Scheduling and Codeword Length Optimization in Time Varying Wireless Networks

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Abstract*— In this paper, a downlink scenario in which a single-antenna base station communicates with K single antenna users, over a time-correlated fading channel, is considered. It is assumed that channel state information is perfectly known at each receiver, while the statistical characteristics of the fading process and the fading gain at the beginning of each frame are known to the transmitter. By evaluating the random coding error exponent of the time-correlated fading channel, we show that there is an optimal codeword length which maximizes the throughput. We examine the throughput of conventional scheduling that transmits to the user with the maximum signal to noise ratio using both fixed length codewords and variable length codewords. Although optimizing the codeword length improves the performance, it is shown that using the conventional scheduling, a gap of $\Omega(\sqrt{\log\log\log K})$ exists between the achievable throughput and the maximum possible throughput of the system. We propose a simple scheduling that considers both the signal to noise ratio and the channel time variation. In this case, among users which their fading gain is above a threshold, user that has minimum channel time variation is selected. We show that by using this scheduling, the gap between the achievable throughput and maximum throughput of the system approaches $o(1)$.

I. Introduction

In wireless networks, diversity is a means to combat the time varying nature of the communication link. Conventional diversity techniques over point-to-point links, such as spatial diversity and frequency diversity, offer performance improvements. In multiuser wireless systems, there exists another form of diversity, called *multiuser diversity* [1]. In a broadcast channel where users have independent fading and feedback their Signal to Noise Ratio (SNR) to the Base Station (BS), system throughput is maximized by transmitting to the user with the strongest SNR.

Multiuser diversity was introduced first by Knopp and Humblet [2]. It is shown that the optimal transmission strategy in the uplink of multiuser system using power control is to only let the user with the largest SNR transmit. A similar result is shown to be valid for the downlink [3]. Multiuser diversity underlies much of the recent works for downlink scheduling [4]–[7]. The opportunistic transmission is proposed in Qualcomm's High Data Rate (HDR) system [4]. In [5], [7], the scheduling is based on the achievable data rate reported by users to the BS. Distributed scheduling is proposed in an uplink scenario, where full Channel State Information (CSI) is not required at the transmitter [8], [9].

Multiuser diversity is extended as an opportunistic downlink scheduling for multiple antenna systems [1]. Also, multiuser diversity has been studied in the context of ad-hoc networks [10].

In wireless networks, the rate of channel variations is characterized by maximum Doppler frequency which is proportional to the velocity. Utilizing multiuser diversity in such an environment needs to be revisited since the throughput depends not only on the received SNR, but also on how fast the channel varies over time.

In this paper, we consider a broadcast channel in which a BS transmits data to a large number of users in a time correlated flat fading environment. It is assumed that the Channel State Information (CSI) is perfectly known to the receivers, while BS only knows the statistical characteristics of the fading process for all the users (which is assumed to be constant during a long period). Moreover, each user feeds back its channel gain to the BS at the beginning of each frame. Based on this information, BS selects one user for transmission in each frame, in order to maximize the throughput. For the case of Additive White Gaussian Noise (AWGN) or block fading, it is well known that increasing the codeword length results in improving the achievable throughput. However, in a time varying channel, it is not possible to obtain arbitrary small error probabilities by increasing the codeword length. In fact, increasing the codeword length also results in increasing the fading fluctuations over the frame, and consequently, the throughput will decrease. Therefore, it is of interest to find the optimum codeword length which maximizes the throughput.

In this paper, a downlink scenario in which a single-antenna base station communicates with K single antenna users, over a time-correlated fading channel, is considered. We analyze different user selection strategies; i) the BS transmits data to the user with the strongest SNR using fixed length codewords (conventional multiuser scheduling), ii) the BS transmits data to the user with the strongest SNR using variable length codewords, and iii) the BS transmits data to the user that achieves the maximum throughput using variable length codewords. We show that in all cases the achievable throughput scales as $\log\log K$. Moreover, in cases (i) and (ii), the gap between the achievable throughput and the maximum throughput scales as

$\sqrt{\log\log\log K}$, while in case (iii), this gap behaves like $o(1)$.

The rest of the paper is organized as follows. In Section II, the model of time correlated fading channel is described. In Section III, different user selection strategies are discussed and the corresponding throughput of the system is derived for each strategy, for $K \to \infty$. Section **??** is devoted to the delay analysis of the system for each strategy. Finally, in Section IV, we conclude the paper.

