf{u}\|^2\} \quad (17)$$

$$= E\{\mathbf{u}^H\left(\alpha^2\mathbf{A}^H\mathbf{A} + \alpha\mathbf{A}^H\mathbf{H}\mathbf{H}^H\mathbf{A}\right)\mathbf{u}\} \quad (18)$$

$$= E\{\mathbf{u}^H\left(\mathbf{A}^H\left(\alpha\mathbf{H}\mathbf{H}^H + \alpha^2\mathbf{I}\right)\mathbf{A}\right)\mathbf{u}\} \quad (19)$$

$$= E\{\alpha\mathbf{u}^H\mathbf{A}^H\mathbf{u}\} \quad (20)$$

$$= E\{\alpha\left\|\sqrt{\mathbf{A}}\mathbf{u}\right\|^2\} \quad (21)$$

As $\mathbf{A}$ is positive definite, the matrix square root always exits. The vector $\mathbf{p}$ is chosen to minimize the new quadratic equation for the MSE.

$$\mathbf{p} = \underset{\mathbf{p}}{\operatorname{argmin}}\left\|\sqrt{\left(\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}\right)}(\mathbf{u}_0 + \tau\mathbf{p})\right\|^2 \quad (22)$$

We have now proposed two improvements to the Vector Precoding method of [3]:

1) α is selected to satisfy the MMSE criterion, which is equal to $\alpha = \frac{2N\sigma^2}{P}$ rather than $\frac{\sigma^2}{5P}$.
2) $\mathbf{p}$ is selected to minimize the MSE, rather than the transmit power.

To quantify the improvement, we first compare the SINR of the two methods. Figure 1 shows the simulated SINR over the fast fading channel. Gains of several dB are achieved at low to medium SNR. These improvements in the SINR carry over to gains in the BER. In [3], a BER of 10^{-5} was found at a transmit SNR of 12 dB, looking at figure 1, we can see an improvement of 2 dB at this power level. Therefore, using this new method we would expect to achieve the same BER at an SNR of 10 dB. The results presented in figure 2 verify this statement.

Figure 2 compares the original vector perturbation method from [3] with this improved version. Rate 1/2 Turbo codes over 16 QAM modulation are used with feedback polynomial $(1 + D^2 + D^3)$ and feedforward polynomial $(1 + D + D^3)$ over blocklengths of 4000. The channel is simulated with four transmit antennas over fast fading environment.

V. THP as an Approximation to Vector Precoding

Vector Precoding is a very powerful tool; however, it can be very computationally expensive for large channel matrices. Large matrices occur when there are many users, DSL lines for instance, or when some dimensions of $\mathbf{H}$ are temporal and

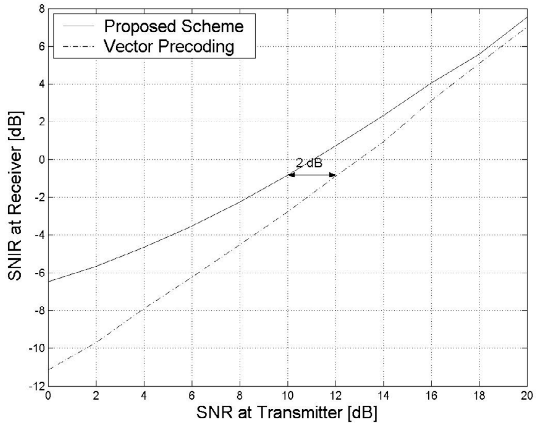

Fig. 1. Simulated SINR for Vector Precoding and the Proposed Scheme

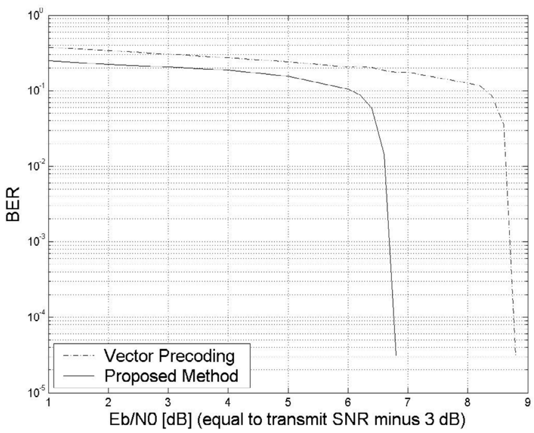

Fig. 2. BER for Turbo coding over the methods considered

$\mathbf{H}$ becomes the size of the entire transmission, such as when ISI is considered.