Throughout this paper, $\mathbb{E}\{.\}$ and var$\{.\}$represents the expectation and variance, respectively, "log" is used for the natural logarithm, rate is expressed in *nats*. For any functions $f(N)$ and $g(N)$, $f(N) = O(g(N))$ is equivalent to $\lim_{N\to\infty}\left|\frac{f(N)}{g(N)}\right| < \infty$, $f(N) = o(g(N))$ is equivalent to $\lim_{N\to\infty}\left|\frac{f(N)}{g(N)}\right| = 0$, $f(N) = \omega(g(N))$ is equivalent to $\lim_{N\to\infty}\frac{f(N)}{g(N)} = \infty$, and $f(N) = \Omega(g(N))$ is equivalent to $\lim_{N\to\infty}\frac{f(N)}{g(N)} = c$, where $0 < c < \infty$.

II. System model

The channel of any given user is modeled as a time-correlated fading process. It is assumed that the channel gain is constant over each channel use (symbol) and varies from symbol to symbol, following a Markovian random process. Assume the fading gain of user k is $\boldsymbol{h}_k = [h_{1,k}, \ldots, h_{N_k,k}]^T$ where $h_{i,k}, 1 \le i \le N_k$ are complex Gaussian random variables with zero mean and unit variance and N_k is the codeword length of user k. The received signal for the k^{th} user is given by

$$\boldsymbol{r}_k = \boldsymbol{S}_k\boldsymbol{h}_k + \boldsymbol{n}_k, \tag{1}$$

where $\boldsymbol{S}_k = \text{diag}(s_{1,k}, s_{2,k}, \ldots, s_{N_k,k})$ is the transmitted codeword with the power constraint[1] $\mathbb{E}\{|s_{i,k}|^2\} \le P$, $\boldsymbol{n}_k$ is AWGN with zero mean and covariance matrix $\boldsymbol{I}$. Assume $h_{0,k}$ is the fading gain at the time instant before $\boldsymbol{S}_k$ is transmitted. The sequence $u_{i,k} = |h_{i,k}|$, $0 \le i \le N_k$, is assumed to be a stationary ergodic chain with the following probability density function [11]:

$$f_{u_{0,k}}(u) = \begin{cases} 2ue^{-u^2} & u \ge 0 \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

$$f(u_{1,k}, u_{2,k}, \cdots, u_{N_k,k}|u_{0,k}) = \prod_{i=1}^{N_k} q_k(u_{i,k}|u_{i-1,k}), \tag{3}$$

where,

$$q_k(u|v) = \begin{cases} \frac{2u}{1-\alpha_k^2}\exp\left(-\frac{u^2+\alpha_k^2v^2}{1-\alpha_k^2}\right)\mathcal{I}_0(\frac{2\alpha_k uv}{1-\alpha_k^2}) & u \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

in which $0 < \alpha_k < 1$ describes the channel correlation coefficient for user k. It is assumed that α_k, $1 \le k \le K$, are i.i.d random variables which remain fixed during the whole transmission, and $\mathcal{I}_0(.)$ denotes the modified Bessel function of order zero.

[1]Obviously, for maximizing throughput, the energy constraint translates to $\mathbb{E}\{|s_{i,k}|^2\} = P$.

It is assumed that the CSI is perfectly known at each receiver, while the statistical characteristics of the fading process and $u_{0,k}$, $1 \le k \le K$ are known to the transmitter.

III. Throughput Analysis

In this section, we derive the achievable throughput of the system in the asymptotic case of $K \to \infty$. We define the user k's throughput per channel use, denoted by T_k, as

$$T_k \triangleq R_k(1 - p_e(k)), \tag{4}$$

where R_k is the transmitted rate per channel use and $p_e(k)$ is the decoding error probability for this user. Using the concept of random coding exponent [12], $p_e(k)$ can be upper-bounded as

$$p_e(k) \le \inf_{0\le\rho\le1} e^{-N(E_k(\rho)-\rho R_k)}. \tag{5}$$

For simplicity of analysis, we use this upper-bound in evaluating the throughput. This bound is tight for rates close to the capacity as used in [13]–[15].

Assuming $s_{i,k}, 1 \le i \le N_k$, are Gaussian and i.i.d., it is shown that the random coding error exponent for user k, $E_k(\rho)$, is given by [15],

$$E_k(\rho) = -\frac{1}{N_k}\log\mathbb{E}_{\boldsymbol{u}_k}\left\{\prod_{i=1}^{N_k}\left(\frac{1}{1+\frac{P}{1+\rho}u_{i,k}^2}\right)^{\rho}\right\}. \tag{6}$$

where $\boldsymbol{u}_k = [u_{1,k}, \ldots, u_{N_k,k}]$.