When the channel matrix is large, the THP precoding provides a good compromise between complexity and performance. It has a worst case complexity of $O(N^2)$ [8], which is significantly lower than the exponential worst case complexity of the sphere encoder [12] used for in Vector Precoding.

The algorithm for THP takes as inputs a matrix $\mathbf{B}$, symbols $\mathbf{y}$, constant τ, and outputs $\mathbf{s}$ such that $\|\mathbf{s}\|^2$ is small and $\mathbf{Bs}$ is equivalent to $\mathbf{y}$ modulo τ. To explain more thoroughly, consider the matrix $\mathbf{B}$, with LQ-decomposition $\mathbf{LQ}$ where $\mathbf{Q}$ is unitary, and $\mathbf{L}$ is lower triangular. Without affecting the power constraint, we can substitute $\mathbf{s}'$ for $\mathbf{Qs}$.

$$\mathbf{y} = \mathbf{Bs} \tag{23}$$
$$= \mathbf{LQQ}^T\mathbf{s}' \tag{24}$$
$$= \mathbf{Ls}' \tag{25}$$

In this way, y_k receives no signal, and hence no interference, from s'_l for $l > k$. Thus, s'_1 can be chosen to communicate to y_1, s'_2 chosen to communicate to y_2 while considering the affect of the interference from s'_1, and so on. At each stage, s'_k is chosen to transmit the symbol $u_k + \tau p_k$, accounting for the effect of the interference from the previously chosen s'_l. Therefore

$$s'_k = \mathbf{L}_{k,k}^{-1}\left(y_k - \sum_{l=1}^{k-1}\mathbf{L}_{k,l}s'_l + \tau p_k\right) \tag{26}$$

where τ is the width of the constellation, and

$$p_k = \text{round}\left(y_k - \sum_{l=1}^{k-1}\mathbf{L}_{k,l}s'_l\right)$$

The matrix given to the THP algorithm does not need to be the channel matrix $\mathbf{H}$. If instead, the transmit wiener filter (14) is the input, the output achieves better performance at low to medium power levels as one might expect [7]. This suffers from the same loss as Vector Precoding, as the interference caused by different symbols are not considered. This can be easily rectified by using the same method suggested earlier in conjunction with Vector Precoding. By giving the THP algorithm the matrix used to calculate the MSE, namely $\left(\sqrt{\mathbf{A}}\right)$, the output will reduce the MSE dramatically. Then, $\mathbf{s}$ is premultiplied by $\mathbf{H}^H\sqrt{\mathbf{A}}$ to determine the correct transmitted signal.

With this modification, gains comparable to those reported earlier for the case of Vector Precoding can be achieved. The only increase in the complexity is an additional matrix multiplication, and the calculation of a matrix square root.

This should not be surprising as the first iteration of the sphere encoder [12] is identical to the algorithm of THP, although the sphere encoder calculates p_k rather than $\mathbf{s}$. In this way, THP can be viewed as an approximate solution to (15), and thus any improvements suggested for Vector Precoding should be applicable to THP as well.

Figure 3 shows the simulated average SINR for the fast fading channel using THP with the transmit Wiener filter, compared to THP with the proposed modification. Equiprobable 16 QAM constellations are used. It is observed that significant gains can be achieved.

VI. Quasi-Static Fading

Vector Precoding, while effective, is not well suited to quasi-static fading. This is because the interference introduced by ϵ is not uniformly distributed across the users for any given $\mathbf{H}$. It is instead, often heavily concentrated in one or two users. This is not a problem in the fast fading environment as different instances of $\mathbf{H}$ favor different users, and consequently, the differences are equalized over different transmissions. However, in the quasi-static case, the users which receive more interference will dominate the overall BER.