In the following, we assume that $u_{0,k} \gg 1$. Since in strategies introduced in this work, a user is selected if the corresponding initial fading gain is maximum or above a certain threshold, this assumption is valid when the number of users is large.

Theorem 1 *For the channel model described in the previous section, and assuming $u_{0,k}$ is known, we have*

$$E_k(\rho) = \frac{1}{N_k}\sum_{i=1}^{N_k}\rho\log\left(1+\frac{Pu_{0,k}^2\alpha_k^{2i}}{(1+\rho)}\right) + O\left(\frac{1}{\sqrt{u_{0,k}}}\right). \tag{7}$$

Proof: Refer to [16]

Minimizing (5) is equivalent to maximize $E_k(\rho) - \rho R_k$. Noting (7), for large values of $u_{0,k}$ we have

$$\begin{aligned} E_k(\rho) - \rho R_k &= \rho\log(\frac{Pu_0^2}{\rho+1}) \\ &+ \rho(N_k+1)\log(\alpha_k) - \rho R_k. \end{aligned} \tag{8}$$

It is easy to show that ρ_k^{opt} which maximizes (8) is

$$\begin{aligned} \log(1+\rho_k^{\text{opt}}) + \tfrac{\rho_k^{\text{opt}}}{1+\rho_k^{\text{opt}}} = \beta_k, &\quad \beta_k < \log(2) + \tfrac{1}{2} \\ \rho_k^{\text{opt}} = 1, &\quad \beta_k \ge \log(2) + \tfrac{1}{2} \end{aligned} \tag{9}$$

where

$$\beta_k = \log(Pu_{0,k}^2) + (N_k+1)\log(\alpha_k) - R_k. \tag{10}$$

Using (4), (5), and (7), we have

$$T_k = R_k \left[1 - e^{-\rho_k^{\text{opt}} N_k \left(\log(\frac{P u_{0,k}^2}{\rho_k^{\text{opt}}+1}) + (N_k+1)\log(\alpha_k) - R_k \right)} \right]. \quad (11)$$

It is easy to show that T_k is a convex function of variables R_k and N_k, and the values of R_k and N_k which maximize the throughput (R_k^{opt} and N_k^{opt}) satisfy the following equations:

$$R_k^{\text{opt}} = \log(\frac{P u_{0,k}^2}{\rho_k^{\text{opt}}+1}) + (2N_k^{\text{opt}}+1)\log(\alpha_k), \quad (12)$$

$$N_k^{\text{opt}} = \sqrt{\frac{\log\left(1+\rho_k^{\text{opt}} N_k^{\text{opt}} R_k^{\text{opt}}\right)}{\rho_k^{\text{opt}} \log(\alpha_k^{-1})}}. \quad (13)$$

It follows that $N_k^{\text{opt}} \to \infty$ and $R_k^{\text{opt}} \to \infty$ as $u_{0,k} \to \infty$. Substituting (12) in (10), we have

$$\beta_k = N_k^{\text{opt}} \log(\alpha_k^{-1}) + \log(\rho_k^{\text{opt}}+1). \quad (14)$$

From (9) and (14), it is concluded that

$$\rho_k^{\text{opt}} = \begin{cases} \frac{N_k^{\text{opt}} \log(\alpha_k^{-1})}{1-N_k^{\text{opt}} \log(\alpha_k^{-1})} & N_k^{\text{opt}} \log(\alpha_k^{-1}) < \frac{1}{2} \\ 1 & N_k^{\text{opt}} \log(\alpha_k^{-1}) \geq \frac{1}{2} \end{cases} \quad (15)$$

Noting (12) and (15), for $\alpha_k = 1$, we have $\rho_k^{\text{opt}} = 0$ and $R_k^{\text{opt}} = \log(P u_{0,k}^2)$ which corresponds to the capacity of quasi-static fading channel.

We obtain the corresponding throughput of the system for two cases. If α_k is fixed and $\alpha_k \neq 1$, $N_k^{\text{opt}} \log(\alpha_k^{-1}) \geq \frac{1}{2}$ for large values of $u_{0,k}$. Consequently, noting (15), we have $\rho_k^{\text{opt}} = 1$. In this case, the corresponding throughput is obtained by substituting (12), (13) in (11) as follows:

$$\begin{aligned} T_k &= \log\left(\frac{P u_{0,k}^2}{2}\right) - 2\sqrt{\log(\alpha_k^{-1}) \log\log\left(\frac{P u_{0,k}^2}{2}\right)} \times \\ &\quad \left(1 + O\left(\frac{\log\log\log(u_{0,k})}{\log\log(u_{0,k})}\right)\right). \end{aligned} \quad (16)$$

Assuming $\alpha_k \to 1$ such that $N_k^{\text{opt}} \log(\alpha_k^{-1}) \ll 1$, we derive the corresponding throughput as follows:

$$\begin{aligned} T_k &= \log(P u_{0,k}^2) - 2\sqrt[3]{\log(\alpha_k^{-1}) \log\log\left(P u_{0,k}^2\right)} \times \\ &\quad \left(1 + O\left(\frac{\log\log\log(u_{0,k})}{\log\log(u_{0,k})}\right)\right) - o(1). \end{aligned} \quad (17)$$

From the above equations, it is concluded that the throughput not only depends on the initial fading gain, $u_{0,k}$, but also on the fading correlation coefficient. Moreover, throughput is an increasing function of the channel correlation coefficient.

In the following, we introduce three scheduling strategies in order to maximize the throughput; i) Traditional scheduling in which the user with the largest channel gain (SNR-based scheduling) is selected and the codeword length is assumed to be fixed. ii) SNR-based scheduling with optimized codeword length regarding the channel condition of the selected user, and iii) Scheduling which exploits both the channel gain and channel correlation coefficient of the users. The asymptotic throughput of the system is derived under each strategy for $K \to \infty$.

A. Strategy I: SNR-based scheduling with fixed codeword length

The BS transmits to the user with the maximum initial fading gain and $N_1 = N_2 = \cdots = N_K = N$ while selecting the data rate to maximize the throughput of the selected user. The following theorem gives the throughput of the system under this scheduling.

Theorem 2 *The asymptotic throughput of the system under Strategy I scales as*

$$\mathcal{T}_1 \sim \log\left(\frac{P\log K}{2}\right) - 2\sqrt{\mathbb{E}\{\log(\alpha^{-1})\}}\sqrt{\log\log(\frac{P\log K}{2})}, \quad (18)$$

as $K \to \infty$.

Proof: For simplicity of notation, we define $\upsilon_k \triangleq u_{0,k}^2$. Let $\upsilon = \max_{1\leq k\leq K} \upsilon_k$ and α be the corresponding correlation coefficient of the selected user. Setting the derivative of (11) to zero, we find the rate of the selected user as follows:

$$R = \log\left(\frac{P\upsilon}{1+\rho^{\text{opt}}}\right) + (N+1)\log(\alpha) - \frac{\log(1+\rho^{\text{opt}} N R)}{\rho^{\text{opt}} N}. \quad (19)$$

It is easy to show that $\rho^{\text{opt}} = 1$ when K is large enough. Substituting (19) in (11),

$$\begin{aligned} \mathcal{T}_1(\upsilon,\alpha) &= \left[\log\left(\frac{P\upsilon}{2}\right) + (N+1)\log(\alpha) - \frac{\log(1+NR)}{N}\right] \times \\ &\quad \left[1 - \frac{1}{1+NR}\right], \end{aligned} \quad (20)$$

where $\mathcal{T}_1(\upsilon,\alpha)$ is the system throughput, conditioned on υ and α. It is easy to see that $p_\upsilon(x) = e^{-x}u(x)$. Noting that $\Pr\{\upsilon \sim \log(K) + O(\log\log K)\} \to 1$ as K tends to infinity [17], we compute the throughput of the system as follows:

$$\begin{aligned} \mathcal{T}_1 &= \mathbb{E}\{\mathcal{T}_1(\upsilon,\alpha)\} \\ &= \log\left(\frac{P\log K}{2}\right) + (N+1)\mathbb{E}\{\log(\alpha)\} \\ &\quad - \frac{\log\log\left(\frac{P\log K}{2}\right)}{N} - \frac{\log N}{N} + O\left(\frac{\log\log\log K}{\log\log K}\right) \end{aligned} \quad (21)$$

The codeword length N is computed such that the system throughput achieved in (21) is maximized. Setting the derivative of (21) with respect to N to zero, we show that the maximizing value of N, namely N^{opt}, scales as

$$N^{\text{opt}} \sim \sqrt{\frac{\log\log\left(\frac{P\log K}{2}\right)}{\mathbb{E}\{\log(\alpha^{-1})\}}}. \quad (22)$$

The proof is completed by substituting (22) in (21). ■

B. Strategy II: SNR-based scheduling with adaptive codeword length

In this scheme, the BS transmits to the user with the maximum initial fading gain. The rate and codeword length are selected to maximize the corresponding throughput.