To deal with this imbalance, we should change our goal from minimizing the MSE, to minimizing the overall BER. Let us assume that the BER of a particular user can be written as a continuous differentiable function F of the average MSE for that user. Defining ϵ_i and MSE_i to be the interference

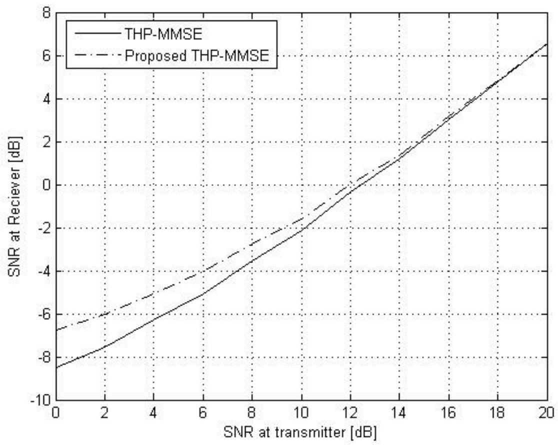

Fig. 3. The SNR for THP using the transmit Wiener Filter [7] and the Proposed Scheme

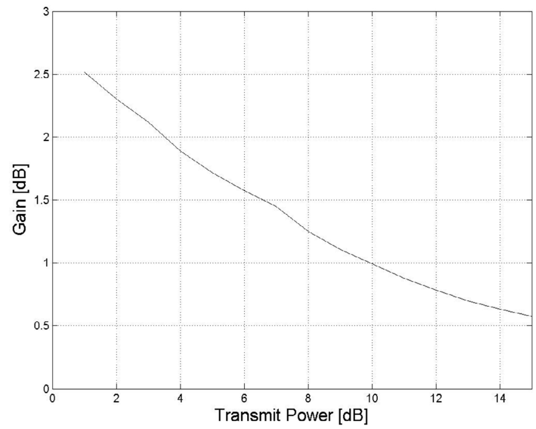

Fig. 4. The gain for the worst user for Equalizing Interference vs Transmit Power

and the MSE for user i, the sum of the BER for all the users can be written as

$$\text{BER} = \sum_{i=1}^{N} F(MSE_i) \quad (27)$$

$$MSE_i = E\{|\epsilon_i|^2\} + \frac{\sigma_n^2}{c^2} \quad (28)$$

This function is significantly more difficult to minimize than the MSE above. To minimize (27) with respect to ϵ, the first step is to take the derivative as

$$\begin{aligned} \frac{\partial}{\partial \epsilon}\text{BER} &= \sum_{i=1}^{N} F'(MSE_i)\frac{\partial}{\partial \epsilon} MSE_i \\ &= \sum_{i=1}^{N} F'(MSE_i)\frac{\partial}{\partial \epsilon}\left(|\epsilon_i|^2 + \frac{\sigma^2}{c^2}\right) \\ &= 2\mathbf{D}\epsilon + tr(\mathbf{D})\frac{\sigma_n^2}{\overline{P}}\frac{\partial}{\partial \epsilon}\gamma^2 \\ &= \mathbf{D}\epsilon + \frac{tr(\mathbf{D})}{2N}\alpha(\mathbf{H}\mathbf{H}^H)^{-1}(\mathbf{u}+\epsilon) \end{aligned}$$

We have defined $\mathbf{D}$ to be a diagonal matrix with $\mathbf{D}_{i,i} = F'(MSE_i)$, and $tr()$ to be the standard trace function. Solving this equation, we can write ϵ as a function of $\mathbf{D}$,

$$\epsilon = -\alpha(\mathbf{H}\mathbf{H}^H\frac{\mathbf{D}2N}{tr(\mathbf{D})} + \alpha\mathbf{I})^{-1}(\mathbf{u}+\epsilon) \quad (29)$$

It remains to find a matrix $\mathbf{D}$ which minimizes (27). Finding $\mathbf{D}$ exactly would be extremely complicated as, in general, there are no known closed form expressions for the BER. A more tractable problem is to solve for the matrix $\mathbf{D}$ which would result in an equal amount of interference for different users. This is equivalent to assuming that the BER will be dominated by the user with the worst MSE. Even with this assumption, the closed form solution to this problem is not tractable. In the following, we present a numerical algorithm for this purpose. Simulation results show that the proposed algorithm converges in two or three iterations.