Theorem 3 *Assuming $K \to \infty$, the asymptotic throughput of the system under Strategy II scales as follows:*

$$\mathcal{T}_2 \sim \log\left(\frac{P\log K}{2}\right) - 2\mathbb{E}\{\sqrt{\log(\alpha^{-1})}\}\sqrt{\log\log\left(\frac{P\log K}{2}\right)}. \tag{23}$$

Proof: The throughput of the system can be written as

$$\mathcal{T}_2 = \mathcal{T}_{2,\mathcal{M}}\Pr\{\mathcal{M}\} + \mathcal{T}_{2,\mathcal{M}^C}\Pr\{\mathcal{M}^C\}, \tag{24}$$

where $\mathcal{M}$ represents the event that $\rho^{\text{opt}} = 1$, $\mathcal{T}_{2,\mathcal{M}}$ denotes the throughput conditioned on $\mathcal{M}$, and $\mathcal{T}_{2,\mathcal{M}^C}$ is the throughput of system, conditioned on $\mathcal{M}^C$, the complement of $\mathcal{M}$. Using (16), we can write

$$\begin{aligned}\mathcal{T}_{2,\mathcal{M}} &= \mathbb{E}\{\log\left(\frac{P\upsilon}{2}\right) - 2\sqrt{\log(\alpha^{-1})\log\log\left(\frac{P\upsilon}{2}\right)} \\ &\quad \times\left(1 + O\left(\frac{\log\log\log(\upsilon)}{\log\log(\upsilon)}\right)\right)|\mathcal{M}\},\end{aligned} \tag{25}$$

where $\upsilon = \max_{1\le k\le K}\upsilon_k$, and α is the channel correlation coefficient of the selected user. Noting that $\upsilon \sim \log K + O(\log\log K)$, with probability one, and υ and α are independent, we have

$$\begin{aligned}\mathcal{T}_{2,\mathcal{M}} &\sim \log\left(\frac{P\log K}{2}\right) - 2\mathbb{E}\left\{\sqrt{\log(\alpha^{-1})}\Big|\mathcal{M}\right\}\times \\ &\quad\sqrt{\log\log\log K}\left(1 + O\left(\frac{\log\log\log\log K}{\log\log\log K}\right)\right)\end{aligned} \tag{26}$$

Using (13) and (15), we can write

$$\begin{aligned}\mathcal{M} &\equiv N^{\text{opt}}\log(\alpha^{-1}) \ge \frac{1}{2} \\ &\equiv \sqrt{\log(\alpha^{-1})}\sqrt{\log(1+NR)} \ge \frac{1}{2} \\ &\cong \sqrt{\log(\alpha^{-1})}\sqrt{\log\log\log K} \ge \frac{1}{2}.\end{aligned} \tag{27}$$

Assuming uniform distribution for α, $X \triangleq \log(\alpha^{-1})$ follows the exponential distribution, i.e., $f_X(x) = e^{-x}u(x)$. Hence,

$$\begin{aligned}\mathbb{E}\left\{\sqrt{\log(\alpha^{-1})}\Big|\mathcal{M}\right\} &= \frac{\int_{\mathcal{M}}\sqrt{x}e^{-x}dx}{\Pr\{\mathcal{M}\}} \\ &\sim \frac{\int_{\epsilon}^{\infty}\sqrt{x}e^{-x}dx}{\Pr\{\log(\alpha^{-1}) \ge \epsilon\}} \\ &\sim \mathbb{E}\{\sqrt{\log(\alpha^{-1})}\}(1+O(\epsilon)),\end{aligned} \tag{28}$$

where $\epsilon \triangleq \frac{1}{4\log\log\log K}$.

Using (13) and (15), $\mathcal{T}_{2,\mathcal{M}^C}$ can be written as

$$\begin{aligned}\mathcal{T}_{2,\mathcal{M}^C} &= \mathbb{E}\left\{\log\left(P\upsilon[1 - N^{\text{opt}}\log(\alpha^{-1})]\right)\Big|\mathcal{M}^C\right\} - \\ &2\mathbb{E}\left\{\sqrt{\frac{[1-N^{\text{opt}}\log(\alpha^{-1})]\log(1+\rho^{\text{opt}}N^{\text{opt}}R^{\text{opt}})}{N^{\text{opt}}}}\Bigg|\mathcal{M}^C\right\}.\end{aligned} \tag{29}$$

Having the fact that $\mathcal{M}^C \equiv N^{\text{opt}}\log(\alpha^{-1}) < \frac{1}{2}$, we have

$$\begin{aligned}N^{\text{opt}} &= \sqrt{\frac{\log(1+\rho^{\text{opt}}N^{\text{opt}}R^{\text{opt}})}{\rho^{\text{opt}}\log(\alpha^{-1})}} \\ &\ge \sqrt{\frac{2\log(1+\rho^{\text{opt}}N^{\text{opt}}R^{\text{opt}})N^{\text{opt}}}{\rho^{\text{opt}}}} \\ &\ge \sqrt{2\log(1+\rho^{\text{opt}}N^{\text{opt}}R^{\text{opt}})N^{\text{opt}}} \\ \Rightarrow N^{\text{opt}} &\ge 2\log(1+\rho^{\text{opt}}N^{\text{opt}}R^{\text{opt}}).\end{aligned} \tag{30}$$

Substituting (30) in (29), and noting $\upsilon \sim \log K + O(\log\log K)$, with probability one, yields,

$$\mathcal{T}_{2,\mathcal{M}^C} \gtrsim \log(P\log K) - \log(2) - 2. \tag{31}$$

Moreover, we have

$$\mathcal{T}_{2,\mathcal{M}^C} \lesssim \log(P\log K). \tag{32}$$

Combining (26), (28), (31), and (32), and substituting in (24), yields the result of Theorem 3.

■

Remark 1- Since $\mathbb{E}\{\sqrt{x}\} \le \sqrt{\mathbb{E}\{x\}}$, for $x > 0$, it is concluded that the achievable rate of Strategy II is higher than that of Strategy I. More precisely,

$$\begin{aligned}\mathcal{T}_2 - \mathcal{T}_1 &\sim 2\left(\sqrt{\mathbb{E}\{\log(\alpha^{-1})\}} - \mathbb{E}\{\sqrt{\log(\alpha^{-1})}\}\right) \\ &\quad\times\sqrt{\log\log\log K}.\end{aligned} \tag{33}$$

For the case of uniform distribution for α, we have

$$\mathcal{T}_2 - \mathcal{T}_1 \sim 0.228\sqrt{\log\log\log K}. \tag{34}$$

Remark 2- Although $\lim_{K\to\infty}\frac{\mathcal{T}_1}{\mathcal{T}_{\max}} = \lim_{K\to\infty}\frac{\mathcal{T}_2}{\mathcal{T}_{\max}} = 1$, where $\mathcal{T}_{\max} \sim \log(P\log K)$ is the maximum achievable throughput for a quasi-static fading channel [17], there exists a gap of $\Omega(\sqrt{\log\log\log K})$ between the achievable throughput of Strategies I and II, and the maximum throughput. As we show later, this gap is due to the fact that the channel correlation coefficients of the users are not considered in the scheduling. In fact, this gap approaches $o(1)$ by exploiting the channel correlation, which is discussed in Strategy III.

C. Strategy III: Scheduling based on both SNR and channel correlation coefficient with adaptive codeword length

To maximize the throughput of the system, the user which maximizes the expression in (16) should be serviced. Here, for simplicity of analysis, we propose a sub-optimum scheduling that considers the effect of both SNR and channel correlation in the user selection. In this strategy, each user is required to feed back its initial fading gain only if it is greater than a pre-determined threshold $\sqrt{\Theta}$, where Θ is a function of the number of users. Among these users, the BS selects the one with the maximum channel correlation coefficient. The data rate and codeword length are selected to maximize the corresponding throughput. The following theorem gives the system throughput under this strategy.

Theorem 4 *Let α_k, $k = 1, \cdots, K$, be i.i.d. random variables with uniform distribution. Using Strategy III, with Θ satisfying*

$$\log K - o(\log K) \lesssim \Theta \lesssim \log K - \log\log\log K - \omega(1), \quad (35)$$

the throughput of the system scales as

$$\mathcal{T}_3 \quad \gtrsim \quad \log(P \log K) - o(1) \quad (36)$$

Proof- Define $\mathcal{A} \triangleq \{k | \upsilon_k \geq \Theta\}$ and $\alpha_{\max} \triangleq \max_{k \in \mathcal{A}} \alpha_k$. Let υ be the squared initial fading gain of the user corresponding to α_{max}. We define the event $\mathcal{G}$ as follows:

$$\mathcal{G} = \Pr\{N^{\text{opt}} \log(\alpha_{\max}^{-1}) \sim o(1)\}, \quad (37)$$

where N^{opt} is the corresponding codeword length and computed from (13). Using (17) and (37), we can write

$$\mathcal{T}_3 \geq \Pr\{\mathcal{G}\} \mathbb{E}\{\mathcal{T}_3(\upsilon, \alpha_{\max})\} \quad (38)$$

where following (17),

$$\begin{aligned}\mathcal{T}_3(\upsilon, \alpha_{\max}) \quad = \quad & \log(P\upsilon) - 2\sqrt[3]{\log(\alpha_{\max}) \log\log(P\upsilon)} \times \\ & \left(1 + O\left(\frac{\log\log\log(\upsilon)}{\log\log(\upsilon)}\right)\right) - o(1). \quad (39)\end{aligned}$$