A. Algorithm

initialize $\mathbf{D}_0$=**I**
repeat until desired accuracy is reached

1) Calculate $E\{\epsilon_i^2\}$
2) $\mathbf{D}_{i+1} = \sqrt{E\{\epsilon_i^2\}}\mathbf{D}_i$
3) Normalize $\mathbf{D}_{i+1} = \frac{\mathbf{D}_{i+1}2N}{\text{tr}(\mathbf{D}_{i+1})}$

The advantage of this method over an alternative method which balances the interference [13] is that it takes into account the structure of the nonlinear precoder proposed earlier. The vector **p** introduces correlation among the users, which negatively impacts the performance of [13].

To quantify the performance improvement, we again assume that the BER is dominated by the user with the worst MSE. This means that the gain achieved by the proposed method is equal to the amount of extra power required for the MSE of the worst user in the proposed method to be equal to the MSE of the worst user using the method proposed in section IV). Figure 4 shows the average multiplier required for the worst user, using Vector Precoding, to have the same MSE as the worst user using the new proposed method. The vector **p** remains unchanged as the minimum of (22). As it can be observed, the achieved gain decreases as the total transmit power increases. This is expected as less interference exists at higher power levels, and thus, the impact of the unbalanced interference decreases.

At a transmit power of 10 dB, the gain in figure fig:MMSErankings is approximately 1.0 dB. One would expect some of this gain to be lost due to the increased interference experienced by the other users. To examine this loss, we have simulated a quasi-static channel over block lengths of

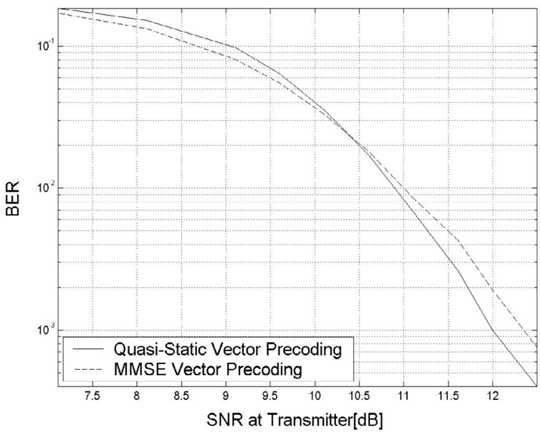

Fig. 5. BER vs SINR for the quasi static fading case for Vector Precoding

100 transmissions of 16-QAM using rate 1/2 Turbo codes with feedback polynomial $(1 + D^2 + D^3)$ and feedforward polynomial $(1 + D + D^3)$.

As can be seen, the entire gain is not realized, with a 0.5 dB gain appearing only at high power levels. This can be misleading. If each transmission block is considered individually, the full gain does appear at the waterfall region. Note that the new method performs worse prior to the waterfall region. On the other hand, the waterfall region appears at different power levels for different iterations of $\mathbf{H}$, and consequently, the BER when averaged over many different iterations appears worse until high power levels. By applying power control more of the gain should be realizable.

VII. Conclusion

By considering the MSE directly when choosing the symbols to transmit, we can achieve significant improvements at low to medium power levels, with a marginal increase in the complexity. This is true for both the Vector Precoding and THP scenarios considered. Using the capacity results reported in [3], this is now only 2.8 dB away from the capacity of the fast fading Channel. The proposed methods are also applicable to other precoding methods, such as Lattice Reduction [14] or Trellis Precoding [15] [16].

This improvement can be further increased for the quasi-static channel by considering the BER rather than the MSE. Gains of 1 to 2 dB can be achieved depending on the power level. However, this technique is only useful when powerful coding schemes are used which can compensate for the increased BER of the other users. The proposed techniques can be easily applied to cases where there exists some channel estimation error. Using the MSE techniques above and adding a term accounting for the channel estimation error, we arrive at the same form for the final solution (the only difference is the value of α). As channel estimation error exists at all power levels, it is expected that the resulting benefits would be observed at all power levels.

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