Noting that $\mathcal{T}_3(\upsilon, \alpha_{\max})$ in (39) is an increasing function of υ, we have

$$\begin{aligned}\mathbb{E}\{ \ \mathcal{T}_3(\upsilon, \alpha_{\max})\} \geq & \ \mathbb{E}\left\{\log(P\upsilon) - 2\sqrt[3]{\log(\alpha_{\max}^{-1})}\right. \\ & \left.\sqrt[3]{\log\log(P\upsilon)}\left[1 + O\left(\frac{\log\log\log(\upsilon)}{\log\log(\upsilon)}\right)\right]\right\} \\ \geq & \log(P\Theta) - 2\mathbb{E}\{\sqrt[3]{\log(\alpha_{\max}^{-1})}\} \\ & \sqrt[3]{\log\log(P\Theta)}\left[1 + O\left(\frac{\log\log\log(\Theta)}{\log\log(\Theta)}\right)\right] \\ \geq & \log(P\Theta) - 2\sqrt[3]{\mathbb{E}\{\log(\alpha_{\max}^{-1})\}} \\ & \sqrt[3]{\log\log(P\Theta)}\left[1 + O\left(\frac{\log\log\log(\Theta)}{\log\log(\Theta)}\right)\right] \quad (40)\end{aligned}$$

$\mathbb{E}\{\log(\alpha_{max}^{-1})\}$ can be derived as follows

$$\mathbb{E}\{\log(\alpha_{\max}^{-1})\} = \sum_{n=1}^{K} \mathbb{E}\{\log(\alpha_{\max}^{-1}) \big| \, |\mathcal{A}| = n\} \Pr\{|\mathcal{A}| = n\} \quad (41)$$

Since α_k, $k = 1, \cdots, K$, are i.i.d. random variables with uniform distribution over $[0, 1]$, we can write

$$\begin{aligned}F_{\alpha_{\max}}(\alpha \, ||\mathcal{A}| = n) \quad &= \quad \alpha^n \\ \Rightarrow \mathbb{E}\{\log(\alpha_{\max}^{-1}) \big| \, |\mathcal{A}| = n\} \quad &= \quad \int_0^1 \log(\alpha^{-1}) n \alpha^{n-1} d\alpha \\ &= \quad \frac{1}{n}, \quad (42)\end{aligned}$$

where $F_X(.)$ denotes the cumulative density function of the random variable X. Indeed, $|\mathcal{A}|$ is a binomial random variable with parameters K and $e^{-\Theta}$. (Note that $\Pr(\upsilon_k \geq \Theta) = e^{-\Theta}$.) Hence,

$$\Pr\{|\mathcal{A}| = n\} = \binom{K}{n} e^{-n\Theta} (1 - e^{-\Theta})^{K-n} \quad (43)$$

Substituting (42) and (43) in (41), we have

$$\mathbb{E}\{\log(\alpha_{\max}^{-1})\} = \sum_{n=1}^{K} \binom{K}{n} \frac{1}{n} e^{-n\Theta} (1 - e^{-\Theta})^{K-n}. \quad (44)$$

After some manipulations, we obtain

$$\mathbb{E}\{\log(\alpha_{\max}^{-1})\} = \sum_{n=1}^{K} \frac{1}{n} (1 - e^{-\Theta})^{K-n} - (1 - e^{-\Theta})^K \sum_{n=1}^{K} \frac{1}{n}. \quad (45)$$

For large values of K, we can approximate (45) as

$$\begin{aligned}\mathbb{E}\{\log(\alpha_{\max}^{-1})\} \quad \backsimeq \quad & \frac{1}{Ke^{-\Theta}}\left(1 + O\left(\frac{1}{Ke^{-\Theta}}\right)\right) \\ + \quad & e^{-Ke^{-\Theta}} (\Theta - \log K). \quad (46)\end{aligned}$$

Noting (40) and (46), for values of Θ satisfying

$$\log K - o(\log K) \lesssim \Theta \lesssim \log K - \log\log\log K - \omega(1), \quad (47)$$

we have

$$\mathbb{E}\{\mathcal{T}_3(\upsilon, \alpha_{\max})\} \geq \log(P \log K) - o(1). \quad (48)$$

We use

$$\Pr\left\{|z - \mathbb{E}\{z\}| < \sqrt{\log\log\log K \text{var}\{z\}}\right\} > 1 - \frac{1}{\log\log\log K} \quad (49)$$

to compute $\Pr\{\mathcal{G}\}$ defined in (37), where $z = N^{\text{opt}} \log(\alpha_{\max}^{-1})$. Noting (13) and (46), we have

$$\begin{aligned}\mathbb{E}\{N^{\text{opt}} \log(\alpha_{\max}^{-1})\} \quad &\leq \quad \mathbb{E}\{\sqrt[3]{\log(\alpha_{\max}^{-1}) \log\log(P\upsilon_{\max})}\} \\ &= \quad \mathbb{E}\{\sqrt[3]{\log(\alpha_{\max}^{-1})}\} \mathbb{E}\{\sqrt[3]{\log\log(P\upsilon_{\max})}\} \\ &\leq \quad \sqrt[3]{\mathbb{E}\{\log(\alpha_{\max}^{-1})\}} \sqrt[3]{\log\log(P \log K)} \\ &= \quad o(1) \quad (50)\end{aligned}$$

We obtain $\text{var}\{\log(\alpha_{\max}^{-1})\}$ as follows:

$$\text{var}\{\log(\alpha_{\max}^{-1}) | |\mathcal{A}| = n\} = \frac{1}{n^2}, \quad (51)$$

and consequently, we have

$$\begin{aligned}\text{var}\{\log(\alpha_{\max}^{-1})\} \ &= \sum_{n=1}^{K} \binom{K}{n} \frac{1}{n^2} e^{-n\Theta} (1 - e^{-\Theta})^{K-n} \\ &= (1 - e^{-\Theta})^K \sum_{i=1}^{K} \frac{1}{i} [(1 - e^{-\Theta})^{-i} - 1] \sum_{n=i}^{K} \frac{1}{n} \quad (52)\end{aligned}$$

For large values of K, we can write

$$\text{var}\{\log(\alpha_{\max}^{-1})\} \backsimeq \frac{1}{K^2 e^{-2\Theta}} - \frac{e^{-Ke^{-\Theta}}}{2} (\log(Ke^{-\Theta}))^2 \quad (53)$$

Noting (53), we have

$$\begin{aligned}\mathrm{var}\{N^{\mathrm{opt}}\log(\alpha_{\max}^{-1})\} &= \mathrm{var}\{\sqrt[3]{\log(\alpha_{\max}^{-1})\log\log(P\upsilon)}\} \\ &\leq \mathrm{var}\{\sqrt[3]{\log(\alpha_{\max}^{-1})}\}\mathrm{var}\{\sqrt[3]{\log\log(P\upsilon)}\} \\ &< O(\frac{\sqrt[3]{\log\log\log K}}{(\log\log\log K)^2}) \end{aligned} \quad (54)$$

Substituting (50) and (54) in (49), we have

$$\begin{aligned}\Pr\left\{|N^{\mathrm{opt}}\log(\alpha_{\max}^{-1}) - o(1)| < O\left(\frac{1}{\log\log\log K}\right)\right\} \\ > 1 - \frac{1}{\log\log\log K}\end{aligned} \quad (55)$$

Using (38), (48) and (55), the result of the theorem follows. ■

Remark 1- The uniform distribution of the correlation coefficients is not a necessary condition for Theorem 4. In fact, Theorem 4 is valid if $\Pr\{\mathcal{G}\} \to 1$.

$$\begin{aligned}\Pr\{N^{\mathrm{opt}}\log(\alpha_{\max}^{-1}) < g(K)\} &= \Pr\{\alpha_{\max} > e^{\frac{-g(K)^3}{\log\log(P\log K)}}\} \\ &= 1 - (F_\alpha(e^{\frac{-g(K)^3}{\log\log(P\log K)}}))^K \end{aligned} \quad (56)$$

where $g(K) \sim o(1)$. Noting (56), we must have $F_\alpha(e^{\frac{-g(K)^3}{\log\log(P\log K)}}) \sim 1 - \omega(\frac{1}{K})$ to satisfy $\Pr\{\mathcal{G}\} \to 1$. Hence, there exists a larger class of distributions that satisfy the requirements for this theorem.

IV. Conclusion

A multiuser downlink communication over a time-correlated fading channel has been considered. We have proposed three scheduling schemes in order to maximize the throughput of the system. Assuming a large number of users in the system, we show that using SNR-based scheduling, a gap of $\Omega(\sqrt{\log\log\log K})$ exists between the achievable throughput and the maximum throughput of the system. We propose a simple scheduling considering both the SNR and channel correlation of the users. We show that the throughput of the proposed scheme reaches the maximum throughput of the system as the number of users tends to infinity.

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